Frequency Response Analysis of an Ocean Wave Energy Converter

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A theoretical analysis is presented for the dynamic behavior and energy conversion efficiency of a wave energy converter which is oscillating and absorbing power in an incident sinusoidal wave train. The energy converter consists of two floating bodies which have different configuration and are connected by a rigid link. Basic equations governing the floating bodies contained in the energy converter are obtained by assuming two dimensional motions and by considering the interactions between the two bodies and hydrodynamic and damping forces, and they have been solved numerically by using Lewis form as the configuration of the floating bodies. Energy absorption is assumed to be proportional to the square of the relative velocity between the oscillating body and the connecting link. It is shown that nearly 100 percent of wave energy is converted into mechanical energy in a wide frequency band.

1 Introduction

The ocean, as the storage of solar energy, possesses an enormous amount of energy in the form of ocean waves, tidal currents, water temperature differences, and salt concentration differences etc.. The idea of getting energy from the waves is not new. The first patent was taken out by Girard in France in 1799 [1] and Stahl proposed several ideas in the United States in 1892 [2].

But wave energy has been ignored for about 100 years because our energy needs were satisfied by easily obtainable fossil energy and later by nuclear energy. However, since the so-called energy crisis in 1973, the limit and price of fossil energy have given an impetus to the interests of many researchers of alternate source of energy-solar, wind, wave, geothermal, and ocean thermal energy.

In 1974, S. H. Salter of Edinburgh University published a novel wave energy converter, Salter's Duck [3], with absorption efficiency of more than 80 percent. Some other good energy devices were proposed about this time such as Cockerell's raft, the Russell Rectifier, and Masuda's floating buoy [4]. In 1976, the National Engineering Laboratory in the United Kingdom published a comprehensive survey [5] on the development of wave and tidal power.

Particularly in Japan, Masuda's buoy-type electric generating units of scores of watts have been well known and have been extensively used as beacon or lighthouse power sources. Its extension, the large power-generating ship "KAIMEI" of the Japanese Marine Science and Technology Center of Science and Technology Agency, has been under experiment with international cooperation [6].

For the theoretical aspects, M. Bessho of the Japanese Defense Academy proposed a device having one floating body by utilizing two modes of relative motion between the floating body and some fixed point secured to coastal ground or the sea bed and showed 100 percent of wave energy absorption efficiency in the two dimensional problem [7].

Evans showed the general theory of energy absorption in two and three dimensions [8] and Evans and Srokosz [9] obtained a device by using the motion of two vertical flat plates which are constrained only to rotate in two dimensions.

Recently, one of the authors proposed a wave energy converter having two or more identical floating bodies connected by links and showed by two dimensional linear theory that almost all the energy in the incident wave would be absorbed with wide bandwidth when three or four bodies are adopted [10].

Since this system does not utilize fixed points such as coastal groud or the sea bed and relative motion between fixed points and floating bodies, it has a high strength against tidal ebb and flow, variation of wave height, and typhoons.

This paper presents a modified type of energy converter having simpler construction than the previous one. This system has only two floating bodies connected by a link, but their configurations are not identical. It is shown that more than 95 percent of incident wave energy will be converted into mechanical energy in the similar wide bandwidth as the author's previous work.

2 Derivation of Fundamental Equations

2.1 *n* Bodies Floating in the Wave. Basic assumptions for this research are listed below.

(1) Fluid is nonviscous, incompressible and flow is irrotational. The depth of water is infinite.

(2) Incident wave is sinusoidal, its amplitude is infinitesimal and the amplitudes of the bodies and the agitation of the fluid due to body motion are also infinitesimal.

(3) Motions of the waves and bodies are all periodic with the same frequency as the incident wave and two dimensional.

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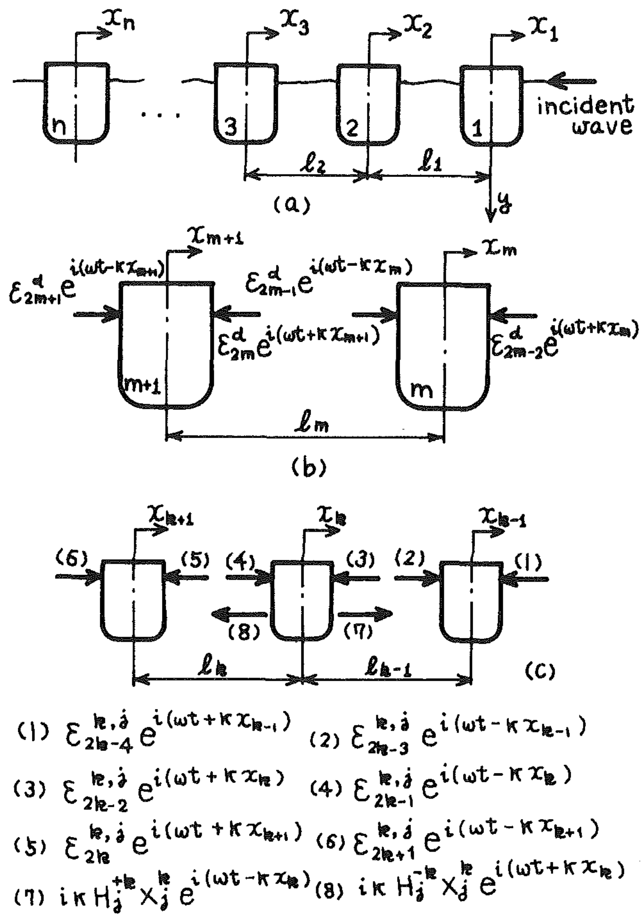

Fig. 1 Interaction of traveling waves in n body system

(4) The effects of local waves caused by the movements of the floating bodies are neglected and only the traveling waves are considered [12].

(5) Configuration of the floating bodies is all expressible by Lewis form.

2.1.1 Basic Relations of One Floating Body. First, consider one floating body which is subjected to an incident wave with single frequency. There exists a velocity potential $\Phi(x,y,t)$ which expresses fluid motion

$$\Phi(x,y,t) = \mathrm{Re}[\varphi(x,y)e^{i\omega t}] \tag{1}$$

Under the assumption of linearity, velocity potential $\varphi(x,y)$ which expresses total fluid motion yields by superposing the following velocity potentials.

$$\varphi = \sum_{i=0}^{4} \varphi_i$$

φ_1: radiation potential due to swaying motion
φ_2: radiation potential due to heaving motion
φ_3: radiation potential due to rolling motion
φ_0: velocity potential which shows the motion of incident wave
φ_4: diffraction potential of incident wave diffracted by the body

Then, each velocity potential is normalized as

$$\varphi_1 = i\omega X\phi_1,\ \varphi_2 = i\omega Y\phi_2,\ \varphi_3 = i\omega Z\phi_3$$
$$\varphi_0 = \frac{igA}{\omega}\phi_0,\ \varphi_4 = \frac{igA}{\omega}\phi_4 \tag{2}$$

where

A: amplitude of incident wave
g: acceleration of gravity

X,Y,Z are complex amplitudes of heaving motion $h(t)$, swaying motion $s(t)$, and rolling motion $r(t)$, respectively.

$$h(t) = \mathrm{Re}[Ye^{i\omega t}],\ s(t) = \mathrm{Re}[Xe^{i\omega t}]$$
$$r(t) = \mathrm{Re}[Ze^{i\omega t}] \tag{3}$$

