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# COMPARATIVE ASSESSMENT OF DIRECT AND INDIRECT PROBABILISTIC METHODS FOR THERMOMECHANICAL ANALYSIS OF STRUCTURAL COMPONENTS IN GAS TURBINES

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## ABSTRACT

Various sources of uncertainty greatly affect the life of structural components of gas turbines. Probabilistic approaches provide a means to evaluate these uncertainties; however, the accuracy of these approaches often remains unknown. Published quantitative studies of the effectiveness of various uncertainty quantification techniques are usually based on very simple examples. This is contrasted by the large-size finite element models that are used for complex geometries of critical structural parts such as turbine blades or nozzles. In such real-life applications the expenses of the "function calls" (runs of these models) preclude systematic studies of probabilistic methods. These expenses are attributed not only to the actual runs of the model, but to the difficulties in parametrically changing the model as well. Such a "complexity gap" leads to a justifiable concern over whether the trends identified in academic studies are relevant to these industrial applications. As a result, structural engineers end up with the number of function calls that they can afford rather than what would be needed for the required level of accuracy. The present effort intends to bridge this gap by studying a mid-level problem: a simplified notional finite element model of a gas turbine component is presented. Despite its simplicity, the model is designed to reflect the major features of more realistic models. The parametric changes of the model are fully automated, which allows for performing an extensive set of benchmark tests that help to determine the relative merits of various existing probabilistic techniques for component life assessment Several meta-modeling techniques are investigated and their performance compared based on direct sampling methods. In this context, various Design of Experiments (DoE) methods are studied. The results are used to construct the Response Surface Equations (RSE) as well as the kriging models. It is emphasized that changes in the relative locations of the critical points induced by variation of independent parameters can critically affect the overall fidelity of the modeling; the means of remedying such a degradation in precision are discussed. Finally, it is shown that when the ranges of independent variables are large, kriging generally provides precision that is an order of magnitude better than RSE for the same DoE.

## INTRODUCTION

Probabilistic use of Finite Element Analysis (FEA)is becoming more and more common, and some of the basic techniques are included in standard commercial software such as ANSYS. In addition, some specialized packages (*e.g.,* ProFES) have been developed explicitly for this purpose. Such packages combine solid geometry modeling (CAD), FEA, and life assessment [1]. However, the practical implications of availability of such tools are far from clear. The problem is not unique to the life assessment of gas turbine components, and similar issues exist in such remote fields as biomechanics, (see for example [2]). An excellent recent paper [3] surveys a wide range of uncertainty quantification techniques and provides some general estimates of

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the associated computational costs. Although these works provide valuable information for practicing engineers, the quantitative comparison in those works is provided mostly for very simple geometries such as plates, and it is not clear how the results will scale up to real-life applications.

It is worth pointing out that life assessment of modern gas turbine components such as turbine nozzles and blades involves very complex FEA models that often have the size of more than one million nodes. Moreover, the solutions are very sensitive to the external and internal boundary conditions, which in turn require sophisticated modeling of both hot gas path and cooling flows. Such formidable multi-disciplinary problems make any extensive parametric probabilistic studies all but impossible even using modern parallel computing. The associated challenges are mostly due not to the calculation itself, but to the complexities of parameterization, especially if it involves geometric changes that are very difficult to automate.

In view of the above, it seems to be important to establish certain guidelines that are based on the study of a model that is somewhat more realistic than a plate or a hollow cylinder, and yet quite manageable from a computational standpoint. Perhaps the only field where such studies exist to a large extent is Most-Probable-Point (MPP) methods [4; 5]. The present work attempts to fill the existing void by building a parametric notional FEA model of a turbine nozzle and using this model as a benchmark for comparison of the available probabilistic techniques. A brief overview of several of the most common techniques is provided below, followed by a description of a parametric model.

## Approximation probabilistic methods

Approximation probabilistic techniques can be combined in three general groups [3; 6]:

1. Most-Probable-Point (MPP) methods that target certain critical points on the cumulative distribution;
2. Direct Monte Carlo sampling and its modifications;
3. Decoupled (indirect) techniques that divide the problem into two separate ones: first a sampling is conducted in some orderly fashion; followed by a fitting of the results into some tractable surrogate model.

## MPP methods

MPP methods include First and Second-Order Reliability Methods (FORM and SORM, respectively) [7] and their derivatives such as the Advanced Mean Value (AMV) procedure [4]. These techniques are often collectively referred to as Fast Probability Integration (FPI). A characteristic feature of these methods is that they require an interaction between the FEA and the sampling procedure. As was mentioned in the Introduction, several studies have provided some estimation of the precision of these methods for mid-sized problems, so the present study does not focus on these techniques and the reader is referred to the existing work (see for example[4; 5]). However, in the future side-by-side comparison of these methods with the use of meta-modeling techniques is envisioned.

