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Foreign Exchange Transaction Exposure in a Newsvendor Setting

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Abstract

In the global supply chain where there is a time lag between arrival of the shipment and the sale, the purchase price to the buyer may, on the day of settlement be different from that on the day of the order if the buyer is to pay in the supplier's currency. Either the supplier or the buyer is exposed to the loss due to exchange rate fluctuations. The key questions that arise then are: Does it matter who bears the risk? What aspect of exchange rate fluctuation affects the decisions of the supply chain partners? In this note related to Transaction Exposure, we show that in a classical newsvendor setting where the supplier has full information, the optimal policies are independent of which one of the two bears the risk. Numerical examples are presented to highlight model. This paper provides good scenarios in the case of risk management for manufacturer and retailer.

Keywords

Foreign exchange transaction exposure; newsvendor model; supply-chain management; prices under uncertainty

1. Introduction

In the global supply chain where there is a time lag between arrival of the shipment and the sale, the purchase price to the buyer may, on the day of settlement be different from that on the day of the order if the buyer is to pay in the supplier's currency. This is due to possible changes in the exchange rates during the time period. If each unit of the product costs w in the currency of the manufacturer, and the exchange rate is r units of retailer's currency per unit of manufacturer's currency at the time of sales, then the retailer has to pay $w.r$ for that product in its own currency if settled immediately. However, on the actual settlement date the amount to be paid could be different. If on the other hand the purchase price is denominated in buyer's currency the vendor will realize a different amount than what was perceived by it at the time of sale. If the exchange rate at the settlement date is subject to fluctuation, then the risk associated with that can be captured by defining the future exchange rate as $r.(1+\varepsilon_r)$ where ε_r can follow some statistical distribution. The nature of global trade is that either the buyer or the seller has to bear what is commonly known in International Finance as *Transaction exposure.* (Eiteman, Stonehill, & Moffett, 2010) The key questions that arise then are: Does it matter who bears the risk? What aspect of exchange rate fluctuation affects the decisions of the supply chain partners?

We use a classical Newsvendor framework where the retailer faces uncertain price dependent demand and has to decide order quantity ahead of demand realization. We explore the Stackelberg version of the problem wherein the supplier/manufacturer, who has full information of the exchange rate distribution and knows that the buyer/retailer will be using its expected outflow to determine optimally the order quantity q and its selling price p. Then as a Stackelberg leader, the supplier chooses optimally the selling (retailer's buying) price w. Added to this, the dimension of settlement currency will decide as to who bears the exchange rate risk. Accordingly we have the following scenarios.

(1) The retailer bears the exchange risk, i.e. manufacturer quotes unit price of w to be settled in its currency. The retailer/buyer will choose its q and p based on the expected outflow of $wr(1+\varepsilon_r)$ per unit reflecting the purchase price uncertainty due to exchange rate fluctuations. The manufacturer aware of this information, will characterize the retailer to be using a purchase cost, converted to the manufacturer's currency, of $\frac{wr(1+\varepsilon_r)}{r} = w(1+\varepsilon_r)$. (Please note that to clearly identify the process of converting and reconverting the currencies we use the longer expression throughout.)

(2) The manufacturer bears the exchange risk i.e. he quotes *wr* to be settled in retailer's currency costing the retailer *wr*. Thus, the retailer takes his decision based on *wr*. But the expected realization for the manufacturer is $\frac{wr}{r(1+\varepsilon_r)} = \frac{w}{(1+\varepsilon_r)}$.

2. Mathematical Model

2.1. Retailer bears the risk: Manufacturer quotes *w* and its realization is also *w*

For 2.1, let us denote the expected outflow of the retailer as $w_r = \frac{wr(1+\varepsilon_r)}{r}$.

