Compaction of Multiphase Geomaterials

A.P.S. Selvadurai[1]

Abstract: The compaction of soils can lead to soil profiles that have properties that are less desirable from the point of view of agricultural use of soils. The paper presents elementary models of soil compaction based on continuum theories of poro-elasto-plasticity. Attention is restricted to the consideration of the quasi-static loading of a saturated one-dimensional column of soil that can experience fluid flow in its pore space and reversible and irreversible deformations of the porous skeleton.

Keywords: Soil compaction, poromechanics, elasto-plastic effects, quasi-static cyclic loading, one-dimensional column test, pore pressure generation, irreversible strains

1   Introduction

The topic of soil compaction induced by agricultural machinery is a topic of major interest to modern agricultural engineering practices. The distribution of stresses within a soil profile has been investigated quite extensively in connection with geomechanics (Davis and Selvadurai, 1996). A comprehensive review of the subject that draws on advances made through classical continuum approaches that range from the use of classical elasticity theory (see e.g. Selvadurai and Boulon, 1995; Selvadurai, 2007) to micro-mechanical approaches based on particulate mechanics (see e.g. Rothenburg and Selvadurai, 1981; Misra and Huang, 2009) is presented by Keller and Lamandé (2010). The subject of geomechanics has made important advances in terms of describing the mechanics of porous geomaterials by taking into consideration their multi-phase nature that takes into consideration coupled behaviour involving fluid flow through the pore space, mechanical deformations (both reversible and irreversible effects) and thermal deformations of the separate phases, and these developments are documented by Desai and Siriwardane (1984), Selvadurai and Nguyen (1995), Selvadurai and Boulon (1995), Selvadurai (1996,

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[1] McGill University, Montréal, QC, Canada

2007), Davis and Selvadurai (2004), Pietruszczak (2010), Selvadurai and Selvadurai (2010), and Laloui (2010). The advanced treatments of multi-phasic soils show promise for developing solutions to problems related to void reduction due to compaction caused by agricultural machinery, which can pose a severe constraint in terms of efficiency of use of the compacted soils in terms of allowing root growth and groundwater percolation. The objective of this paper is to examine certain elementary problems related to poromechanics, which can shed light on the relevance of advanced theories of continuum poromechanics in its ability to provide satisfactory answers to soil compaction problems. The complete consideration of all aspects of the soil compaction problem is beyond the scope of this paper. Such problems are invariably three-dimensional, history dependent and have to take into consideration some aspects of uncertainty in terms of the constitutive responses, which can include unsaturation, more complex constitutive relationships related to fluid migration and time dependent skeletal responses and spatial inhomogeneities of properties relevant to the compaction process. In this paper, we consider a relatively elementary one-dimensional problem of a saturated soil column which is subjected to quasi-static cycles of surface loading. The objective of the study is to evaluate the influence of poromechanics on the compaction behaviour of a saturated soil. In particular, the computational modelling addresses issues relate to the coupling of the fluid flow and deformations of the porous skeleton that includes irreversible deformations.

2   Modelling approach

Soils in an agricultural context can possess various physical configurations that depend on the type of basic soil components ranging from clay to sand and the nature of the aggregation of the soil to form a soil cluster or ultimately a continuum. The first passage of agricultural machinery can induce further reorganization of these clusters to from a soil profile with nearly continuous distributions of material or geomechanical properties. Typical stages of void reduction, both at the inter-aggregate and intra-aggregate scales are illustrated in Figure 1. The poro-geomechanics processes that will lead to the creation of a soil continuum from a collection of soil aggregates can be quite complex since the mechanics can involve very large irreversible strains with reversible, irreversible and time-dependent material phenomena and moving boundary type interfaces. The creation of a soil continuum region through void reduction at the inter-aggregate and intra-aggregate levels is conceptually illustrated in Figure 1. Indicated in Figure 1 are typical processes could include (i) the aggregation of soil clusters, (ii) the densification of the aggregate due to inter cluster void reduction, (iii) coalescence of clusters by inter-cluster void reduction (iv) creation of a continuum region with the removal of inter
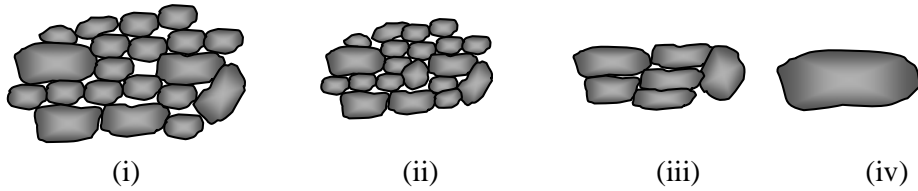
cluster voids.