The foregoing velocity potentials have to satisfy the Laplace equation, free surface conditions, body surface conditions, the condition on the sea bed and radiation conditions and velocity potentials, which satisfy these conditions, are given using Green's function as

$$\phi_j(x,y) = \frac{1}{2\pi}\int_c \left(\frac{\partial\phi_j}{\partial n} - \phi_j\frac{\partial}{\partial n}\right)G(x,y,x',y')$$
$$\cdot ds(x',y') \quad (j=1,\ldots,4) \tag{4}$$

where

Nomenclature

A = amplitude of incident wave
$a(x), \eta(x,t)$ = variation of water level
$a, a_1, a_2, \ldots, a_n$ = reflected coefficients
B_1, B_2 = body width
$b, b_1, b_2 \ldots, b_n$ = transmitted coefficients
c = distance from the connecting point to the center of rotation of link
D = draft
E = mean absorbed energy by the system
E_{in} = mean energy in incident wave of unit amplitude
F_j' = force acting in the direction of j' mode
$f_{11c}, f_{22c}, f_{13c}$ = added mass
f_{33c} = added moment of inertia
G = Green's function
g = acceleration of gravity
H_0 = half width/depth ratio
$H_j^{\pm}$ $(j=1,\ldots,4)$ = Kochin function
$h(t)$ = heaving motion
I_B = moment of inertia of link
i = $\sqrt{-1}$
k_2 = static restoring force
k_3 = static restoring moment
$l, l_1, l_2, \ldots, l_{n-1}$ = distance between each bodies
$M = [m_{ij}]$ = complex matrix
M_B = mass of the link
M = mass of body
M_3 = moment of inertia of body
n = total number of bodies, ratio of body width $(=B_2/B_1)$, normal perpendicular surface of body
$N = [n_i]$ = complex vector
P_1, P_2 = horizontal force acting on body 1, 2
Q_1, Q_2 = vertical force acting on body 1, 2
R = complex amplitude of reflected wave
$r(t)$ = rolling motion
S_0 = cross sectional area of the body under the draft
$s(t)$ = swaying motion
s = surface of body
T = complex amplitude of transmitted wave

$$G(x,y,x',y') = \frac{1}{2}\ln\frac{(x-x')^2+(y-y')^2}{(x-x')^2+(y+y')^2}$$

$$-2\int_0^\infty \frac{e^{-k(y+y')}\cos k(x-x')}{k-\kappa}dk + 2\pi i e^{-k(y+y')} \cdot \cos\kappa(x-x') \qquad (5)$$

n: outward normal to the body surface c

$\int_c \cdot ds$: integral over surface of body

κ: ω^2/g

The velocity potential which expresses the motion of the incident wave is given as

$$\phi_0 = e^{-\kappa y + i\kappa x} \qquad (6)$$

Then, Kochin function is defined as

$$H_j^\pm \langle\kappa\rangle = \int_c \left(\frac{\partial}{\partial n}\phi_j - \phi_j\frac{\partial}{\partial n}\right)e^{-\kappa y \pm i\kappa x}ds(x,y)$$

$$(j=1,\ldots,4) \qquad (7)$$

From the property of $G(x,y,x',y')$, we get

$$\phi_j(x,y) \to \begin{cases} iH_j^+(\kappa)e^{-\kappa y - i\kappa x} & x \to +\infty \\ iH_j^-(\kappa)e^{-\kappa y + i\kappa x} & x \to -\infty \end{cases} \qquad (8)$$

When the variation of water level is expressed

$$\eta(x,t) = \mathrm{Re}[a(x)e^{i\omega t}] \qquad (9)$$

$a(x)$ is given by using velocity potential φ as

$$a(x) = -\frac{i\omega}{g}\varphi(x,0) \qquad (10)$$

By utilizing these relations, the wave forms of radiation waves at $x \to \pm\infty$ caused when the body moves at each mode with unit amplitude are given as

For sway motion: $i\kappa H_1^\pm e^{i(\omega t \mp \kappa x)}$

For heaving motion: $i\kappa H_2^\pm e^{i(\omega t \mp \kappa x)}$ (11)

For rolling motion: $i\kappa H_3^\pm e^{i(\omega t \mp \kappa x)}$

2.1.2 Incident Waves to Each n Floating Body [*11, 12*]. By using the results of Section 2.1.1, the incident waves to each floating body are obtained when the bodies lie one behind the other independently as shown in Fig. 1(*a*). The amplitude of each floating body is normalized by dividing by the incident wave amplitude A. The rotational amplitudes of the floating body and of the connecting links which will be discussed later are normalized by dividing by A and further by using the body width. In the following analysis, we consider two problems: the first is the diffraction problem in which n fixed floating bodies are subject to incident waves and the second is the radiation problem in which the kth body will be moved in j mode in the still water and the other bodies are fixed. x_1, x_2, ..., x_n are the coordinates of place taken positive in the direction of the arrows at each body, y is the vertical coordinate at the first body 1, and l_1, l_2, ... the distances between each body as shown in the figure.

(*a*) *Diffraction Problem.* Consider that this system is subject to an incident wave in the negative direction of x when n bodies are fixed in space. At first, when body 1 is subject to the incident wave, one portion of the wave is reflected and the remaining portion is transmitted, and the transmitted one is incident to body 2, which wave is further separated into reflected and transmitted waves, reflected one is incident to body 1 and transmitted one is to body 3, and so on. Thus, there exists interaction of travelling waves between the bodies.

Now, define the total incident waves to body 1 as $\epsilon_0^d e^{i(\omega t+\kappa x_1)}$, $\epsilon_1^d e^{i(\omega t-\kappa x_1)}$ and to body 2 as $\epsilon_2^d e^{i(\omega t+\kappa x_2)}$, $\epsilon_3^d e^{i(\omega t-\kappa x_2)}$ and so on. Figure 1(*b*) shows a general case where body m and body $m+1$ are indicated.

When one fixed body is subject to an incident wave of unit amplitude, the reflected wave $\mathrm{R}e^{i(\omega t-\kappa x)}$ and the transmitted wave $Te^{i(\omega t+\kappa x)}$ yield

$$R = iH_4^+,\ T = 1 + iH_4^- \qquad (12)$$

and these R and T are defined, respectively, as reflected coefficient a and transmitted coefficient b. Then, in Fig. 1(*b*),

(i) The sum of the waves existing to the left of body m is

$$\epsilon_{2m-1}^d e^{i(\omega t-\kappa x_m)} + \epsilon_{2m-1}^d a_m e^{i(\omega t+\kappa x_m)} + \epsilon_{2m-2}^d b_m e^{i(\omega t+\kappa x_m)} \qquad (13)$$

(ii) The sum of waves to the right of body $m+1$ is

$$\epsilon_{2m}^d e^{i(\omega t+\kappa x_{m+1})} + \epsilon_{2m}^d a_{m+1} e^{i(\omega t-\kappa x_{m+1})} + \epsilon_{2m+1}^d b_{m+1} e^{i(\omega t-\kappa x_{m+1})} \qquad (14)$$

Using the fact that these two sums are equal and that the relation $x_m + l_m = x_{m+1}$, we can obtain

Nomenclature (cont.)

t = time
$[u_{ij}]$ = complex matrix
$[w_i]$ = complex vector
X = complex amplitude, complex vector
X_4 = complex amplitude of rolling motion of link
$x,y,x',y',x_1,\ldots,x_n$ = cartesian coordinates
x_1,x_2,x_3 = nondimensionalized parameter in App. 1, 2
Y = complex amplitude
Z = complex amplitude
α = $a_1a_2e^{-i2\kappa l}$
δ_{ij} = kronecker's delta
ϵ^d = diffraction coefficient
ϵ^{kj} = radiation coefficient
η = energy conversion efficiency
η_1,η_2 = energy conversion efficiency due to damper placed at body 1, 2
κ = wave number ($=\omega^2/g$)
μ_1,μ_2 = coefficient of damping
μ_R,μ_L = dimensionless coefficient of damping
ξ_D = ω^2D_1/g
ξ_{DO} = tuning frequency
ρ = density of fluid
σ = section modulus
$\Phi(x,y,t),\phi(x,y)$ = velocity potentials
φ_0 = velocity potential which shows motion of incident wave
$\varphi_1,\varphi_2,\varphi_3$ = radiation potentials
φ_4 = diffraction potential
$\phi_0,\ldots,\phi_4$ = normalized velocity potentials
ω = angular frequency
ω_0 = reference frequency

