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An Induced-Convection Effect Upon the Peak-Boiling Heat Flux[1]

An induced-convection effect upon the peak pool-boiling heat flux is identified and described. A method is developed for correlating this effect under conditions of variable gravity, pressure and size, as well as for various boiled liquids. The effect is illustrated, and the correlation verified, with a large number of peak heat-flux data obtained on a horizontal ribbon heater. The data, obtained in a centrifuge, embrace an 87-fold range of gravity, a 22-fold range of width, a 15-fold variation of reduced pressure, and five liquids.

Introduction

The peak heat flux in saturated pool boiling, q_{max}, is a transition value dictated by the onset of a hydrodynamic instability in the removal of vapor from a heater element. The Zuber-Kutateladze equation for this transition on an infinite flat plate, q_{maxF}, references [1–3][2] is[3]

$$q_{maxF} = 0.131 h_{fg}\rho_g^{1/2} \sqrt[4]{\sigma g(\rho_f - \rho_g)} \sqrt{1 + \frac{\rho_g}{\rho_f}} \tag{1}$$

Two kinds of parameters are entirely absent from equation (1). There is no characteristic length and there are no transport properties. Accordingly, Lienhard and Schrock [4] were able to use the Law of Corresponding States to write equation (1) in terms of a reduced peak heat flow, $F(p_r)$, such that

$$q_{maxF} = \lambda F(p_r) \tag{1a}$$

where λ is a complicated function of gravity critical data, and other constants characteristic of the liquid being boiled, and p_r is the reduced pressure.

In 1965, Lienhard and Watanabe [5] found that a previous analytical expression for the minimum heat flux on horizontal wires [6] could be written as the *product* of the minimum heat flux for a flat plate, and a function of the geometric scale parameter, L', defined as

$$L' = L\sqrt{\frac{g(\rho_f - \rho_g)}{\sigma}} \tag{2}$$

They then provided broad experimental support for the notion that a similar separation would describe the q_{max} in finite geometries. Their correlating equation was

$$q_{max} = \lambda F(p_r)\cdot f(L') \tag{3}$$

or

$$\frac{q_{max}}{\lambda F(p_r)} = f(L') \tag{3a}$$

Very little attention has been given to date to the fact that the bubbles rising above a heater of finite size should drag upon the surrounding fluid and induce a secondary flow about the heater. In 1964, the late Costello and his co-workers [7] found that a flat ribbon heater, mounted on a slightly wider block, induced strong side flows. When the side flow was blocked by vertical walls, q_{max} was much lower than it was when the side flows were allowed. Furthermore, they observed an increase of q_{max} with decreasing ribbon width when the side flow was permitted.

Borishanski (see, e.g., [2]) proposed a parameter to describe such an effect as early as 1956. He suggested that his peak heat flux data could be well represented by an expression of the form

$$q_{max} = (0.131 + 4N^{-0.4})h_{fg}\rho_g^{1/2} \sqrt[4]{\sigma g(\rho_f - \rho_g)} \tag{4}$$

where

$$N \equiv \frac{\rho_f \sigma}{\mu^2}\sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}} \tag{5}$$

and μ is the viscosity. We shall see shortly that the parameter, N, characterizes buoyant effects. Borishanski's expression indicates that the influence of the parameter, N, is really pretty small in the flat plate geometry since 0.131 is generally $\gg 4N^{-0.4}$.

The present study is motivated by a specific interest in the in-

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[2] Numbers in brackets designate References at end of paper.

[3] Symbols not defined in context are explained in the Nomenclature section.

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Nomenclature

- $f(\ )$ = an arbitrary function of ( )
- $F(p_r)$ = a function of p_r equal to q_{maxF}/λ
- g = acceleration of gravity or other force field
- h_{fg} = latent heat of vaporization
- I = induced-convection scale parameter, $\sqrt{\rho_f L\sigma/\mu^2}$
- L = characteristic length
- L' = dimensionless size, equation (2)
- M = molecular weight
- N = induced-convection buoyancy parameter, I^2/L'
- P = the parachor, $M\sigma^{1/4}/(\rho_f - \rho_g) \simeq$ constant, for any fluid
- p_c = critical pressure
- p_r = reduced pressure, system pressure $\div\ p_c$
- R = ideal gas constant
- T_c = critical temperature
- q_{max} = the peak pool-boiling heat flux
- q_{maxF} = q_{max} for an infinite horizontal flat plate, as given by Zuber's equation (1)
- W = width of horizontal ribbon heaters
- W' = dimensionless size based on W
- λ = $g^{1/4}p_c(P/M)(8Mp_c/3RT_c)^{3/4}$
- μ = liquid viscosity
- ρ_f = density of saturated liquid
- ρ_g = density of saturated vapor
- σ = surface tension between a liquid and its vapor