(i) (ii) (iii) (iv)

Figure 1: Discrete to continuum generation during compaction of cohesive soils.

The modelling of the transition of subsets of micro-continua to create another continuum region with distinct properties is a non-routine exercise in continuum mechanics. The concept is not restricted to compaction of agricultural soils; it also is of interest to offshore land reclamation by placement of dredged material to form the reclaimed area (Selvadurai and Ghiabi, 2008). The reverse of the procedure, that involves the fragmentation of an initially continuum region is also interest to modelling gross failure and degradation of a continuum (Selvadurai and Sepehr, 1999; Selvadurai, 2009). The objective of the research is to examine responses of the compacted geomaterial in the transformed state Figure 1(iv) in the light of poro-mechanics that takes into consideration the fully saturated condition of the geomaterial and both elastic and incrementally elasto plastic responses of the porous skeleton.

2.1 Poroelastic behaviour of the compacted soil

The mechanical behaviour of a fluid saturated porous medium undergoing infinitesimal elastic strains was first developed by Biot (1941), taking into consideration Darcy's law (Selvadurai, 2000) to describe the flow of the fluid through the pore space and Hookean elastic behaviour of the porous skeleton Davis and Selvadurai, 1996). The basic equations governing the mechanical and fluid transport behaviour of an isotropic poroelastic medium consisting of non-deformable solid matter, which is saturated with an incompressible fluid, can be written in the forms

$$\mathbf{v}_f = -(\mathbf{K}/\mu)\nabla p \tag{1}$$

$$\sigma = G(\nabla \mathbf{u} + \mathbf{u}\nabla) + (\lambda \nabla . \mathbf{u})\mathbf{I} + p\mathbf{I} \tag{2}$$

where σ is the total stress tensor, **u** is the displacement vector of the skeletal phase, $\mathbf{v}_f$ refers to the velocities of the fluid, p is the fluid pressure in the pore space, **K** is the saturated permeability matrix, G and λ are Lamé elastic constants of the porous skeleton, μ is the dynamic viscosity of water, ∇ is the gradient operator and **I** is the unit matrix. In (1) the velocity of the porous matrix is neglected.

2.2 *Poroelasto-plastic behaviour of the compacted soil*

In extending the studies to include poroelasto-plasticity effects, we need to select an appropriate constitutive response for saturated clay-type materials. There are a variety of constitutive relations that have been proposed in the literature and for the purposes of illustration, we select an elasto-plastic skeletal response of the Modified Cam Clay type (see. e.g. Desai and Siriwardane, 1984; Davis and Selvadurai, 2004; Pietruszczak, 2010). Attention is restricted to an isotropic elasto-plastic material defined by the yield function

$$(\tilde{\sigma} - a)^2 + (q/M)^2 - a^2 = 0 \tag{3}$$

where q is the von Mises stress, a is the radius of the yield surface, $\tilde{\sigma}$ is the mean effective stress, M is the slope of the critical state line and these are defined by

$$q = \sqrt{3\tilde{s}_{ij}\tilde{s}_{ij}/2}; \quad \tilde{\sigma} = -(\tilde{\sigma}_{kk}/3); \quad \tilde{s}_{ij} = \tilde{\sigma}_{ij} + \tilde{\sigma}\,\delta_{ij} \tag{4}$$

The hardening rule is defined by

$$\tilde{\sigma} = \tilde{\sigma}(\varepsilon^{pl}_{kk}) = \tilde{\sigma}^0_c + \tilde{\sigma}_c(\varepsilon^{pl}_{kk}) \tag{5}$$

and the incremental plastic strains are defined by an associated flow rule of the type

$$d\varepsilon^{pl}_{ij} = d\lambda\,\frac{\partial G}{\partial \tilde{\sigma}_{ij}}; \quad G = \sqrt{\left(\tilde{\sigma} - \frac{\tilde{\sigma}^o_c}{2} - \frac{\tilde{\sigma}_c(\varepsilon^{pl}_{kk})}{2}\right)^2 + \left(\frac{q}{M}\right)^2} \tag{6}$$

where the hardening rule takes the form

$$\tilde{\sigma}_c = \tilde{\sigma}_c(\varepsilon^{pl}_{kk}) = H(-\varepsilon^{pl}_{kk}) \tag{7}$$

where *H* is a positive constant. In addition to the elasto-plastic constitutive response for the porous skeleton, we assume that the fluid flow through the porous skeleton remains unchanged during yield and subsequent hardening of the porous skeleton. It is recognized that both internal damage and elastoplastic effects can contribute to changes in the permeability of geomaterials (Selvadurai, 2004). Furthermore, the compaction process can induce stratifications at the macro-level of the fabric that can also give rise to transversely isotropic properties in all mechanical and transport phenomena.

