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About the Perturbing Factors Influence in the Spacecraft Motion Simulation Model

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Abstract

The new simulation model of the spacecraft's center of mass motion with taking into account of gravitational potentials of the Earth, the Moon and the Sun is presented in this paper. The simulation model was created on the basis of analysis of the present mathematical models of gravitational potentials of the Earth, the Moon and the Sun and different methods of numeral calculation. Influence of the Earth's atmosphere, which causes the spacecraft motion deceleration, was taken into account. By means of the created simulation model the analysis of the environment perturbation factors influencing on the spacecraft motion trajectory was carried out. It has been revealed that deviation of the calculated orbit from the given one depends on orbit height, and this dependence has been defined. The estimation of relative influence of the moments of various forces on the mass center motion was carried out.

Keywords: spacecraft, simulation model, gravitational potentials, perturbation factors, motion trajectory

1 Introduction

Due to historical reasons Kazakhstan has a spaceport Baikonur on its territory and after independence obtaining Kazakhstan develops its own Space Program, where the creation of own space engineering is provided. At present simulation modeling is used practically in all fields of researching work, especially in such field as space branch. Using of simulation models lets reduce both the time for a spacecraft development and designing and the material inputs for its realization.

Simulation modeling is the method, which allows building of the models describing processes like they should be going on in reality. Such model can be “played” in time both for one test and for the given set of tests. Besides, simulation model allows to obtain detailed statistics on the different aspects of the system functioning depending on input data.

Construction of the adequate mathematical models, development of algorithms and software of simulation modeling of the spacecraft motion and its control system are the major parts of the spacecraft design process. Adequate simulation model guarantees the reliability of all the spacecraft segments functioning.

There are many mathematical models for the spacecraft orbital parameters calculation. All of them have their own advantages and disadvantages depending on the preset problem goals. For valid simulation modeling of the spacecraft motion it is necessary to take into account of a great number of acting forces. Such models provide high accuracy of the orbital parameters calculation, but their realization is very difficult and requires a great calculation resources.

Complexity of selecting orbital model, as for any model, depends on some factors where accuracy of necessary prediction is the most fundamental. So far as we’ll compute prediction on the base of this orbital model we need to decrease the computing complexity of this model. Unfortunately these two missions are conflict so we’ll search for the model which attains appropriate balance of the model accuracy and required computing resources for prediction.

To obtain the appropriate level of model accuracy we need to determine the types and comparative values of the forces acting on the spacecraft. For our orbital model we have the Earth’s attraction including perturbations due to inhomogeneous distribution of its mass, gravitational Sun’s attraction, gravitational Moon’s attraction and atmospheric disturbance. Importance of these forces depends as well on its comparative values as on its periodical or long-term acting.

2 Mathematical model of gravitational potential of the Earth

The new simulation model of the spacecraft’s center of mass motion with taking into account of gravitational potentials of the Earth, the Moon and the Sun is presented in this paper. The simulation model was created on the basis of analysis of the present mathematical models of gravitational potentials of the Earth, the Moon and the Sun and different methods of numeral calculation. For the Earth’s gravitational potential the model Earth Gravitational Model 2000 (EGM2000) was accepted as the basis one. It allows expanding of the Earth gravitational potential into series till the 360th member. Numerical experiments showed that using of the first 21 members of expansion is enough for numerical calculation with accuracy 10^{-6}. For taking into account of the gravitational potentials of the Sun and the Moon it is required to expand their force functions into series on spherical functions till the fourth member.

For the spacecraft motion analysis we need to create mathematical model which is appropriate to the genuine nature of its motion. The easiest model is the spacecraft replaced by mass point *m*, which contains mass of the body.

To study the influence of gravitational field of the Earth it's used in theoretical mechanics the concept of force function $U_E\,(x,y,z)$ [1], its gradient determines the vector of force $\overline{F}_E = gradU_E$.

We can also use the concept of potential function. It is accepted in celestial mechanics to use the concept of force function of field of attraction which is also named as potential. If the body of unit mass is influenced by the gravitational field then the acting force equals the acceleration of attractive force. So we can consider it as the potential of gravitational acceleration field. Essentially the spacecraft's motion takes place under the influence of attractive forces or gravitational forces. These forces are determined by Newtonian law of gravitation [1].