Subscripts

j,j' = mode of motion
m = number of body
θ = position of body
$'$ = non-dimensionalized value except j'

Superscript

k = number of body

$$-b_{m+1}\epsilon_{2m+1}^{d}-a_{m+1}\epsilon_{2m}^{d}+e^{i\kappa l m}\epsilon_{2m-1}^{d}=0$$
$$e^{i\kappa l m}\epsilon_{2m}^{d}-a_{m}\epsilon_{2m-1}^{d}-b_{m}\epsilon_{2m-2}^{d}=0 \tag{15}$$

where $n-1\geq m\geq 1$ and $\epsilon_0^d=1$ for an incident wave of unit amplitude, $\epsilon_{2n-1}^d=0$ for body n.

(*b*) *Radiation Problem.* In Fig. 1(*c*), consider that all bodies are fixed in space except body k and body k is oscillated as $X_j^k e^{i\omega t}$ $(j=1,2,3)$. Then from equation (11) travelling waves $i\kappa H_j^{-k}X_j^k e^{i(\omega t+\kappa x_k)}$ and $i\kappa H_j^{+k}X_j^k e^{i(\omega t-\kappa x_k)}$ will be caused on the left and right side of body k, respectively, and these waves become the incident waves to the other floating bodies and interference will be caused between each body. Instead of ϵ^d, $\epsilon^{k,j}$ is used here, where superscripts k, j mean that the body k oscillates in j mode.

Using the fact that the waves on the left side of body $k-1$ and on the right of body k are equal and also using the relation $x_{k-1}+l_{k-1}=x_k$, we get

$$-b_k\epsilon_{2k-1}^{k,j}-a_k\epsilon_{2k-2}^{k,j}+e^{i\kappa l_{k-1}}\epsilon_{2k-3}^{k,j}=i\kappa H_j^{+k}X_j^k$$
$$e^{i\kappa l_{k-1}}\epsilon_{2k-2}^{k,j}-a_{k-1}\epsilon_{2k-3}^{k,j}-b_{k-1}\epsilon_{2k-4}^{k,j}=0 \tag{16}$$

In the same way, from the body k and body $k+1$, we get

$$-b_{k+1}\epsilon_{2k+1}^{k,j}-a_{k+1}\epsilon_{2k}^{k,j}+e^{i\kappa l_k}\epsilon_{2k-1}^{k,j}=0$$
$$e^{i\kappa l_k}\epsilon_{2k}^{k,j}-a_k\epsilon_{2k-1}^{k,j}-b_k\epsilon_{2k-2}^{k,j}=i\kappa H_j^{-k}X_j^k \tag{17}$$

From equations (15), (16), and (17),

$$-b_{m+1}\epsilon_{2m+1}^{k,j}-a_{m+1}\epsilon_{2m}^{k,j}+e^{i\kappa l_m}\epsilon_{2m-1}^{k,j}=i\kappa H_j^{+k}X_j^k\delta_{m+1,k}$$
$$e^{i\kappa l m}\epsilon_{2m}^{k,j}-a_m\epsilon_{2m-1}^{k,j}-b_m\epsilon_{2m-2}^{k,j}=i\kappa H_j^{-k}X_j^k\delta_{m,k} \tag{18}$$
$$(n\geq k\geq 1,\ n-1\geq m\geq 1,\ j=1,2,3)$$

where

$$\delta_{i,j}=\begin{cases}0 & (i\neq j)\\ 1 & (i=j)\end{cases} \tag{19}$$

2.1.3 *Reflected and Transmitted Waves.* The total reflected and transmitted waves of an n body system subject to an incident sinusoidal wave train of unit amplitude are respectively the wave existing to the right of body 1 and the wave existing to the left of body n, and they are, resepectively, total reflected wave:

$$a_1e^{i(\omega t-kx_1)}+b_1\left(\epsilon_1^d+\sum_{k=1}^{n}\sum_{j=1}^{3}\epsilon_1^{k,j}\right)e^{i(\omega t-kx_1)}+\sum_{j=1}^{3}i\kappa H_j^{+1}X_j^1X_j^1e^{i(\omega t-\kappa x_1)} \tag{20}$$

Total transmitted wave:

$$b_n\left(\epsilon_{2n-2}^d+\sum_{k=1}^{n}\sum_{j=1}^{3}\epsilon_{2n-2}^{k,j}\right)e^{i(\omega t+\kappa x_n)}+\sum_{j=1}^{3}ikH_j^{-n}X_j^ne^{i(\omega t+\kappa x_n)}$$

2.2 System Consisting of Two Floating Bodies

2.2.1 *Diffraction and Radiation Coefficients.* Since two floating bodies are used in this energy converter, diffracted and radiated waves will be easily obtained by putting $n=2$ in the previous section.

When the two bodies are fixed in space with distance l and a unit amplitude sinusoidal wave is incident to body 1, the coefficients of the diffracted wave are obtained from equation (15).

$$\epsilon_1^d=\frac{a_2b_1e^{-i2\kappa l}}{1-\alpha},\quad \epsilon_2^d=\frac{b_1e^{-ikl}}{1-\alpha} \tag{22}$$

where

$$\alpha=a_1a_2e^{-i2\kappa l} \tag{23}$$

From equations (18) and (19), the coefficients of the radiation wave when body 2 is fixed and body 1 is moved as $X_j^1e^{i\omega t}$ are given by

$$\epsilon_1^{1,j}=\frac{i\kappa a_2H_j^{-1}X_j^1e^{-i2\kappa l}}{1-\alpha},\quad \epsilon_2^{1,j}=\frac{i\kappa H_j^{-1}X_j^1e^{-i\kappa l}}{1-\alpha}$$
$$(j=1,2,3) \tag{24}$$

Similarly the coefficients of the radiation wave when body 1 is fixed and body 2 is moved as $X_j^2e^{i\omega t}$ are given by

$$\epsilon_1^{2,j}=\frac{i\kappa H_j^{+2}X_j^2e^{-i\kappa l}}{1-\alpha},\quad \epsilon_2^{2,j}=\frac{i\kappa a_1H_j^{+2}X_j^2e^{-i2\kappa l}}{1-\alpha}$$
$$(j=1,2,3) \tag{25}$$

Then, the total incident waves to body 1 when a unit amplitude sinusoidal wave is incident to this system are given by the foregoing results

$$e^{i(\omega t+\kappa x_1)}+\frac{a_2b_1e^{-i2\kappa l}}{1-\alpha}e^{i(\omega t-\kappa x_1)}+\sum_{j=1}^{3}i\kappa\frac{a_2e^{-i2\kappa l}}{1-\alpha}H_j^{-1}X_j^1e^{i(\omega t-\kappa x_1)}+\sum_{j=1}^{3}i\kappa\frac{e^{-i\kappa l}}{1-\alpha}H_j^{+2}X_j^2e^{i(\omega t-\kappa x_1)} \tag{26}$$