fluence of variable gravity upon q_{max}. Accordingly the implications of the induced flow observed by Costello, et al., are serious since the effects of this flow stand to be magnified by gravity. We have therefore created a configuration comparable to theirs and studied its performance under variable gravity. Before considering this experiment, however, let us first develop a correlation, similar to equation (3), which accounts for induced flow.

Method of Correlation

The viscosity of the liquid, which has not appeared to be of great importance in prior studies of q_{max}, must now be considered. It is only through the viscous drag exerted by rising bubbles upon the surrounding liquid that flow can be induced. Thus the dependent variable q_{max} will depend primarily upon 7 independent variables[4]: ρ_f, ρ_g, h_{fg}, σ, L, g, and μ. These 8 variables are expressible in four dimensions: Btu, ft, sec, and lb_m. Therefore, we must write down (8-4) or four independent dimensionless groups to describe the phenomenon, in accordance with the Buckingham pi theorem.

The first and most obvious of these groups might be ρ_g/ρ_f. This has generally taken the following form in q_{max} predictions:

$$\sqrt{1 + \rho_g/\rho_f}$$

The second group is a dimensionless *dependent* variable:

$$q_{max}/h_{fg}\rho_g^{1/2}(\rho_f\sigma g)^{1/4}$$

the denominator of which can be combined with $\sqrt{1 + \rho_g/\rho_f}$ to form Zuber's q_{maxF}:

$$\frac{q_{max}}{q_{maxF}} \equiv \frac{q_{max}/h_{fg}\rho_g^{1/2}(\rho_f\sigma g)^{1/4}}{0.131\sqrt{1 + \rho_g/\rho_f}(2 - [\sqrt{1 + \rho_g/\rho_f}]^2)^{1/4}} \quad (6)$$

A third group, $L\sqrt{g\rho_f/\sigma}$, can be multiplied by $(2 - [\sqrt{1 + \rho_g/\rho_f}]^2)^{1/2}$ to get L'.

The fourth group can then be obtained by elimination. The dependent variable, q_{max}, and the only other variable containing the units of Btu, namely, h_{fg}, will be eliminated. So too will ρ_g be deleted as unrelated to the viscous drag problem. If the method of indices is being used to establish the groups, this will leave 4 equations in 5 unknown indices, so one more choice must be made. Setting the index of g equal to zero gives the new parameter

$$I = \sqrt{\rho_f L\sigma/\mu^2} \equiv \sqrt{NL'} \quad (7)$$

We shall call this the "induced-convection *scale* parameter." It is analogous to the ratio of the square root of a Grashof number divided by L', and it characterizes the relevant forces in the following way:

$$I = \frac{[(\text{inertia force})(\text{surface tension force})]^{1/2}}{\text{viscous force}}$$

If gravity is left in this parameter and the size, L, is deleted, the resulting group will be $(\sigma/\mu^2)\sqrt{\sigma\rho_f/g}$. This in turn can be divided by $(2 - [\sqrt{1 + \rho_g/\rho_f}]^2)^{1/2}$ to give Borishanski's N for the fourth group. We shall call N the "induced-convection *buoyancy* parameter" since it replaces L with g. It characterizes the relevant forces in the following rather complicated way:

$$N = \frac{(\text{inertia force})(\text{surface tension force})^{3/2}}{(\text{viscous force})^2(\text{buoyant force})^{1/2}}$$

Combining the four groups, we obtain the desired correlating relation:

$$\frac{q_{max}}{q_{maxF}} = f(L', I, \sqrt{1 + \rho_g/\rho_f}) \quad \text{or} \quad f(L', N, \sqrt{1 + \rho_g/\rho_f}) \quad (8)$$

or, if we choose to use the Law of Corresponding States,

$$\frac{q_{max}}{\lambda F(p_r)} = f(L', I, \sqrt{1 + \rho_g/\rho_f}) \quad \text{or} \quad f(L', N, \sqrt{1 + \rho_g/\rho_f}) \quad (9)$$

Equation (9) implies that the generalized function, $F(p_r)$, should be used to approximate q_{maxF}. This function was established in [4], based upon the flat plate data of Cichelli and Bonilla [8]. Equation (1*a*) is therefore inexact, and equations (8) and (9) represent slightly different means for correlating data.