All simplified models of the Earth and the spacecraft in the form of a mass point grows inadequate in case of Earth's gravitational potential modeling. In this case we must use the models in which the Earth is considered as the body restricted with the spheroid or another more complex model.

There are many another models of the Earth gravitational field potential. Geoid is used in the capacity of close approximation to the Earth's surface in case of exact calculation of the spacecraft's orbit parameters [1]. Geoid is the hypothetical level surface coincided with the surface of still ocean and continuing under the continent. Sometimes geoid is considered as a body confined by the ocean's surface with the middle water level without perturbations. Attractive potential have the same value in all points of geoid. The considered models of Earth's field of attraction are inaccurate and don't take into account a non-symmetry and non-sphericity of terrestrial globe.

The most precise potential model of Earth's field of attraction is considered in [1] where attractive potential of the Earth is presented in the form of series expansion by spherical functions:

$$U_E = \frac{fm_E}{r}\left\{1 - \sum_{n=2}^{\infty} J_n \left(\frac{r_0}{r}\right)^n P_n(\sin\varphi) + \sum_{n=2}^{\infty}\sum_{k=1}^{n}\left(\frac{r_0}{r}\right)^n P_n^k(\sin\varphi)[C_{nk}\cos k\lambda + S_{nk}\sin k\lambda]\right\} \qquad (1)$$

Here f is a gravitational constant, $r = |\vec{r}|$ is the position vector of the spacecraft, λ is the spacecraft longitude and φ is the spacecraft latitude, m_E is the Earth's mass, r_o is the equatorial radius of the Earth, $P_n(\sin\varphi)$ is the Legendre polynomial of degree n (when $k = 0$), n and k are degree and the index of the associated Legendre function correspondingly, $P_n^k(\sin\varphi)$ - associated Legendre function of index k and degree n (when $k \neq 0$), J_n, S_{nk}, C_{nk} are the coefficients defined with a help of geodesic measurements, their values are specified in the special tables.

3 Mathematical model of lunisolar perturbations

Influence of the Moon and the Sun gravitation fields usually is called lunisolar perturbations. Mathematical model of lunisolar perturbations are described in details in [1, 6]. The sum of perturbation functions corresponding to the Moon and the Sun attraction of the spacecraft equals to:

$$U_{LS} = \frac{\mu_L}{r_L} \sum_{n=2}^{4} \left(\frac{r}{r_L} \right)^n P_n(\cos\theta_L) + \frac{\mu_S}{r_S} \sum_{n=2}^{4} \left(\frac{r}{r_S} \right)^n P_n(\cos\theta_S) \tag{2}$$

where $\mu_L = fm_L$, $\mu_S = fm_S$ are the gravitational parameters of the Moon and the Sun correspondingly, r_L, r_S are the position vectors of the Moon and the Sun, r is the satellite's position vector, θ_L, θ_S are the angles between the satellite's position vector and the Moon and the Sun correspondingly.

Lunisolar perturbations have the same infinitesimal order as perturbations caused by zonal harmonic (except the second harmonic), that is with the order of 10^{-6} [1], consequently we can take into account only the first harmonics with infinitesimal order below 10^{-6}. Hence, gravitational potential of the Sun and the Moon was expanded into series by spherical functions with account of its first four terms.

4 Mathematical model of the atmosphere

Influence of Earth's atmosphere on the spacecraft's motion was taken into account. Atmosphere causes inhibition of the spacecraft. Environmental resisting force is caused by continuous collision of the spacecraft with molecules and atoms of the air. Atmosphere resistance must be considered as perturbation force on the heights of 150 - 1500 km. An analysis of existing models of atmosphere used in the National Aeronautics and Space Administration (NASA) and the European Space Agency was made. These models allow precise modeling of atmosphere influence on the spacecraft's motion in dependence on orbit height, time of year and a day. Any of them can be used in considered simulation model. It must be noted that perturbations caused by atmosphere resistance differ from perturbations determined by the harmonics of Earth's field of attraction.