3 Poroelastic and poroelasto-plastic response of a one-dimensional column

We consider the problem of a one-dimensional fluid saturated column, which is saturated with an incompressible fluid and the porous skeleton can possess either an elastic response characterized by Hooke's law or an elasto-plastic constitutive response characterized by a soil plasticity model similar to modified Cam Clay model with an associated flow rule and an isotropic hardening rule. The flow properties of the porous medium are defined by Darcy's law. The properties characterizing these models are shown in Figure 2. The surface of the one-dimensional column is subjected to a quasi-static normal traction that can be applied as a Heaviside step function of time or in a cyclic loading unloading mode in a time-dependent fashion as shown in Figure 2. The upper surface of the column is maintained at zero pore fluid pressure. The constitutive models are implemented in a general purpose computational multi-physics code and the initial boundary value problems are analyzed via a finite element technique with finite difference-based time integration. The accuracy of the code is also verified by comparing the results obtained by the code for elastic, poroelastic and elasto-plastic problems.

The objective of the analysis is to determine the influence of the constitutive responses of the skeleton, both poroelastic and poroelasto-plastic on the quasi-static time-dependent displacement behaviour of the column. In particular, attention is focused on the illustration of the one-dimensional surface displacement of the column during the application of the quasi-static load cycling. Specific results will be presented in a complete version of the paper.

4 Conclusions

The process of soil compaction during repeated quasistatic load cycling can be approached as a problem in continuum poroelasto-plasticity that takes into both mechanical deformations and fluid transport characteristics. The initial compaction from a soil aggregate is a more complex problem that requires knowledge of more advanced concepts of continuum poroelasto-plasticity that cannot be obtained con-
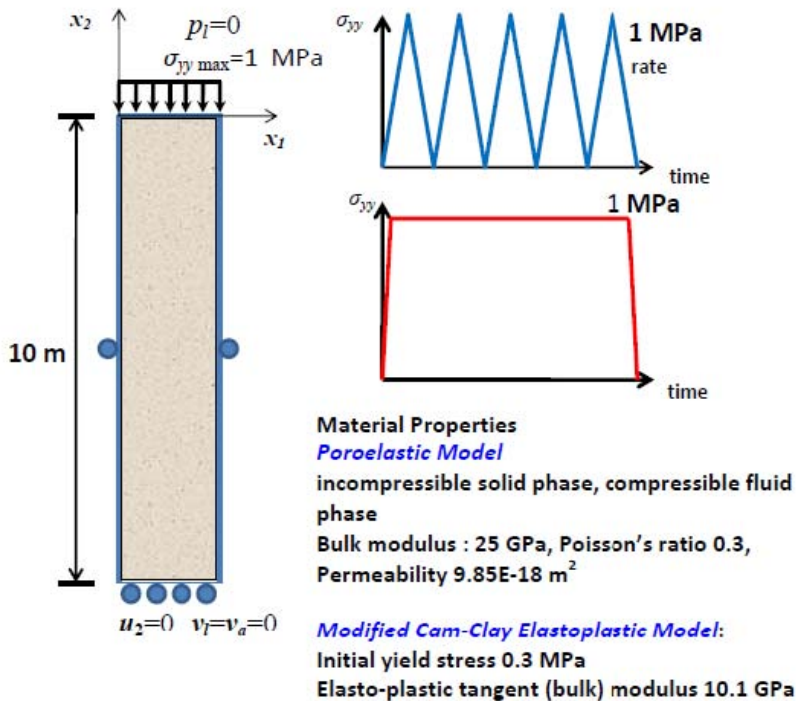
Figure 2: The problem of a one-dimensional poro-elastic and poroelasto-plastic elements subjected to time-dependent surface loadings.

veniently, particularly in the case of agricultural soils. Subsequent compaction can, however, be examined by appeal to an incremental formulation, with the assumption that the initial compaction results in the densification of the soil cluster arrangement to the virtual elimination of the inter cluster void space.

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