The choice of I or N as the correlating parameter for induced convection is arbitrary. If one is interested strictly in the influence of gravity, I is probably more convenient because it leaves the influence in the coordinates, q_{max}/q_{maxF} and L', without introducing it in a third parameter. The Borishanski parameter, N, however characterizes the buoyancy forces explicitly.

Equation (3*a*) is of this form. The term $\sqrt{1 + \rho_g/\rho_f}$, which is almost exactly unity for pressures up to the neighborhood of the critical point, appears to be irrelevant at lower pressures and was not needed by Lienhard and Watanabe. Furthermore, their experiments were made on horizontal wires which provided little obstruction to, or interaction with, the movement of any flow that might have been induced. In their case (as in Borishanski's) viscosity was no longer a relevant variable and the fourth-dimensionless group did not appear. Vliet and Leppert [9] found that strong forced-convection currents over cylinders altered q_{max} significantly, but no such evidence has been given for induced convection.

The present dimensional analysis therefore vindicates the separation of q_{maxF} from a function of geometric scale that was assumed to exist in previous studies. It also shows that the previous correlation equation (3*a*) was limited by two implicit assumptions. In the present work we shall retain the assumption that $\sqrt{1 + \rho_g/\rho_f}$ does not contribute, but we shall look for the influence of an additional parameter, I, or N. Our correlation equation will accordingly be

$$\frac{q_{max}}{q_{maxF}} = f(I, W') \quad \text{or} \quad f(N, W') \quad (10)$$

where W' is an L' based upon the width, W, of horizontal heaters, and q_{maxF} is computed from equation (1).

Experiment

At least three things must now be checked experimentally. The first is whether or not equation (8) is based upon the correct physical variables and whether it will succeed in correlating data. The second is whether or not the parameter, I, really will exert significant influence on q_{max}. Finally, experimental evidence will be needed to locate the point at which scale effects vanish as the scale parameters are increased.

Our experiments were made in the University of Kentucky Boiling and Phase-Change Laboratory on a general purpose centrifuge capable of developing as much as 100 earth-normal gravities. The centrifuge facility is described fully in reference [10]. Full details of the experiments, and the raw data, are given in reference [11]. Fig. 1 shows the centrifuge facility. The test capsule appears through an open hatch on the left-hand side. The peak heat-flux transition was observed with the help of a synchronized strobe-light through a plexiglas viewing window on the right-hand side.

Fig. 2 shows a typical flat ribbon heater in place on a special ribbon mount within the test capsule. The inside of the capsule was 7 in. long, by 3 in. high, by 3¾ in. wide. It is equipped with a vacuum manifold and a vacuum measuring line, a thermo-

[4] *Since the induced flow will act to disrupt the hydrodynamic process of vapor removal, we are using the word "convection" to describe a fluid mechanical action—not a heat removal process. The transport variables of conductivity or diffusivity are therefore not introduced.*

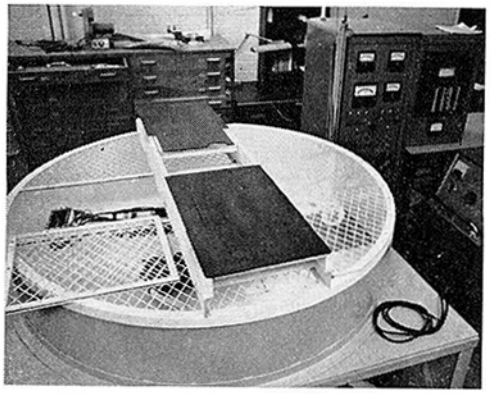

Fig. 1 Gravity-boiling centrifuge facility

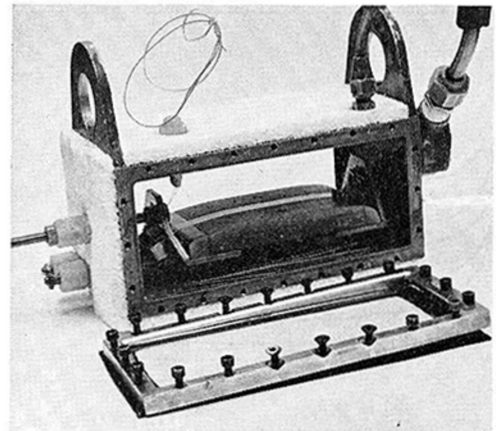

Fig. 2 Test capsule with ribbon heater mount in place

couple to monitor the liquid temperature, and power leads for both the test heater and a preheater on the bottom. These lines pass through appropriate vacuum seals and slip-rings to a pump, a power supply, and a meter panel, outside.