Atmospheric density on the certain height is proportional to the pressure of overlying atmosphere layer, so density is distributed along the height as the atmospheric pressure in the case when temperature is constant. It means that the more is height the less is pressure decrease in case of lifting on the same height [4].

In fact temperature and composition of the air changes with the height what can cause the declination in the pressure and density distribution along the height. Thus, density has a complex dependence on time.

Information about the air density obtained with the help of satellites and contained in Standard Atmosphere is used for real computations of the spacecraft's motion in atmosphere. It consists from the tables and formulas allowing to find the density on the certain height and at certain moment. Standard CIRA-96 [4] was used for solving this problem.

The detail analysis of non-potential forces was made in [5]. Elementary force *dF* acting on the area *dS* may be written in the form:

$$d\overrightarrow{F_A} = \frac{1}{2} c\rho V \cos\varepsilon \vec{v} ds \tag{3}$$

where c is the drag coefficient, ρ is the atmosphere's density, V is the spacecraft velocity relative to the incident flow, ε is an angle between the normal to the element *dS* and the velocity vector relative to the incident flow, $\vec{v}$ is the unit vector, directed along the vector of the incident flow.

Integrating along the part of the spacecraft's surface S washing with the air flow we'll obtain a force acting on the surface of spacecraft:

$$\overrightarrow{F_A} = \int_S \frac{1}{2} c\rho V \cos\varepsilon \vec{v}\, ds \tag{4}$$

Numerical experiment was made by means of the spacecraft's trajectory calculating on the geostationary, middle and low orbits for the purpose of accuracy control of mathematical models, used methods of numerical calculation and developed algorithms.

5 Mathematical model of spacecraft's center of mass motion

Let's consider the spacecraft's center of mass motion in geocentric coordinate system as the motion of a mass point with infinitesimal mass under the influence of forces determined by potential function *U* and non-potential force *F*.

Following equations of the spacecraft's center of mass motion corresponds to this mathematical model:

$$\frac{d^2\vec{r}}{dt^2} = f(r, t, U, F) \tag{5}$$

where $\bar{r}$ is the spacecraft's radius vector, t is time, U signifies the potential perturbing forces and F – the non-potential perturbing forces.

This equation in the projections on the axis of absolute coordinate system is in the form [2], [3]:

$$\begin{aligned}
&\frac{dr^2}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2 - r\left(\frac{d\lambda}{dt}\right) cos^2\varphi = \frac{\partial U}{\partial r} + \frac{F_r}{m}, \\
&\frac{d}{dt}\left(r^2\frac{d\varphi}{dt}\right) + r^2\left(\frac{d\lambda}{dt}\right)^2 sin\,\varphi\, cos\,\varphi = \frac{\partial U}{\partial \varphi} + \frac{F_\varphi}{m}, \\
&\frac{d}{dt}\left(r^2\frac{d\lambda}{dt} cos^2\varphi\right) = \frac{\partial U}{\partial \lambda} + \frac{F_\lambda}{m}
\end{aligned} \tag{6}$$

where r, φ, λ are the spacecraft's spherical coordinates.

Potential of perturbation forces is determined by the expression:

$$U = U_E + U_{LS} \tag{7}$$

6 Results of programmatically-mathematical support of the simulating modeling system of the spacecraft's motion

Some numerical experiments of the spacecraft's trajectory computing were made for the control of developed models adequacy. Geostationary and low orbits were used in capacity of experimental samples.

Numerical experiments for the geostationary spacecrafts were made in two stages to demonstrate the influence of all perturbations on the spacecraft. Trajectory of the spacecraft's center of mass was obtained by the help of Maple 11on the first stage. Its equations are written in when only the interaction under Newtonian law takes place so it is a motion without perturbations. Differential equations of motion in the form (6) were integrated with account of terms *U, F* on the second stage. These terms correspond to perturbations from non-sphericity of the Earth and gravitational influence of the Moon and the Sun. Trajectories of Moon and Sun motion were computed in line with ephemeris in the time of the spacecraft observation.