Similarly for body 2,

$$\frac{b_1e^{-i\kappa l}}{1-\alpha}e^{i(\omega t+\kappa x_2)}+\sum_{j=1}^{3}i\kappa\frac{e^{-i\kappa l}}{1-\alpha}H_j^{-1}X_j^1e^{i(\omega t+\kappa x_2)}+\sum_{j=1}^{3}i\kappa\frac{a_1e^{-i2\kappa l}}{1-\alpha}H_j^{+2}X_j^2e^{i(\omega t+\kappa x_2)} \tag{27}$$

2.3 Fundamental Equations When Two Bodies Are Arranged Independently. By using the relations (26), (27) and the relation of Huskind, the force $F_{j'}^1$, acting in the direction of j' mode of the body 1 when a unit amplitude sinusoidal wave is incident is given by

$$F_{j'}^1=-\rho gH_{j'}^{+1}-\rho g\frac{a_2b_1e^{-i2\kappa l}}{1-\alpha}H_{j'}^{-1}-\rho g\left(\sum_{j=1}^{3}i\kappa\frac{a_2e^{-i2\kappa l}}{1-\alpha}H_j^{-1}X_j^1\right)H_{j'}^{-1}-\rho g\left(\sum_{j=1}^{3}i\kappa\frac{e^{-i\kappa l}}{1-\alpha}H_j^{+2}X_j^2\right)H_{j'}^{-1} \tag{28}$$

where ρ is the density of the fluid and $j'=1,2,3$ [13].

In the same way, the force $F_{j'}^2$, acting in the direction of the j' mode of the body 2 is given by

$$F_{j'}^2=-\rho g\frac{b_1e^{-i\kappa l}}{1-\alpha}H_{j'}^{+2}-\rho g\left(\sum_{j=1}^{3}i\kappa\frac{e^{-i\kappa l}}{1-\alpha}H_j^{-1}X_j^1\right)H_{j'}^{+2}-\rho g\left(\sum_{j=1}^{3}i\kappa\frac{a_1e^{-i2\kappa l}}{1-\alpha}H_j^{+2}X_j^2\right)H_{j'}^{+2} \tag{29}$$

Then, the fundamental equations governing the dynamics of this system are

$$\begin{aligned}D_1^\theta X_1^\theta+D_{13}^\theta X_3^\theta&=F_1^\theta/\rho\omega^2\\ D_2^\theta X_2^\theta&=F_2^\theta/\rho\omega^2\\ D_{13}^\theta X_1^\theta+D_3^\theta X_3^\theta&=F_3^\theta/\rho\omega^2\quad \theta=1,2\end{aligned} \tag{30}$$

where $D_j^\theta=d_j^\theta+i\,|H_j^{+\theta}|^2$, $D_{13}^\theta=d_{13}^\theta+iH_1^{+\theta}\overline{H_3^{+\theta}}$ (31)
$(j=1,2,3)$

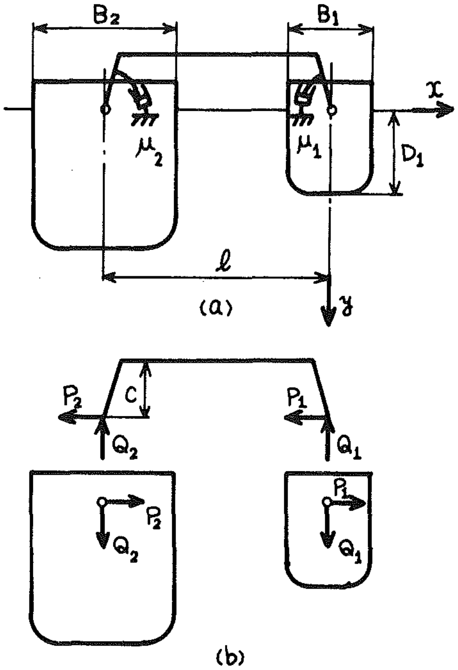

Fig. 2 Schematic diagram of two body system

(– shows complex conjugate number)

$$d_1^\theta = (-\omega^2 M^\theta - \rho\omega^2 f_{11c}^\theta)/\rho\omega^2 \quad d_{13}^\theta = -f_{13c}$$

$$d_2^\theta = (k_2^\theta - \omega^2 M^\theta - \rho\omega^2 f_{22c}^\theta)/\rho\omega^2$$

$$d_3^\theta = (k_3^\theta - \omega^2 M_3^\theta - \rho\omega^2 f_{33c}^\theta)/\rho\omega^2$$

$f_{11c}, f_{22c}, f_{13c}$: added mass

M: mass of the body

M_3: moment of inertia of the body

f_{33c}: added moment of inertia

k_2, k_3: static restoring force and moment, respectively

Substituting equations (28), (29) into equation (30) and nondimensionalizing the resulting equations by using float widths B_1 and B_2, we get

$MX = N$ namely

$$[m_{ij}][X_1^1 X_2^1 X_3^1 X_1^2 X_2^2 X_3^2]^T = [n_i] \tag{32}$$

where $B_1 X_3^1$ is replaced by X_3^1 and $B_2 X_3^2$ is also replaced by X_3^2 for nondimensionalizing. See Appendix 1.

2.4 Fundamental Equations When the Two Floating Bodies Are Connected by a Link. Consider the two body system connected by a link and having two viscous dampers with the coefficients of damping μ_1 and μ_2 placed between the float and the link as shown in Fig. 2(*a*).

Connecting points between the body and the link are considered to be placed at the center of rotation. Each of the two floating bodies moves in three modes subject to the constraint of a connecting link when an incident wave is incident to this system. It is assumed that the wave energy is absorbed by the two viscous dampers and that the viscous dampers cause only moments in proportion to the relative rotational velocity.

Considering the amplitude of motion is small, we get

$$\text{for heaving: } X_2^2 = X_2^1 - X_4 l$$
$$\text{for swaying: } X_1^2 = X_1^1 \tag{33}$$

where X_4: the amplitude of rolling motion of the link.

Then, constraining forces caused by the connection of link are considered in Fig. 2(*b*). Adding these constraining forces and counteracting moment due to the viscous dampers to equation (32) and taking into consideration the amplitude constraint (33), we get

$$[m_{ij}]\begin{bmatrix} X_1^1 \\ X_2^1 \\ X_3^1 \\ X_1^1 \\ X_2^1 - l'X_4 \\ X_3^2 \end{bmatrix} = [n_i] + \begin{bmatrix} P_1/\rho\omega^2 B_1^2 \\ Q_1/\rho\omega^2 B_1^2 \\ -i\mu_R'(X_3^1 - X_4) \\ P_2/\rho\omega^2 B_2^2 \\ Q_2/\rho\omega^2 B_2^2 \\ -i\mu_L'(X_3^2 - nX_4) \end{bmatrix} \tag{34}$$

where

P_1: horizontal force acting on body 1

Q_1: vertical force acting on body 1

$-i\mu_R'(X_3^1 - X_4)$: rolling moment

P_2: horizontal force acting on body 2

Q_2: vertical force acting on body 2

$-i\mu_L'(X_3^2 - nX_4)$: rolling moment

$$\mu_R = \mu_1/(\rho\omega_0 B_1^4), \mu_L = \mu_2/(\rho\omega_0 B_2^4)$$
$$\mu_R' = \mu_R\omega_0/\omega, \mu_L' = \mu_L\omega_0/\omega \tag{35}$$
$$n = B_2/B_1, \; l' = l/B_1$$

ω_0: reference frequency which will be explained later.

$B_1 X_4$ is replaced by X_4.