Carefully cleaned nichrome ribbons, between 0.046 and 1.000 in. wide, with a 4.00 heated length, were cut from 0.009, 0.002, or 0.001-in. stock and stretched out on the $2^1/_2$-in-wide mounting block. The capsule was charged with reagent grade acetone, methanol, benzene, isopropanol, or double distilled water, and mounted on the centrifuge. During a run, the liquid level was held in the range 1.0 to 1.5 in. above the ribbon surface and noted to make a hydrostatic head correction, and the temperature held to within a degree of saturation. The angular speed of the centrifuge was read, and (with the preheater turned off) the capsule pressure was recorded. Finally, the power supplied to the ribbon was increased until the peak heat-flux transition was observed, and there it was read.

About 874 observations were made—roughly 419 in acetone, 262 in methanol, 104 in isopropanol, 81 in benzene, and 8 in water. These raw data are tabulated fully in reference [11] and will not be reproduced here. The probable errors of the variables, computed in [11], are about ± 6 percent for $q_{\max}$, ± 4 percent for W', and ± 2 percent for I. The variability of observed $q_{\max}$ values was on the order of ± 15 percent, as is typical of $q_{\max}$ data. Data were measured over a reduced pressure range from 0.0016 to 0.0246, and a range of gravity from 1 to 87 times earth normal gravity.

Results and Correlation

Equation (10) indicates that these data, obtained over a very broad range of conditions, should correlate onto a single surface in $q_{\max}/q_{\max F}$, W', and I or N, coordinates. And if our supposition that induced convection is a significant effect in this configuration, then the surface should show that $q_{\max}/q_{\max F}$ varies significantly with either I or N.

To create these surfaces, we plotted all of the present data, and three points for water at one atmosphere and one gravity, from [7], twice: first on $q_{\max}/q_{\max F}$ versus W' coordinates for comparatively narrow ranges of I; then on $q_{\max}/q_{\max F}$ versus I coordinates for ranges of W'. Fig. 3 shows a typical example of one of these crossplots. An additional 20 crossplots not shown here can be made available on request.[5]

Fig. 3 reveals some things that were generally true of all of the data. The great majority of the data for any substance clustered within ± 15 percent of a mean surface through them, and a

[5] The method of correlation used in reference [11] was based on equation (9) instead of (8). This resulted in a doubling of those errors introduced by the Law of Corresponding States and an awkward scaling of $q_{\max}$. The present correlation overcomes these difficulties but required replotting the data instead of using the curves in [11]. The effectiveness of equation (8) was made clear by Sun [12] who used it with great success to correlate cylinder data. The shape of our resulting surface differs somewhat from that plotted in reference [11].