The geostationary spacecraft. The elements of the orbit obtained from Zonal catalogue for geostationary satellites in period of 1996 – 1999 are assumed as the input data. Its values were equal to:

- *Ps* (sidereal period around the Earth) = 1436.19;
- *e* (eccentricity) = 0.00093;
- *i* (orbit inclination) = 3.053°;
- *W* (longitude of ascending node) = 78.138°;
- *w* (ascending node-perigee angle) = 271.083°;
- *M0* (mean anomaly) = 99.41°.

Right ascension and declinations were equal to (-1.0h, -49m, -2.3s) and (13.0°, 5.0', 56.7") correspondingly. These data were equal to (-21.0h, -50m, -58.8s) and (16.0°, 30.0', 38.48") for the Sun.

Results of modeling without perturbations and with perturbations are presented at the figure 1. We can see that external perturbation factors influence at the spacecraft's motion enough strong so the computed trajectory with account of external factors corresponds to real trajectory more precisely.

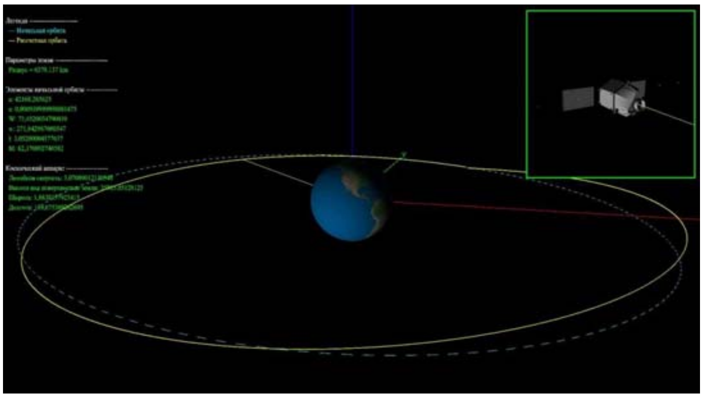

Fig.1. Trajectory of the spacecraft's motion with account of external perturbation factors

The low-orbital spacecraft. The spacecraft's coordinates in rectangular coordinate system obtained from navigation message are considered as input data for this spacecraft. Its values were equal to:

- $X = 2438.985643$ km;
- $Y = -2773.417039$ km;
- $Z = -5642.110483$ km;
- $V_x = -3.9209266597$ km/sec;
- $V_y = 5.0714967832$ km/sec;
- $V_z = -4.1924904816$ km/sec.

Table 1 – Results of the spacecraft's center of mass motion modeling with account of external perturbations

The values	R	φ	λ
Initial data	6195.1783	1.1450	-0.3070
Computation data (C)	6743.4628	1.0166	1.6437
Observation data (O)	6743.2661	0.8932	1.4812
Deviation (O-C)	0.19665	0.1234	0.1625

The spacecraft's trajectory with account of all external perturbations is presented at the figure 2 and at the figure 3.

