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Technical Briefs

PSL – An Economical Approach to the Numerical Analysis of Near-Wall, Elliptic Flow

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The paper points out that, in the numerical computation of elliptic or three-dimensional turbulent flows, the neglect of pressure-variations across the very thin viscosity affected region near the wall allows a fine-grid analysis of this sublayer without prohibitive penalties in core or computational time. The scheme has been successfully applied to the three-dimensional flow around a U-bend.

1 Introduction

In the numerical study of complex, two-dimensional turbulent flows near walls, one commonly finds that a different approach to handling the near-wall low-Reynolds-number region (i.e., the viscous sublayer and "buffer" layer) is adopted, depending upon whether the flow as a whole is of boundary-layer or "recirculating" type. In the former case, because an economical, once-through marching solution can be applied, the near-wall zone is often analyzed by adopting a fine grid to cover the low-Reynolds-number region. Such an approach is rarely practicable in an elliptic flow, however, because the coupling of the velocity and pressure fields requires an iterative solution; this feature means not only that computer times may typically be two orders of magnitude greater than for a boundary-layer study, but also that the dependent variables must be stored over the whole solution domain. Because of the substantial core and computer time requirements, the near-wall region is usually handled by way of wall functions [1] in which wall adjacent nodes are placed relatively far from the surface so that they lie in the region of fully turbulent fluid. The wall functions attempt to embody, through a mixture of analysis and experimental data, the integrated effects of the near-wall sublayer; in this way, no substantial near-wall mesh refinement is needed. This feature is crucial in keeping the overall core and computing time requirements to manageable levels and is the reason why, despite the crudeness of the physical treatment, wall functions are nearly universally adopted. The above remarks apply with even greater force to three-dimensional flows.

The problem with such a simple approach to the physics is that it is not adequate to account for the diversity of the phenomena displayed by turbulent flow near walls. There are, for example, many situations where the velocity vector parallel to the wall undergoes strong skewing across the low-Reynolds-number region, a feature which no wall-function approach appears to have mimicked successfully.

The purpose of the present note is to recommend a new numerical practice that facilitates the use of fine near-wall mesh in computing elliptic and three-dimensional flows. This development thus opens the way to more refined modelling of the physics of the near-wall region than has hitherto been employed.

2 The PSL Scheme and Its Application

The PSL scheme is based on the idea that, while the flow as a whole must be regarded as elliptic, there is a thin parabolic sublayer (whence the acronym) immediately adjacent to the wall across which static pressure variations are negligible or, in the case of highly-curved surfaces, where the variation may be obtained by assuming radial equilibrium. This parabolic sublayer is taken to extend over the whole of the low-Reynolds-number region where the turbulent transport properties exhibit such a strongly nonlinear variation. If, as we have argued is desirable on physical grounds, a fine grid treatment is employed across this region, then major simplifications may be made to the conventional, incompressible elliptic treatment [2]. Our own implementation of the idea has been within the context of finite volume procedures employing a staggered arrangement of dependent variables, Fig. 1 (analogous simplifications can clearly be adopted with an orthodox finite difference method). Within the PSL:

(i) the pressure does not require storing (it is given by the pressure just outside the region);

(ii) thus, no Poisson or pressure-perturbation equation has to be solved;

(iii) the velocity component normal to the wall may be obtained very rapidly by cell continuity rather than by solving the normal momentum equation. Thus, referring to Fig. 1, for a Cartesian mesh and a two-dimensional flow,

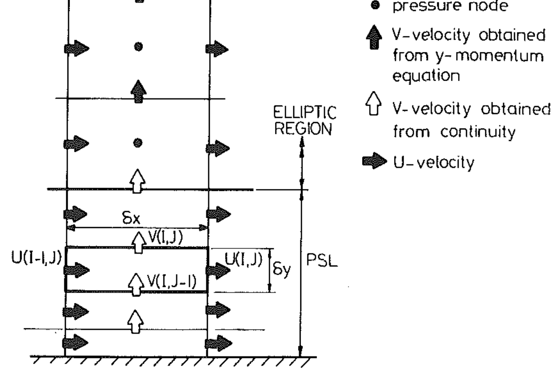

Fig. 1

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$$V(I,J) = (U(I-1,J) - U(I,J))\,\delta y/\delta x + V(I,J-1)$$