In the classical newsvendor problem, the retailer's profit function is decomposable into two parts, depending upon whether the retailer's order quantity q exceeds or falls short of the demand D for the product. When q exceeds D, the retailer sells D units at p per unit, disposes of the rest at salvage value of v per unit and incurs an acquisition cost of *wr* as explained above, for each of the q units ordered. If q is below D then the retailer buys and sells the q units at a profit margin of *p-wr* per unit and pays a shortage penalty of s per unit on unfilled demand, where p, s and v are expressed in manufacturer's currency. Further, the demand error and the exchange risk error are assumed to be independent of each other. Then using w_r as defined earlier, the retailer's profit function in manufacturer's currency will be:

$$\begin{aligned}\pi &= -q.w_r + D(p,\varepsilon).p + \big(q - D(p,\varepsilon)\big).v \qquad if D(p) \le q \\ &= -q.w_r + q.p - (D(p,\varepsilon) - q).s \qquad if D(p) > q\end{aligned} \tag{1}$$

where $D(p,\varepsilon) = g(p) + \varepsilon$ if the error is additive
$= g(p)\varepsilon$ if the error is multiplicative

and $D(p)$ is the price dependent demand with deterministic component $g(p)$ and the stochastic component ε which is distributed with mean μ and support [A,B]. Following the customary conventions of the literature on the subject, the relationship between g and ε is assumed to be either additive (Mills, 1958) or multiplicative (Karlin & Carr, 1962), with the former (latter) exhibiting a constant (variable) error variance and a variable (constant) coefficient of variation. Chan, Shen, Simchi-Levi, and Swan (2004), Petruzzi and Dada (1999), Yao (2002), and Yao, Chen, and Yan (2006) discuss the implications of these assumptions and provide a review of the extant works on the field.

The demand function is presented in a very general form. For a unique optimal solution the only conditions needed are that g be downward sloping and at least twice differentiable, with respect to p. Most of the demand distributions normally used in the sales-promotion field, i.e. linear, iso-elastic, log-concave or concave in p and the like fulfil this requirement (Yao, 2002; Yao, et al., 2006). Similarly stochastic demand component, ε is also presented in a general form. All that is needed for unique optimal solution is that it belongs to GSIFR family defined over a finite range *[A,B]* and have a mean of μ, a standard deviation of σ, a density function of *f(.)* and a cumulative density function of *F(.)* (Yao, 2002; Yao, et al., 2006). The GSIFR family includes the most widely used in the literature such as uniform, normal, beta, gamma and the like.

Further, we can define the stocking factor, z, *expected number of shortage Φ and the expected left over Λ as follows;*

$$\begin{aligned}\Phi &= \int_z^B (\varepsilon - z) f(\varepsilon) d\varepsilon \\ \Lambda &= \int_A^z (z-\varepsilon) f(\varepsilon) d\varepsilon = \Phi + z - \mu \\ z &= q - g, \quad if\ additive \\ &= q/g, \quad if\ multiplicative\end{aligned} \tag{2}$$

Φ and *Λ* represent the expected number of shortages and leftovers, respectively, as a result of demand fluctuations. With respect to the stocking variable, *z*, it was introduced by Petruzzi and Dada (1999) and subsequently used by Arcelus, Kumar, and Srinivasan (2005), among many others, as a replacement for another decision variable, namely the order quantity. It represents the expected level of leftover and shortages, generated by the demand uncertainty and by the retailer's optimal policies.

The retailers expected profit $E\pi_r(p,q)$ can be written as follows:

$$E\pi_r(p,q) = (p - w_r)(g(p)+\mu) - (w_r - v)\Lambda - (p + s - w_r)\Phi \qquad if\ additive$$

$$= g(p)\mu - g(p)(w_r - v)\Lambda - g(p)(p + s - w_r)\Phi \qquad if\ multiplicative \tag{3}$$

where $w_r = \frac{wr(1+\varepsilon_r)}{r}$

The objective is to find the levels of p and q that maximizes $E\pi_r(p,q)$ given that the manufacturer quotes *w*.
On the other hand, the manufacturer's profitability can be expressed by

$$\pi_m = (w - c)q^* \tag{4}$$

where q^* is the optimal ordering policy followed by the retailer who bears the exchange rate risk and uses w_r to optimize. Here *c* is the purchase cost per unit to the manufacturer.
The retailer's optimal ordering policies are given by

$$q^* = g + z = g + F^{-1}\left(\frac{p^* + s - w_r}{p + s - v}\right) \qquad if\ additive\ error$$
$$= gz = gF^{-1}\left(\frac{p^* + s - w_r}{p + s - v}\right) \qquad if\ multiplicative\ error \tag{5}$$