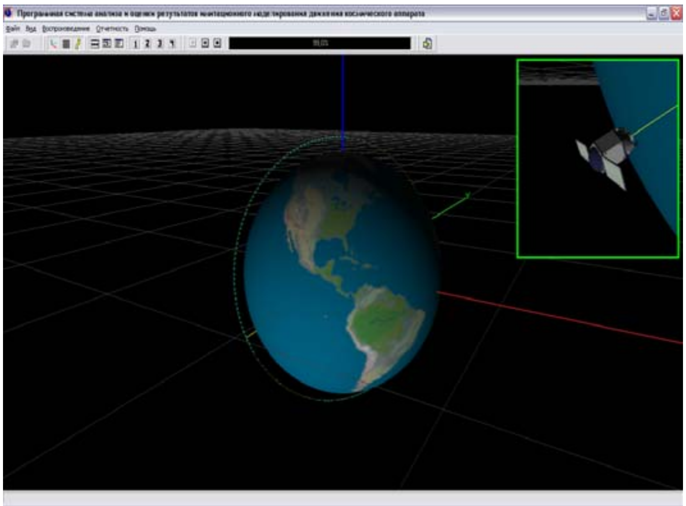

Fig.2. Computed trajectory of the low-orbital spacecraft

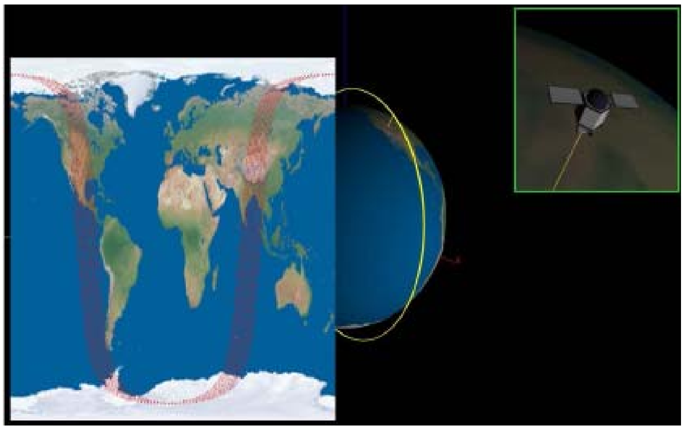

Fig.3. Program module for the spacecraft trajectory computing

On the basis of the developed algorithms and accepted mathematic model of the spacecraft motion a program complex was elaborated, with using of visual programming environment Delphi 7 and application package Maple.

Program modules of visualization of the environment objects (the Earth, the Moon and the Sun) were developed for increase of presentation and perception improvement of the results obtained in process of mathematical and numerical modeling.

The program module of motion visualization is elaborated for display of a trajectory of the spacecraft motion in orbit under influence of external factors. It allows:

- to display initial orbit and calculated trajectory of the spacecraft motion in two-dimensional view on the Earth map and in stereoscopic picture;
- to display information about the spacecraft current state during its motion reproduction.

7 Numerically estimation of external perturbations

By means of the created simulation model the analysis of the environment perturbation factors influencing on the spacecraft motion trajectory was carried out. It has been revealed, that deviation of the calculated orbit from the given one depends on orbit height, and this dependence has been defined. For the satellite in orbit with height less than 1600 km the effects, caused by the Moon and the Sun, are very small. But they cannot be neglected, if it is necessary to obtain data about the Earth potential harmonics of a high order from the satellite surveillances. The estimation of relative influence of the moments of various forces on the mass center motion was carried out.

The obtained results demonstrate high efficiency of the elaborated algorithms, which may be of interest for engineers designing space vehicles.

Considered methods and algorithms of premises forces and moments computation were tested with the help of Maple 11 and assumed as the basis of developed software of simulation modeling of external perturbations. Software is implemented in programming environment Delphi with help of high-level language.

Deviation of the actual spacecraft's trajectory from the preset trajectory in the certain moments is effected by the long-term influence of perturbations because of the predominance of one of considered perturbations.

Compensation of perturbation acting on the spacecraft during its flight because of inaccuracy of injection or providing the spacecraft's flight along the preset trajectory is the main mission of control system of the spacecraft [2].

Before solving the problems of control of motion we need to study the character of motion under various conditions, particularly under the influence of various external perturbations showed above.

Let's estimate the values of moments of various forces acting on the spacecraft. We'll estimate the maximal values of gravitational and aerodynamic moments for the spacecraft. Results of forces comparison are shown at the figure 4. At this figure we can see that the gravitational force of the Earth and the force of atmospheric influence have the maximal values.