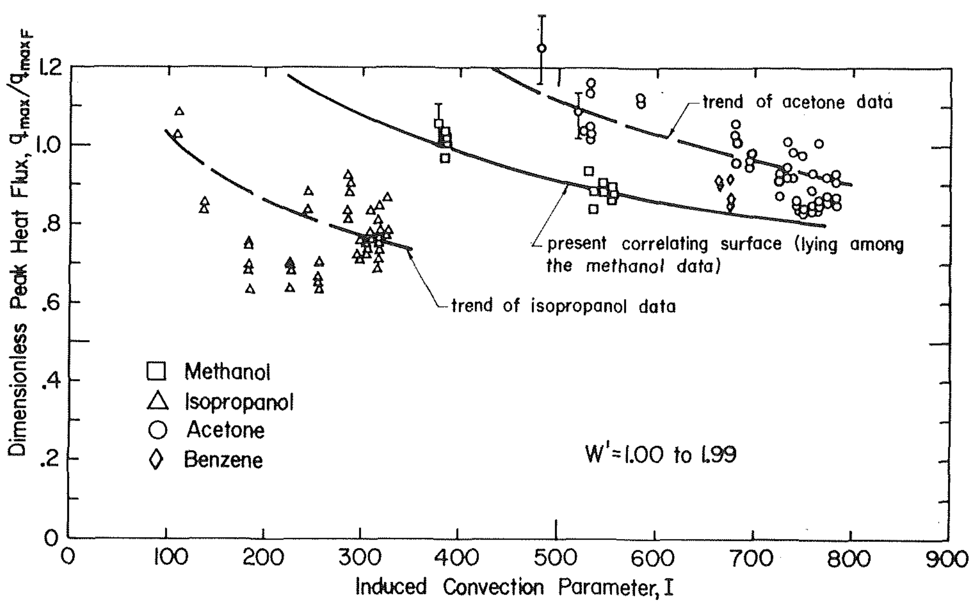

Fig. 3 A typical crossplot for peak heat-flux data in a range of W′

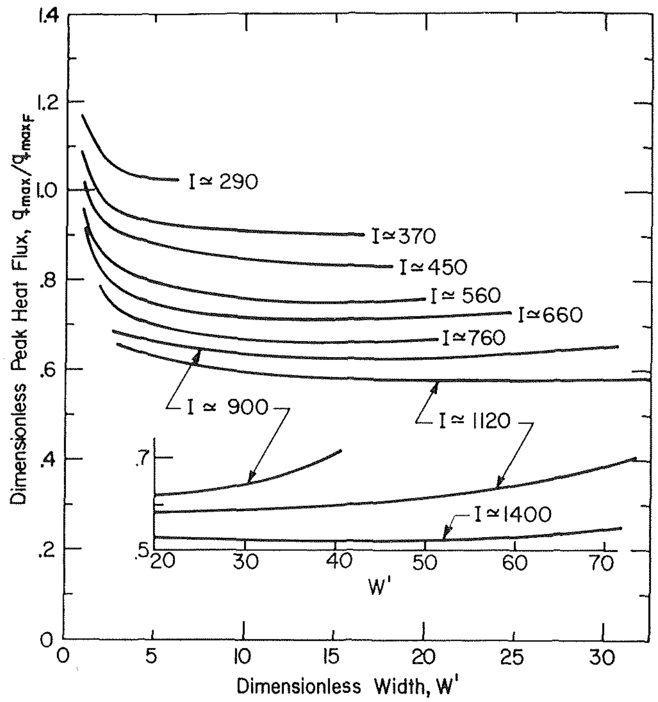

Fig. 4 q_{max}/q_{maxF} versus W' contours for 9 values of I

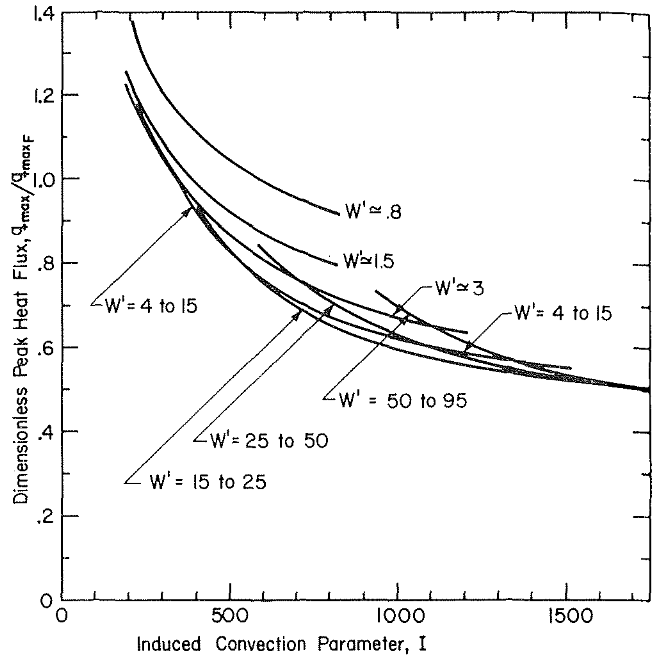

Fig. 5 q_{max}/q_{maxF} versus I contours for 8 values of W'