2.2. Manufacturer bears the risk: Manufacturer quotes *w* and its realization is $\frac{wr}{r(1+\varepsilon_r)}$.

For 2.2, let us denote the expected inflow of the manufacturer as $w_m = \frac{wr}{r(1+\varepsilon_r)}$. The outflow per unit for the retailer will be wr in its currency.
In this case, the retailer's expected profit $E\pi_r(p,q)$ can be written as follows:

$$E\pi_r(p,q) = (p - w)(g(p) + \mu) - (w - v)\Lambda - (p + s - w)\Phi \qquad if\ additive$$
$$= (p - w)g(p)\mu - g(p)(w - v)\Lambda - g(p)(p + s - w) \qquad if\ multiplicative \tag{6}$$

The objective is to find the levels of p^* and q^* that maximizes $E\pi_r(p,q)$ given that the manufacturer quotes *w*.

On the other hand, since the manufacturer bears the exchange risk, its profitability can be expressed by

$$E\pi_m = \left(\frac{wr}{r(1+\varepsilon_r)} - c\right)q^* = (w_m - c)q^* \tag{7}$$

where q^* is the optimal ordering policy followed by the retailer who uses *w* to optimize.
Observe that in this instance the manufacturer's profit is an expected profit. Table 1 presents the unit inflow and outflow to both the parties expressed in manufacturer's currency and the corresponding optimal order quantity when the manufacturer quotes a price w.

Table 1: Across the Case Unit Flows and Optimal Policies

	Case 1	Case 2
Exchange risk bearer	Retailer	Manufacturer
Outflow per unit for the retailer	$w_r = \frac{wr(1+\varepsilon_r)}{r}$	w
Realization per unit for the manufacturer	w	$w_m = \frac{wr}{r(1+\varepsilon_r)}$
Optimal ordering policy of the retailer	$g[p_{w_r}] + F^{-1}\left(\frac{p + s - w_r}{p + s - v}\right)$	$g[p_w] + F^{-1}\left(\frac{p + s - w}{p + s - v}\right)$

3. Theorem

If the manufacturer is risk neutral and a Stackelberg leader then the optimal policies under both the cases are identical.

Proof: For every value of w_1 and corresponding q_1 of case 2.1 there exists a $w_2=w_1 r(1+\varepsilon_r)$ in retailer's currency that will produce order quantity $q_2= q_1$. This is because the retailer's unit cost evaluation is the same in both the cases. Furthermore, the expected value of w_2 *in manufacturer's currency* $= w_1$ and hence the profit under case 1 and the expected profit under case 2 will be the same. If the manufacturer is risk neutral then it will place identical value for a dollar profit of case 2.1 and a dollar *expected* profit of case 2.2. The optimal profit π_{m1}^* can be achieved in case 2.2.

We need to show that expected profit in case 2.2 cannot be more than that. This is proved by contradiction. Using similar logic we can show that for every price $w_2 r(1+\varepsilon_r)$ in retailer's currency there exist a price $w_1 = \frac{w_2}{1+\varepsilon_r}$ that will give identical quantity and profit. If an expected profit greater than π_{m1}^* is achievable in case 2.2, then such a profit is achievable is case 2.1 also which is a contradiction as π_{m1}^* is optimal.

It can be observed that for any particular case, in the currency of the manufacturer,

$$(outflow\ for\ the\ retailer) = (realization\ for\ the\ manufacturer)(\frac{r(1+\varepsilon_r)}{r})$$

Hence, the optimal ordering policy of the retailer for both cases is being governed by the realization of the manufacturer in that particular case multiplied by the factor $\frac{r(1+\varepsilon_r)}{r}$.

The relationship between the realizations for the manufacturer (MR) in both the cases can be written as

$$MR_2 = MR_1 . \frac{r}{r(1+\varepsilon_r)} \tag{8}$$

3. Numerical Analysis

This section presents a numerical illustration to highlight the key results of the paper. Given the central objective of the paper, our numerical analysis centres on showing the irrelevance of who bears the fluctuations in the exchange rate risk *r*, on their optimal profit-maximizing pricing and ordering policies. All computations were carried out with MAPLE's optimization capabilities.