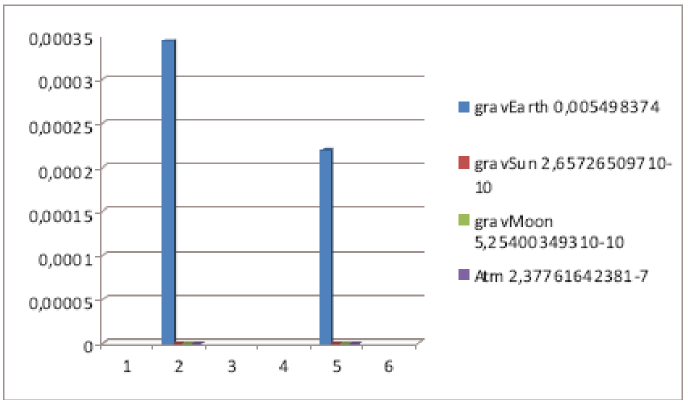

Fig. 4. Comparison of external perturbation factors

Including perturbation forces in equation of motion we obtained the spacecraft's trajectories of perturbed and unperturbed motion. Trajectory of unperturbed motion is shown at the figure 5. Trajectories of perturbed motion with account of external perturbation forces are shown at the figures 6, 7.

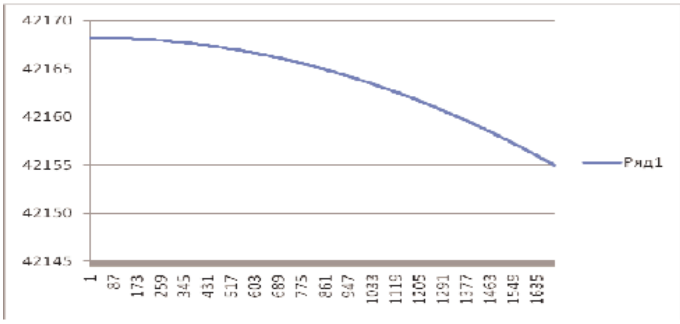

Fig. 5. Change of the spacecraft's trajectory without perturbations

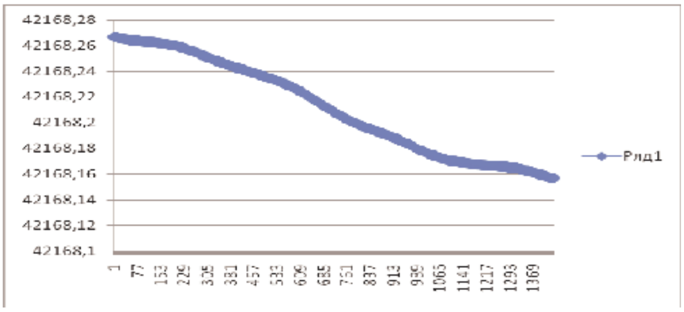

Fig.6. Change of the spacecraft's trajectory because of perturbations from non-sphericity of the Earth

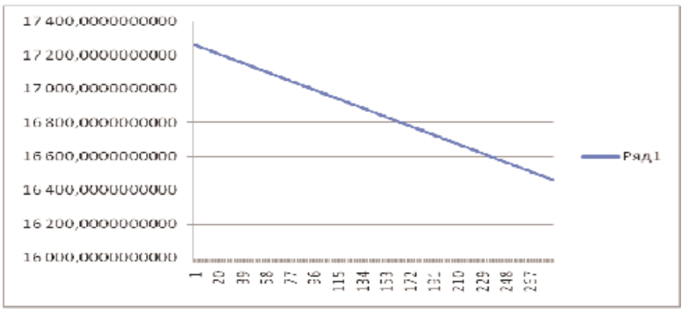

Fig. 7. Atmosphere influence on the spacecraft's trajectory

8 Conclusion

Existing mathematical models, methods and algorithms of computation of gravitational potential of the Earth, Moon and the Sun are considered and analyzed. Model EGM2000 is considered as the basic model for taking into account gravitational potential of the Earth. This model allows us to expand the gravitational potential of the Earth into series to 360th term. Numerical experiments showed that using of the first 21 members of expansion is enough for numerical calculation with accuracy 10^{-6}. For taking into account of the gravitational potentials of the Sun and the Moon it is required to expand their force functions into series on spherical functions till the fourth member.

Analysis of used modeling methods of Earth's atmosphere showed that CIRA96 is the main model used for computation in NASA and European space agency. This model allows to model atmospheric influence on the spacecraft's motion in dependence on orbit height, time of year and a day.

Set of numerical methods, algorithms and program modules allowing to take into account the influence of gravitational forces of the Earth, the Sun and the Moon on the spacecraft's motion was developed.

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