Assuming that the mass of the link is M_B, the moment of inertia is I_B, center of gravity of the link is at the center of the link and the distance from the connecting point is c, the dynamic relations for forces acting on the connecting link will be given.

$$M_B' X_1^1 = (P_1 + P_2)/\rho\omega^2 B_1^2,$$

$$M_B'\left(X_2^1 - \frac{l'}{2}X_4\right) = (Q_1 + Q_2)/\rho\omega^2 B_1^2$$

$$-I_B' X_4 + i\mu_R'(X_4 - X_3^1) + i\mu_L' n^4\left(X_4 - \frac{X_3^2}{n}\right)$$

$$= c_1'(P_1 + P_2)/\rho\omega^2 B_1^2 + \frac{l'}{2}(Q_2 - Q_1)/\rho\omega^2 B_1^2 \tag{36}$$

where

$$M_B' = M_B/\rho B_1^2, \; I_B' = I_B/\rho B_1^4, \; c' = c/B_1 \tag{37}$$

Then, the fundamental equations of the float-link system yield

$$[u_{ij}][X_1^1 X_2^1 X_3^1 X_3^2 X_4]^T = [\omega_i] \tag{38}$$

For the element of $[u_{ij}]$ and $[w_i]$, see Appendix 2.

Solving equations (38), we get the amplitude of motion of the bodies and the connecting link. The mean energy E absorbed per unit time per unit length by the two viscous dampers of this system is given as

$$E = \frac{1}{2}\rho\omega^3 B_1^2(\mu_R' |X_3^1 - X_4|^2 + n^2\mu_L' |X_3^2 - nX_4|^2) \tag{39}$$

If we consider the input energy $E_{in} = \rho g^2/(4\omega)$ in the incident wave of unit amplitude per unit time per unit length, the energy conversion efficiency $\eta = E/E_{in}$ is given by

$$\eta = 32H_0^2\xi_D^2(\mu_R' |X_3^1 - X_4|^2 + n^2\mu_L' |X_3^2 - nX_4|^2) \tag{40}$$

where H_0 = half width/depth ratio (See Section 3.2) of body 1.

$$\xi_D = \omega^2 D_1/g \tag{41}$$

where D_1 is the draft of body 1.

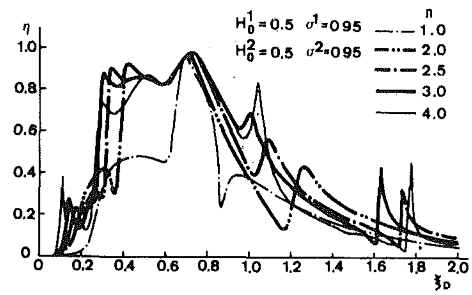

Fig. 3 Frequency response of conversion efficiency for $n = B_2/B_1$

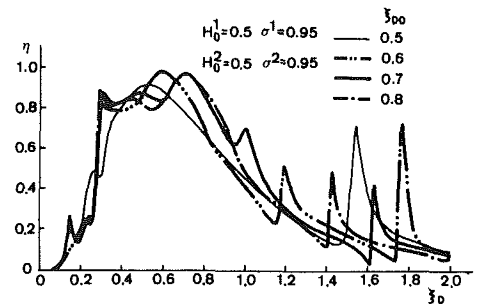

Fig. 4 Frequency response of conversion efficiency showing the effects of tuning frequency ξ_{DO}

Table 1 Optimum values of l', μ_R, and μ_L

n	ℓ'	μ_R	μ_L
1.0	2.69	0.330	0.310
2.0	4.72	0.379	0.209
2.5	5.44	0.273	0.129
3.0	5.94	0.228	0.0813
4.0	6.12	0.126	0.0251

Table 2 Optimum values of l', μ_R, and μ_L at each tuning frequency

ξ_{DO}	ℓ'	μ_R	μ_L
0.5	4.26	0.292	0.124
0.6	7.52	0.32	0.358
0.7	5.94	0.228	0.0813
0.8	6.55	0.218	0.103

2.5 Total Reflected and Total Transmitted Waves. The amplitudes of the total reflected and transmitted waves of this energy converter system are given by using equations (20), (21) as

$$|R| = \Bigg| a_1 + \frac{a_2 b_1^2 e^{-i4H_0\xi_D l'}}{1-\alpha} + \sum_{j=1}^{3} i2H_0\xi_D \left(H_j^{+1\prime} + \frac{b_1 a_2 e^{-i4H_0\xi_D l'}}{1-\alpha} H_j^{-1\prime} \right) X_j^1 + \sum_{j=1}^{3} i2H_0\xi_D n \frac{b_1 e^{-i2H_0\xi_D l'}}{1-\alpha} H_j^{+2\prime} X_j^2 \Bigg| \tag{42}$$

$$|T| = \Bigg| \frac{b_1 e^{-i2H_0\xi_D l'}}{1-\alpha} + \sum_{j=1}^{3} i2H_0\xi_D \frac{e^{-i2H_0\xi_D l'}}{1-\alpha} H_j^{-1\prime} X_j^1 + \sum_{j=1}^{3} i2H_0\xi_D n \left(H_j^{-2\prime} + \frac{a_1 b_2 e^{-i4H_0\xi_D l'}}{1-\alpha} H_j^{+2\prime} \right) X_j^2 \Bigg| \tag{43}$$

3 Results of Numerical Analysis

3.1 Selection of the Optimal Parameter. As seen from the foregoing analysis, a number of system parameters are contained in this energy converter. But, when the profile of the floating body is specified beforehand, the width ratio of the two floating bodies is given, and the mass of the connecting link is neglected for simplicity, only three parameters l', μ_R, and μ_L are remained. In order to get optimal parameter values so that the energy converting efficiency will be maximum on some specified wave frequency, the Powell method has been adopted. However, since this Powell method is used on the premise that the function has only one peak value, there is no information whether the parameter values are optimum or not even if the parameter values providing extreme value of efficiency are obtained in some searching region when the function has several peak values.

Then, from the results of a numerical search for various initial values, the existence of several peak values is confirmed. But, as it is known that the number of peaks is small and that the maximum energy conversion efficiency is 100 percent, the parameter value is adopted when the value is found to be more than 90 percent.

The value of ω_0 which is needed in normalizing μ_1 and μ_2 in equation (35) will be obtained from the tuning frequency $\xi_{DO}\,(=\omega_0^2 D_1/g)$ used in the numerical search by the Powell method.

3.2 Configuration of the Body. Lewis form, which is well known in the field of naval architecture, is used for the two floating bodies which facilitates the numerical analysis. In most of the following analysis, half width/depth ratio $H_0 = B/(2D) = 0.5$, and section modulus $\sigma = S_0/(BD) = 0.95$ are used; here B is the width of the body, D is the draft of the body, and S_0 is the cross-sectional area of the body under the draft.

Since the dynamics of this floating system are influenced by the mass distribution within the body, it is assumed in the following that a homogeneous mass distribution is localized in the lower half of the body as a typical model when various equipments of the energy converter are taken into the body.

3.3 Effects of the Ratio $n = B_2/B_1$. In order to investigate the possibility of improving energy converting efficiency, a three dimensional search, that is, a search for the value of the length of the link and coefficients of damping μ_R and μ_L, has been done for various values of $n = 1.0 \sim 4.0$. The results are shown in Fig. 3. The tuning point is specified at $\xi_{DO} = 0.7$ in all cases. It is found from this figure that appreciable improvement of energy converting efficiency η will be noticed in the wide range of dimensionless frequency $\xi_D = 0.3 \sim 0.7$ for large values of n. Optimal parameter values of l', μ_R and μ_L are shown on Table 1.