These are, of course, the classic boundary-layer simplifications known for 80 years; what appears to be novel is their application to a very thin sublayer in a shear flow that is overall not analyzable under the boundary-layer approximation. The adaptations required to most elliptic solving schemes to incorporate the PSL treatment will be trivial. In the codes used at UMIST the momentum equation (or equations) for the component(s) parallel to the wall are solved simultaneously over both the elliptic region and the PSL; the velocity component normal to the wall within the PSL is found next by applying continuity to the pressure cells; thereafter, the momentum equation for this component is solved over the elliptic region only. Finally the pressure or pressure-correction equation is solved over the elliptic region with corresponding adjustments also being made to the velocity field.

There is also often scope for reducing storage associated with velocity-component information. In many cases, at the expense of somewhat more code reorganization, the solution can be arranged so that velocities in the PSL are stored only on the row contiguous with the elliptic region and along two, one-dimensional columns of nodes (which are successively overwritten), rather than in a full two-dimensional array.

A referee has queried the use of the PSL approach in the vicinity of a stagnation point where the variation of pressure normal to the wall is relatively rapid. Perhaps the first thing to emphasize is that any errors associated with pressure variations across the buffer region will affect coarse-grid wall-function schemes at least as much as a PSL approach. Our experience at UMIST suggests in fact that even on the axis of an impinging jet the PSL approximation can be applied over most of the low-Reynolds-number region. Although in our applications to date the PSL treatment has been applied to the same number of cells in each column (viz Fig. 1), this practice is neither necessary nor optimal in some applications. In the impinging jet, for example, a thin PSL at the stagnation point could be expanded to cover the full height of the domain once the jet had been deflected into a radial wall jet. Indeed a self-adjusting scheme for the number of nodes in the PSL at any x-position could readily be devised.

The most important field of application of the approach is perhaps in three-dimensional flows describable by the partially parabolic equations for there only the pressure field requires three-dimensional storage. The scheme has been successfully applied by the authors to the turbulent flow in a circular tube around a 90 deg bend. Nine nodes have been put in the parabolic sublayer along each radial string of nodes. For identical grid densities in the fully turbulent region, computing times are no longer (in fact somewhat less) than with our previously used wall-function approach; core requirement is also little affected because most of this is associated with the pressure field which (alone) has to be held on a three-dimensional array (and which is identical for the two approaches since there are no pressure nodes within the PSL). The PSL scheme has also been applied in high-Reynolds-number *laminar* flow around pipe bends where again near the wall the velocity field undergoes very rapid changes but the pressure is obtained adequately via radial equilibrium.

The scheme has since been adopted by two of our colleagues who had hitherto been using a fine-grid low-Reynolds-number approach within two-dimensional fully elliptic treatments. When there were no flow reversals in the near-wall layer, the introduction of PSL reduced their computational times, in one case by a factor of two and in the other by a factor of three.[2] Benefits were much reduced when reverse flow was present but it seems likely that these can be substantially restored by reorganizing the solution in the PSL so that the direction of marching is always that indicated by the velocity at the outer edge of this sublayer.

3 Conclusion

The PSL approach allows a fine-grid resolution to be applied to the near-wall sublayer with no significant increase in computer time or storage, compared with a conventional wall-function treatment. Because the former scheme facilitates a better modelling of the turbulent transport processes, it is thought that in many cases it may supplant the latter.

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[2] Their numerical results obtained with PSL are insignificantly different from those given by the fully elliptic solution using an identical mesh.