The probability density function of the general, four-parameter beta distribution, denoted by *f(y/a,b,α,β)*, and its corresponding standard beta density function, are as follows.

$$\mathrm{B}(\alpha,\beta) = \int_0^1 t^{(\alpha-1)}(1-t)^{(\beta-1)}\,dt \qquad (A1)$$

The generalized beta density function is ,

$$f(y/a,b,\alpha,\beta) = \frac{(y-a)^{(\alpha-1)}(b-y)^{(\beta-1)}}{\mathrm{B}(\alpha,\beta)(b-a)^{(\alpha+\beta-1)}}, \quad a \le y \le b; \quad \alpha,\beta > 0, \qquad (A2)$$

and the standard beta density function, is :

$$f(x/0,1,\alpha,\beta) = \frac{x^{(\alpha-1)}(1-x)^{(\beta-1)}}{\mathrm{B}(\alpha,\beta)}, \quad 0 \le x \le 1; \quad \alpha,\beta > 0, \qquad (A3)$$

through the variable transformation : $x = \frac{y-a}{b-a}$ $\qquad (A4)$

where *b* (*a*) is the largest (smallest) value of *y*; *α* and *β*, the shape parameters of the beta distribution; and *B*(*α,β*), the beta function, designed to ensure that the total area under the density curves equals 1.

For the numerical example we use a multiplicative form for the error effect on *r* here and hence the term ***(1+ εr)***. Here *r* is the exchange rate and its error $\boldsymbol{\varepsilon_r}$. We assume that the error has a range of ± 10% and follows a transformed Beta distribution as below.

$$r * [1 + \int_0^1 (-0.1 + 0.2u) * \frac{u^{\alpha-1}(1-u)^{\beta-1}}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt} du] \tag{9}$$

The transformation takes care of our requirement/assumption that the error risk fluctuates within 10% of r on either side. i.e. between 0.9r to 1.1r.

Based on the values of α and β, the expected value the exchange rate will be governed by

$$r * [1 + (-0.1 + 0.2\left(\frac{\alpha}{\alpha+\beta}\right)] \qquad (10)$$

Equivalent additive form for such exchange rate error could also be framed. As an example if the exchange rate is taken as r=45, and the error fluctuates within 10% of r on either side i.e. between 41.5 to 49.5, the additive form can be given by

$$r + [\int_0^1 (-4.5 + 9u) * \frac{u^{\alpha-1}(1-u)^{\beta-1}}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt} du] \qquad (11)$$

Based on the values of α and β, the expected value of the exchange rate will be governed by

$$r + [-4.5 + 9\left(\frac{\alpha}{\alpha+\beta}\right)] \qquad (12)$$

For example, $r.(1+\varepsilon_r)$ and $r+\varepsilon_r$ values, representing the expected exchange raterealizations for certain values of the parameters α and β of the Beta distribution are shown in Table 2

Table 2: Expected Realizations

(α,β)	$\frac{\alpha}{\alpha+\beta}$	$r.(1+\varepsilon_r)$	$r+\varepsilon_r$
(1,1)	0.5	r	R
(1,3)	0.25	0.95r	r- 2.25
(3,1)	0.75	1.05r	r + 2.25
(2,5)	0.285	0.957r	r - 1.928
(5,2)	0.714	1.043r	r + 1.928

As both the forms have the same effect, only the multiplicative form is used for the numerical analysis. The mean of the exchange rate risk is assumed to be 0 and the support interval is assumed to be taking care of 10% deviation of the exchange rate r on both the sides of the mean. The random variable representing this risk is assumed to be following the beta distribution. The beta distribution has been selected because of its flexibility to accommodate observed phenomenon, through appropriate changes in the distribution parameters. The values of α and β will vary to simulate whether the distribution is left skewed ($\alpha<\beta$), right skewed ($\alpha>\beta$) or symmetrical ($\alpha = \beta$).

As for the demand distribution on the retailer's side, we consider the linear and the iso-elastic forms of the deterministic portion of the demand and the additive and the multiplicative way the stochastic component of the demand error affects the total demand by taking two cases; one of the linear demand and additive error (AL) and the other of the iso-elastic demand and multiplicative error (IM). In these cases, $D(p,\varepsilon)$ takes the following forms:

$$\begin{aligned} D(p,\varepsilon) &= a - b.p + \varepsilon\,, \quad a>0, b>0, a \gg b, \qquad linear\ demand\ additive\ error \\ &= a.p^{-b}.\varepsilon \qquad a>0, b>1 \qquad iso-elastic\ demand\ multiplicative\ error \end{aligned} \qquad (13)$$

The demand error is considered to be a random variable uniformly distributed over the interval (-3,500, 1,500), for the AL demand model and (0.7, 1.1), for its MI counterpart. Either support interval describes the uniform distribution completely. The values of the parameters a and b and the means μ of the errors and their support values [A,B], the salvage (v) and the shortage costs (s) are mentioned in Table 3.