3.4 Effects of Tuning Frequency ξ_{DO}. Figure 4 shows the energy converting efficiency η for the various values of tuning frequency $\xi_{DO} = 0.5 \sim 0.8$ when the value of n is fixed at 3.0. In this system, nearly 90 percent efficiency is available for $\xi_{DO} = 0.3 \sim 0.8$ at the tuning frequency. Optimal parameter values at each tuning frequency are shown on Table 2.

3.5 Energy Absorption at Each Damper. In the foregoing analysis, the sum of the absorbed energy at the two dampers is considered. Then, at what ratio is energy absorbed at each

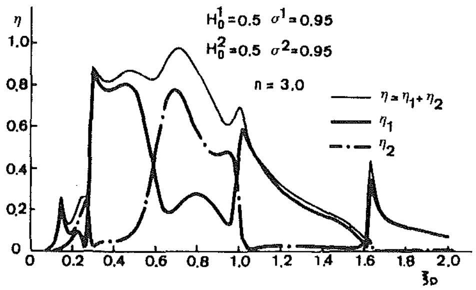

Fig. 5 Energy absorption at each damper

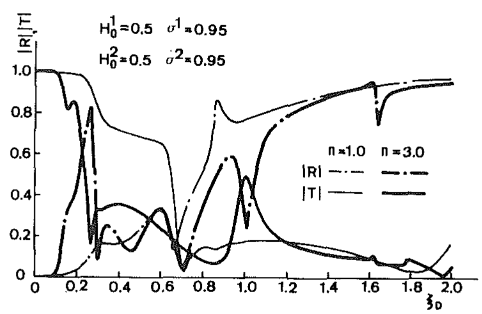

Fig. 7 Total reflected and transmitted waves

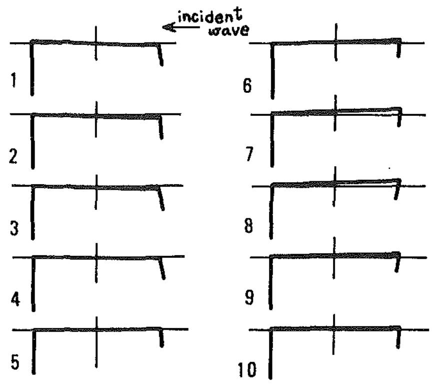

Fig. 6 Dynamic performances of the system subject to an incident wave

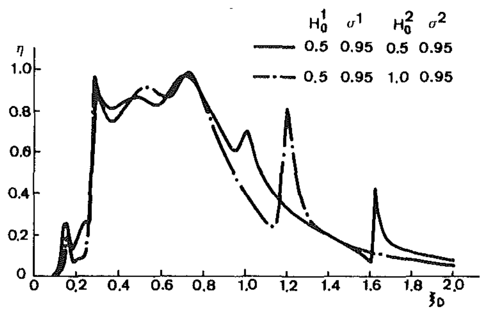

Fig. 8 Effects of configuration of body 2 for $D_2/D_1 = 3.0$

damper? Figure 5 shows this. Let the first conversion efficiency due to the damper placed at body 1 be named η_1 and the second at body 2 be named η_2. This figure shows the result for $n=3.0$ as an example, however, almost the same tendency is observed for $n \geq 2$ which is not shown here.

The dynamic performance of this system is shown below. To simplify the figure, body 1 and 2 are approximated by thin plates each having the corresponding draft. Figure 6 shows, as an example, the movements of the bodies when the draft of the body 1 is five times the amplitude of an incident wave and nondimensional incident wave frequency ξ_D is 0.7. The period of the motion is equally divided by 10 and the movements of the body and of the connecting link at each instant are shown one by one as indicated in the figure.

It is found from the figure that the motion of body 2 is very small and that the energy is absorbed by the relative motion caused by the movement of body 1.

At $\xi_D=0.7$ in Fig. 5, η_2 shows predominant energy absorption at body 2. This is due to the large relative motion between the body and the link at body 2 caused by the large heaving motion of body 1.

In Fig. 3, the energy converting efficiency increases at low value of ξ_D by increasing the ratio n. An examination of the dynamic performance at this frequency ξ_D as discussed above, indicates that the motion of body 2 will be appreciably small by increasing n and that the transmitted wave will be reduced.

In Fig. 5, η_1 predominates at lower value of ξ_D. This fact is supposed to be caused by the large rotational motion of body 1.

3.6 Total Reflected and Transmitted Waves. Figure 7 shows the amplitude $|R|$ of the total reflected wave and the amplitude $|T|$ of the total transmitted wave for the values of $n=1.0$ and 3.0. For $n=3.0$, an appreciably reduced value of $|T|$ is noticed at the low frequency range.

As is discussed in the former section, it is understood that the amplitude of the transmitted wave will be reduced substantially at lower value of ξ_D by increasing the value of n and that accordingly body 2 acts as a wave reflector. For the value $|R|$, appreciable improvement of the characteristics is also noticed for the case $n=3.0$. After all, judging from the low value of $|T|$ on the wide frequency range, we can conclude that this system will be very effective for wave suppression.

3.7 Effect of the Configuration of Body 2. Judging from the discussions thus far, the draft ratio D_2/D_1 is supposed to give deep effect on the energy absorption efficiency. Therefore, to see the effect due to the change of configuration of body 2, the value D_2/D_1 should be fixed. As examples of the configuration of body 2, two kinds of characteristic value, $H_0=0.5$, $\sigma=0.95$ and $H_0=1.0$, $\sigma=0.95$ are tried for fixed value of D_2/D_1.

Figure 8 shows this result. Although this is a limited case, the two curves show that the change of configuration of body 2 seems to have little influence on the energy absorption efficiency.

4 Conclusions

An ocean wave energy converter consisting of two floating bodies which have different configuration is proposed and analysed by using linear theory. Energy conversion efficiency is optimized with respect to two damper coefficients and link length by means of the Powell method.

The results obtained in this paper are summarized as follows:

(1) By increasing the size of body 2, the bandwidth of high energy conversion efficiency becomes wider because the total transmitted wave decreases. Compared with the author's previous paper in which three or more identical bodies are used, the system proposed in this paper shows higher energy conversion efficiency in spite of its simpler construction by properly selecting the body configuration.

(2) In a system having $H_0=0.5$, $\sigma=0.95$ as the characteristic values of the Lewis form and a width ratio of the floating bodies $n=B_2/B_1=3.0$, more than 95 percent of energy conversion efficiency is available if the tuning frequency ξ_{DO} is assigned for $0.3 \sim 0.8$ and better results are obtained for large value of ξ_{DO}.

(3) It is shown that the motion of body 2 is decreased by

increasing the ratio n and that body 2 therefore acts as a wave reflector. This fact is supposed to be the cause of high energy converting efficiency at low value of ξ_D.

(4) This energy converter absorbs most of the energy of the incident wave, so the amplitude of the transmitted wave behind the floating bodies becomes very small. Thus this converter has high wave suppressing effect over a wide range of wave frequency.

(5) Since this system does not utilize relative motion between the body and a fixed point such as coastal ground or the sea bed, it has a high strength against tidal ebb and flow, variations of wave height, and typhoons.