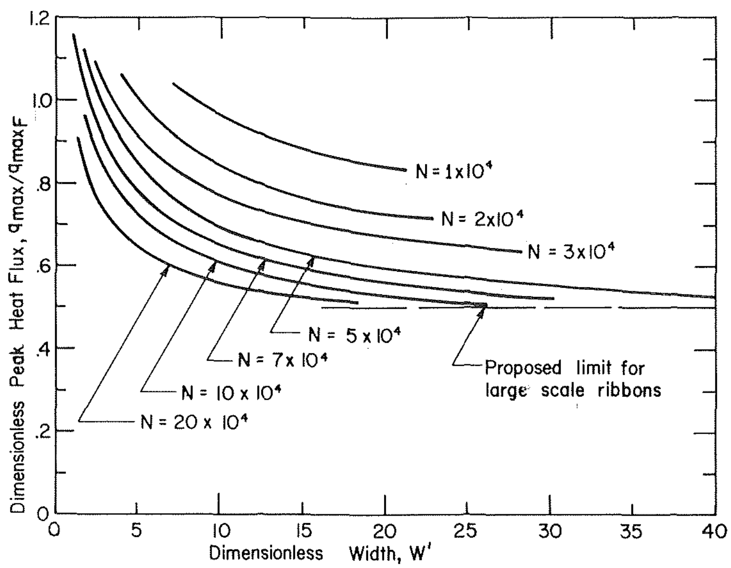

Fig. 6 q_{max}/q_{maxF} versus W' contours for 7 values of N

geometrically similar family of surfaces could be drawn through all of the substances. Methanol was most representative of the substances used. Acetone and water presented the highest q_{max}/q_{maxF} values—about 25 percent above the methanol—and isopropanol the lowest—about 30 percent below the methanol. The benzene data lay between the methanol and acetone data.

The resulting correlating surfaces are each presented here in sets of contours. The correlation function $f(I, W)$ is presented in Figs. 4 and 5 which give q_{max}/q_{maxF} versus W' and q_{max}/q_{maxF} versus I contours, respectively, as obtained from the 21 cross-plots. This surface generally follows the mean of the data for methanol and is consistent with trends that can be identified individually in each of the remaining substances.

The correlation in terms of N was obtained by transforming the curves given in Figs. 4 and 5 with the help of equation (7). It is given in a single plot of q_{max}/q_{maxF} versus W', with N as parameter, in Fig. 6.

Discussion

The correlation surfaces reveal the strong influence of the parameters I and N, that we anticipated, and they also show that this influence vanishes when the *scale* parameters, I and W,' become large. This was to be anticipated since buoyancy completely overbalances both capillary and viscous forces as the scale is increased. Thus

$$\text{limit}\left[\frac{q_{max}}{q_{maxF}}\right] \simeq \frac{1}{2}, \text{ large } I \text{ and } W' \qquad (11)$$

This limit appears to be valid for all $I > 1500$, and W' on the order of 50, depending upon the value of I. Since N is not a scale parameter we cannot propose a proper criterion in terms of it. However, the various lines of constant N seek $q_{max}/q_{maxF} = {}^1/_2$ as an asymptote in Fig. 6.

That this limiting q_{max} is less than the flat plate value might reflect either or both of two factors: (*a*) The finite size of the centrifuge capsule doubtless results in a limiting high Reynolds number circulation which affects the peak heat flux; and/or (*b*) Zuber's equation has not really been subjected to broad testing in a proper flat plate configuration, and some error in the constant, 0.131, could conceivably contribute to a deviation of the present data from the supposed flat plate equation.

When W' falls below unity, surface tension assumes very strong control over the peak heat-flux transition and q_{max} rises sharply. The results of Lienhard and Watanabe, and other authors who have studied q_{max} on cylinders, show a comparable increase of q_{max} as L', based on cylinder diameter, falls below 0.40.

The fact that our data for different substances tend to cluster together, indicates that another parameter is needed to achieve a complete description on the present system. This would have to be a second scale parameter related to the finite size of the capsule. An "I" based upon capsule size, for example, would be smallest for isopropanol and progressively larger for methanol, benzene, acetone, and water—consistent with the order of separation we observed in our crossplotting. Although this effect is a secondary one in the present study, it points up the limitation of the present data to a particular container.

Conclusions

1 Induced (or natural) convection can exert a strong influence on the peak pool-boiling heat flux, if the configuration is one which is susceptible to it.

2 The present data should not be viewed as having broad applicability. They are restricted to a particular configuration of heater and container, and their value lies in that they illustrate the induced-convection effect.

3 Equations (8) and (9) are the appropriate expressions to use to correlate q_{max} data for any heater configuration, in a large container, over ranges of pressure, gravity, and size, and for different liquids.

4 The peak heat flux for a horizontal ribbon heater (and probably for other geometries as well) approaches a constant minimum fraction of q_{maxF} when the scale parameter, I and W', become large. It also approaches this limit as the induced-convection buoyancy parameter, N, increases.

Acknowledgments

Professor R. Eichhorn provided much helpful discussion of this work and K. H. Sun assisted with the apparatus and data reduction.

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