Table 3: Optimal Policies

Linear Demand Additive Error							
Parametric values: a=100000, b=1500, [A,B]=[-3500,1500], μ=-1000, v=10, s=5, c=20, r=45							
Exchange Risk Bearer	*(α,β)* parameters for the Beta distribution of the exchange rate error	w^* M.C	***wr**** R.C	***q****	***p****	**Expected Profit** **Retailer***	**Manufacturer***
Retailer	(1,1)	42.13		17640	53.70	185970	390548
	(1,3)	43.82		18047	53.45	195075	429886
	(3,1)	40.61		17234	53.95	177080	355344
	(2,5)	43.56		17989	53.49	193761	423989
	(5,2)	40.82		17292	53.91	178336	360142
Manufacturer	(1,1)		42.13	17640	53.70	185970	390548
	(1,3)		41.62	18047	53.45	195075	429886
	(3,1)		42.64	17234	53.95	177080	355344
	(2,5)		41.70	17989	53.49	193761	423989
	(5,2)		42.57	17292	53.91	178336	360142

Observe in the numerical example results that for a particular set of values of α and β of the exchange rate risk following the Beta distribution as mentioned above, the optimal policies like p, q and the profits for the retailer and the manufacturer remain the same. To observe this, simply look at rows with risk parameters (1,3) across the cases of the retailer and the manufacturer. Only the purchase price w changes in both the cases. This goes on to show that when both the parties have full information of the actions going to be taken by each of them, it does not really matter who bears the exchange rate risk. Risk borne by either of them will yield the same set of policies.

4. Conclusion

This paper has dealt with the impact of foreign-exchange transaction exposure within a newsvendor setting. At issue is to measure the impact of settling financial obligations, stated in a foreign currency and incurred before the change in the exchange rate, but settled after the change in the rate has occurred. The basic result is that, as long as the retailer is not a risk taker, a risk-neutral manufacturer is indifferent as to which side bears the exchange risk. A paper of this nature may be subject to a wide gamut of generalizations and extensions to adapt the underlying structure to a particular application. Examples include the type of demand distribution to fit alternate modes of demand error structure and the type of functional form of the deterministic portion of the demand to conform to alternate patterns, widely used, especially in the marketing literature.

References

Arcelus, F. J., Kumar, S., Srinivasan, G. (2005). Retailer's response to alternate manufacturer's incentives under a single period, price-dependent, stochastic-demand framework. *Decision Sciences* 36, 599-626.

Chan, L. M. A., Shen, Z. J. M., Simchi-Levi, D., & Swan, J. L. (2004). Coordination of pricing and inventory decisions: A survey and classification. In D. Simchi-Levi, S. D. Wu & Z. J. M. Shen, (Eds.), *Handbook of Quantitative Analysis: Modeling in the E-Business Era*. New York: Kluwer, 335-392.

Eitemann, D. K, Stonehill, A. I., & Moffett, M. H. (2010). *Multinational Business Finance (12th edition)*. Boston: Prentice Hall.

Karlin, S., & Carr, C. R. (1962). Prices and optimal inventory policy. In K. J. Arrow & H. Scarf (Eds.), *Studies in Applied Probability and Management Science*. Palo Alto, CA: Stanford University Press, 159-172.

Mills, E. S. (1958). Uncertainty and price theory. *Quarterly Journal of Economics*, 73, 116-130.

Petruzzi, N. C., Dada, M. (1999). Pricing and the newsboy problem: A review with extensions. *Operations Research*, 47, 183-194.

Yao, L. (2002). Supply chain modelling: Pricing, contracts and coordination. Ph.D. Thesis, Chinese University of Hong Kong.

Yao, L., Chen, Y., Yan, H. (2006). The newsvendor problem with pricing: Extensions. *International Journal of Management Science and Engineering Management*, 1, 3-16.