Acknowledgment

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APPENDIX 1

Substituting equations (28), (29), and (31) into equation (30) and nondimensionalizing, we get six equations. As an example, the derivation of the first equation when $\theta = 1$ is given below.

$$D_1^1 X_1^1 + D_{13}^1 X_3^1 = F_1^1/\rho\omega^2$$

$$= \Big[-\rho g H_1^{+1} - \rho g \frac{a_2 b_1 e^{-i2\kappa l}}{1-\alpha} H_1^{-1} - \rho g \Big(\sum_{j=1}^{3} i\kappa \frac{a_2 e^{-i2\kappa l}}{1-\alpha} H_j^{-1} X_j^{-1} \Big) H_1^{-1} - \rho g \Big(\sum_{j=1}^{3} i\kappa \frac{e^{-i\kappa l}}{1-\alpha} H_j^{+2} X_j^{-1} \Big) H_1^{-1} \Big] \Big/ \rho\omega^2$$

$$= -\frac{1}{\kappa} H_1^{+1} - \frac{1}{\kappa} \frac{a_2 b_1 e^{-i2\kappa l}}{1-\alpha} H_1^{-1} - \Big(i \sum_{j=1}^{3} \frac{a_2 e^{-i2\kappa l}}{1-\alpha} H_j^{-1} X_j^{-1} \Big) H_1^{-1} - \Big(i \sum_{j=1}^{3} \frac{e^{-i\kappa l}}{1-\alpha} H_j^{+2} X_j^2 \Big) H_1^{-1} \qquad (44)$$

In order to nondimensionalize this equation, both sides of equation (44) are divided by B_1^2 and using the characteristics of the Kochin function that $H_{1,3}^- = -H_{1,3}^+, H_2^- = H_2^+$ derived from the symmetrical nature of the configuration of the body, we get

$$\Big(D_1^1{}' + i \frac{a_2 e^{-i4H_0\xi_D l'}}{1-\alpha} (H_1^{+1}{}')^2 \Big) X_1^1 - i \frac{a_2 e^{-i4H_0\xi_D l'}}{1-\alpha} H_2^{+1}{}' H_1^{+1}{}' X_2^1 + \Big(D_{13}^1{}' + i \frac{a_2 e^{-i4h_0\xi_D l'}}{1-\alpha} H_1^{+1}{}' H_3^{+1}{}' \Big) X_3^1$$
$$- i \frac{e^{-i2H_0\xi_D l'} \cdot n}{1-\alpha} H_1^{+2}{}' H_1^{+1}{}' X_1^2 - i \frac{e^{-i2H_0\xi_D l'} \cdot n}{1-\alpha} H_2^{+2}{}' H_1^{+1}{}' X_2^2 - i \frac{e^{-i2H_0\xi_D l'} \cdot n}{1-\alpha} H_3^{+2}{}' H_1^{+1}{}' X_3^2 \qquad (45)$$
$$= -\frac{1}{2\xi_D H_0} \Big(1 - \frac{a_2 b_1 e^{-i4H_0\xi_D l'}}{1-\alpha} \Big) H_1^{+1}{}'$$

where $'$ shows a nondimensionalized value.

Then, by putting as

$$\frac{i a_2 e^{-i4H_0\xi_D l'}}{1-\alpha} = x_1, \quad \frac{i e^{-i2H_0\xi_D l'}}{1-\alpha} = x_2 \qquad (46)$$
$$-\frac{1}{2\xi_D H_0} \Big(1 - \frac{a_2 b_1 e^{-i4H_0\xi_D l'}}{1-\alpha} \Big) = n_1$$

and also by replacing the coefficients of X which appear in the above equation with $m_{11}, m_{12}, \ldots, m_{16}$ respectively, we get

$$m_{11}X_1^1 + m_{12}X_2^1 + m_{13}X_3^1 + m_{14}X_1^2 + m_{15}X_2^2 + m_{16}X_3^2 = n_1 \qquad (47)$$

In the following, the same derivation may be applied to the second equation. To the third equation, the derivation may be carried out by using B_θ^3. Finally, the following matrix equation is derived.

$$[m_{ij}][X_1^1 X_2^1 X_3^1 X_1^2 X_2^2 X_3^2]^T = [n_i] \qquad (48)$$

where

$$m_{11} = D_1^1{}' + x_1 H_1^{+1}{}' H_1^{+1}{}' \qquad m_{21} = -x_1 H_1^{+1}{}' H_2^{+1}{}'$$
$$m_{12} = -x_1 H_2^{+1}{}' H_1^{+1}{}' \qquad m_{22} = D_2^1{}' + x_1 H_2^{+1}{}' H_2^{+1}{}'$$
$$m_{13} = D_{13}^1{}' + x_1 H_3^{+1}{}' H_1^{+1}{}' \qquad m_{23} = -x_1 H_3^{+1}{}' H_2^{+1}{}'$$
$$m_{14} = -x_2 n H_1^{+2}{}' H_1^{+1}{}' \qquad m_{24} = x_2 n H_1^{+2}{}' H_2^{+1}{}'$$
$$m_{15} = -x_2 n H_2^{+2}{}' H_1^{+1}{}' \qquad m_{25} = x_2 n H_2^{+2}{}' H_2^{+1}{}'$$
$$m_{16} = -x_2 n H_3^{+2}{}' H_1^{+1}{}' \qquad m_{26} = x_2 n H_3^{+2}{}' H_2^{+1}{}'$$
$$n_1 = -\frac{H_1^{+1}{}'}{2H_0\xi_D}(1 + i x_1 b_1) \qquad n_2 = -\frac{H_2^{+1}{}'}{2H_0\xi_D}(1 - i x_1 b_1)$$
$$m_{31} = D_{13}^1{}' + x_1 H_1^{+1}{}' H_3^{+1}{}' \qquad m_{41} = -x_2 H_1^{+1}{}' H_1^{+2}{}'/n$$
$$m_{32} = -x_1 H_2^{+1}{}' H_3^{+1}{}' \qquad m_{42} = x_2 H_2^{+1}{}' H_1^{+2}{}'/n$$
$$m_{33} = D_3^1{}' + x_1 H_3^{+1}{}' H_3^{+1}{}' \qquad m_{43} = -x_2 H_3^{+1}{}' H_1^{+2}{}'/n$$
$$m_{34} = -x_2 n H_1^{+2}{}' H_3^{+1}{}' \qquad m_{44} = D_1^2{}' + x_3 H_1^{+2}{}' H_1^{+2}{}'$$
$$m_{35} = -x_2 n H_2^{+2}{}' H_3^{+1}{}' \qquad m_{45} = x_3 H_2^{+2}{}' H_1^{+2}{}'$$
$$m_{36} = -x_2 n H_3^{+2}{}' H_3^{+1}{}' \qquad m_{46} = D_{13}^2{}' + x_3 H_3^{+2}{}' H_1^{+2}{}'$$
$$n_3 = -\frac{H_3^{+1}{}'}{2H_0\xi_D}(1 + i x_1 b_1) \qquad n_4 = \frac{i H_1^{+2}{}'}{2H_0\xi_D} b_1 x_2$$
$$m_{51} = -x_2 H_1^{+1}{}' H_2^{+2}{}'/n \qquad m_{61} = -x_2 H_1^{+1}{}' H_3^{+2}{}'/n$$
$$m_{52} = x_2 H_2^{+1}{}' H_2^{+2}{}'/n \qquad m_{62} = x_2 H_2^{+1}{}' H_3^{+2}{}'/n$$

$$m_{53}=-x_2H_3^{+1\prime}H_2^{+2\prime}/n \qquad m_{63}=-x_2H_3^{+1\prime}H_3^{+2\prime}/n$$
$$m_{54}=x_3H_1^{+2\prime}H_2^{+2\prime} \qquad m_{64}=D_{13}^{2}{}'+x_3H_1^{+2\prime}H_3^{+2\prime}$$
$$m_{55}=D_2^2{}'+x_3H_2^{+2\prime}H_2^{+2\prime} \qquad m_{65}=x_3H_2^{+2\prime}H_3^{+2\prime}$$
$$m_{56}=x_3H_3^{+2\prime}H_2^{+2\prime} \qquad m_{66}=D_3^2{}'+x_3H_3^{+2\prime}H_3^{+2\prime}$$
$$n_5=\frac{iH_2^{+2\prime}}{2H_0\xi_D}b_1x_2 \qquad n_6=\frac{iH_3^{+2\prime}}{2H_0\xi_D}b_1x_2$$

$$n=B_2/B_1,\ x_1=\frac{ia_2e^{-i4H_0\xi_Dl'}}{1-\alpha},\ x_2=\frac{ie^{-i2H_0\xi_Dl'}}{1-\alpha}$$

$$x_3=\frac{ia_1e^{-i4H_0\xi_Dl'}}{1-\alpha},l'=l/B_1,\ B_\theta X_3^\theta\rightarrow X_3^\theta\,(\theta=1,2)$$
$$H_j^{+\theta\prime}=H_j^{+\theta}/B_\theta\ (j=1,2;\theta=1,2),\ H_3^{+\theta\prime}=H_3^{+\theta}/B_\theta^2\,(\theta=1,2)$$
$$D_j^\theta{}'=D_j^\theta/B_\theta^2\,(j=1,2;\theta=1,2),\qquad D_3^\theta{}'=D_3^\theta/B_\theta^4\,(\theta=1,2),$$
$$D_{13}^\theta{}'=D_{13}^\theta/B_\theta^3\,(\theta=1,2) \tag{49}$$

APPENDIX 2

Substituting the constraining condition (33) which is given by connecting the floating bodies with the connecting link and the force balancing equation (36) related to the connecting link into the matrix equation (32) and rearranging, we get the following matrix equation

$$[u_{ij}][X_1^1X_2^1X_3^1X_3^2X_4]^T=[w_i] \tag{50}$$

where

$$u_{11}=D_1^1{}'+x_1H_1^{+1\prime}H_1^{+\prime}-2nx_2H_1^{+2\prime}H_1^{+1\prime}+n^2(D_1^2{}'+x_3H_1^{+2\prime}H_1^{+2\prime})-M_B'$$
$$u_{12}=-x_1H_2^{+1\prime}H_1^{+1\prime}+nx_2H_2^{+1\prime}H_1^{+2\prime}+n(-x_2H_2^{+2\prime}H_1^{+1\prime}+nx_3H_2^{+2\prime}H_1^{+2\prime})$$
$$u_{13}=D_{13}^1{}'+x_1H_3^{+1\prime}H_1^{+1\prime}-nx_2H_3^{+1\prime}H_1^{+2\prime}$$
$$u_{14}=n(-x_2H_3^{+2\prime}H_1^{+1\prime}+n(D_{13}^2{}'+x_3H_3^{+2\prime}H_1^{+2\prime}))$$
$$u_{15}=-l'n(-x_2H_2^{+2\prime}H_1^{+1\prime}+nx_3H_2^{+2\prime}H_1^{+2\prime})$$
$$w_1=-\frac{H_1^{+1\prime}}{2H_0\xi_D}(1+ix_1b_1)+i\frac{nH_1^{+2\prime}}{2H_0\xi_D}b_1x_2$$
$$u_{21}=-x_1H_1^{+1\prime}H_2^{+1\prime}-nx_2H_1^{+1\prime}H_2^{+2\prime}+n(x_2H_1^{+2\prime}H_2^{+1\prime}+nx_3H_1^{+2\prime}H_2^{+2\prime})$$
$$u_{22}=D_2^1{}'+x_1H_2^{+1\prime}H_2^{+1\prime}+2nx_2H_2^{+1\prime}H_2^{+2\prime}+n^2(D_2^2{}'-x_3H_2^{+2\prime}H_2^{+2\prime})-M_B'$$
$$u_{23}=-x_1H_3^{+2\prime}H_2^{+1\prime}-nx_2H_3^{+1\prime}H_2^{+2\prime}$$
$$u_{24}=n(x_2H_3^{+2\prime}H_2^{+1\prime}+nx_3H_3^{+2\prime}H_2^{+2\prime})$$
$$u_{25}=-l'n(x_2H_2^{+2\prime}H_2^{+1\prime}+n(D_2^2{}'+x_3H_2^{+2\prime}H_2^{+2\prime}))+M_B'l'/2$$
$$w_2=-\frac{H_2^{+1\prime}}{2H_0\xi_D}(1-ix_1b_1)+i\frac{nH_2^{+2\prime}}{2H_0\xi_D}b_1x_2$$
$$u_{31}=D_{13}^1{}'+x_1H_1^{+1\prime}H_3^{+1\prime}-nx_2H_1^{+2\prime}H_3^{+1\prime}$$
$$u_{32}=-x_1H_2^{+1\prime}H_3^{+1\prime}-nx_2H_2^{+2\prime}H_3^{+1\prime}$$
$$u_{33}=D_3^1{}'+x_1H_3^{+1\prime}H_3^{+1\prime}+i\mu_R'$$
$$u_{34}=-nx_2H_3^{+2\prime}H_3^{+1\prime}$$
$$u_{35}=l'nx_2H_2^{+2\prime}H_3^{+1\prime}-i\mu_R'$$
$$w_3=-\frac{H_3^{+1\prime}}{2H_0\xi_D}(1+ix_1b_1)$$
$$u_{41}=-nx_2H_1^{+1\prime}H_2^{+2\prime}+x_1H_1^{+1\prime}H_2^{+1\prime}+n(nx_3H_1^{+2\prime}H_2^{+2\prime}-x_2H_1^{+2\prime}H_2^{+1\prime})+\frac{2c'M_B'}{l'}$$
$$u_{42}=nx_2H_2^{+1\prime}H_2^{+2\prime}-D_2^1{}'-x_1H_2^{+1\prime}H_2^{+1\prime}+n(nD_2^2{}'+nx_3H_2^{+2\prime}H_2^{+2\prime}-x_2H_2^{+2}H_2^{+1\prime})$$
$$u_{43}=-nx_2H_3^{+1\prime}H_2^{+2\prime}+x_1H_3^{+1\prime}H_2^{+1\prime}+i2\mu_R'/l'$$
$$u_{44}=n(nx_3H_3^{+2\prime}H_2^{+2\prime}-x_2H_3^{+2\prime}H_2^{+1\prime})+in^3\mu_L'2/l'$$
$$u_{45}=-l'n(n(D_2^2{}'+x_3H_2^{+2\prime}H_2^{+2\prime})-x_2H_2^{+2\prime}H_2^{+1\prime})+2(I_B'-i\mu_R'-in^4\mu_L')/l'$$
$$w_4=i\frac{nH_2^{+2\prime}}{2H_0\xi_D}b_1x_2+\frac{H_2^{+1\prime}}{2H_0\xi_D}(1-ix_1b_1)$$
$$u_{51}=-x_2H_1^{+1\prime}H_3^{+2\prime}+n(D_{13}^2{}'+x_3H_1^{+2\prime}H_3^{+2\prime})$$
$$u_{52}=x_2H_2^{+1\prime}H_3^{+2\prime}+nx_3H_2^{+2\prime}H_3^{+2\prime}$$
$$u_{53}=-x_2H_3^{+1\prime}H_3^{+2\prime}$$
$$u_{54}=n(D_3^2{}'+x_3H_3^{+2\prime}H_3^{+2\prime})+in\mu_L'$$
$$u_{55}=n(-l'x_3H_2^{+2\prime}H_3^{+2\prime}-in\mu_L')$$
$$w_5=i\frac{nH_3^{+2\prime}}{2H_0\xi_D}b_1x_2 \tag{51}$$