## Direct methods

This approach attempts to construct an entire probability density function (PDF) of the response by random sampling. The advantage of the simple Monte Carlo technique is that the Confidence Interval (CI) estimation is straightforward, and therefore the accuracy of the estimation can be assessed quantitatively. In addition, the number of simulations does not scale explicitly with the number of independent variables. However, the drawback of this method is its low sampling efficiency. Thus, a host of methods have been developed to increase sampling efficiency. Such methods can be generally referred to as variation reduction techniques. Latin Hypercube Sampling (LHS) is perhaps the most popular of these techniques [8; 9]. From the point of view of the Design of Experiments, LHS can be also classified as so-called space filling design [9].

## Decoupled methods

The first step of such a procedure is to construct a design of experiments. The results are consequently used to construct a surrogate model (or meta model), which is used to conduct all the probabilistic analysis. Due to the simplicity of the resulting meta model, extensive sampling using a simple Monte Carlo simulation is very cheap computationally, and the main source of the error stems from the discrepancy between the surrogate model and the real one. By far the most popular type of meta-modeling is the combination of Design of Experiments and Response Surface Equations (DoE/RSE) [10; 9] and [8]. In general one can identify two steps of constructing a meta-model:

1. Design of Experiments: a judicious choice of the independent paramaters in the set of runs. Central Composite Designs (CCD) of resolution $V$ are traditionally used, while another common option is Box-Behnken design. The resolution $V$ implies that both main effects and two-term interactions are not aliased with each other [9; 8]. These designs were originally constructed for real experiments rather than computer simulations. Random error, which exists in the real experiment, is absent in the computer simulation, and only biased error due to approximation is present. The full implication of this difference is not well studied, but it is clear that it is significant.
2. Finding an appropriate form of a meta-model and choosing its parameters that fit the results of DoE.

Once the values at the $n$ sampling points are obtained, one can construct a corresponding set of k-dimensional vectors $s_1 \dots s_k$ (here $k$ is the number of input variables). A typical

second-order RSE has the following form

$$y = a_0 + a_i x_i + a_{ij} x_i x_j \quad (1)$$

Where unknown coefficients $a$ are sought based on the least square error minimization. A summation is implied over the repeating indices within appropriate limits. Clearly, part of the reason for RSE popularity is their simplicity. The relative merits of these methods (as well as direct sampling techniques) depend on the problem at hand, but the indisputable advantage of meta-modeling is its flexibility. Once the meta-model is constructed, the changes of the statistical properties of the response due to any changes in the probability distributions of the input parameters can be obtained at no extra cost. The same is true about the evaluation of multiple responses. Thus, for a small number of independent variables, the decoupled methods provide a transparent and efficient means to minimize the number of expensive "function calls," *i.e.*, actual runs of the physical model. A variety of commercial packages, such as JMP[11] or Design Expert[12] exist to facilitate setting up DoEs and constructing RSEs. However, as the number of independent variables increases, the number of function calls for even the most frugal DoEs (such as saturated designs) becomes prohibitively large. The exact number of independent variables depends on the resources, but more likely than not the effective use of DoE/RSE is limited to a single-digit number of independent variables (see for example [3]). Perhaps the most common way to deal with such a "dimensionality curse" is by running a screening (two-level) DoE in order to identify the main contributors. However, this task is not always straightforward[13], and there are cases where a large number of variables are almost equally significant, especially if the problem involves multiple responses. Lately, however, Kriging has been receiving more and more attention as a viable alternative to RSEs, and this technique will be described below in more detail.

### Kriging

This method originates in Geostatics and was popularized by G. Matheron [14], although simultaneously the approach was developed in meteorology by Gandin, under the name "optimum interpolation" (see more detail on the history of Kriging in [15]). The stochastic process $y$ is considered, which depends on $k$ parameters $x_1 \dots x_k$ . Let us assume as before that the results from $n$ samples are known, and denote the corresponding set of k-dimensional vectors as $s_1 \dots s_k$ and a generic predictor for an the value $y$ in a given location of $s_0$ is given by $p(y, s_0)$.

Then "ordinary kriging" refers to a spatial prediction based on the following two assumptions:

$$y(s) = \mu + \delta(s) \quad (2)$$

here $\mu$ corresponds to an unknown scalar (this is referred to as *model assumption*), and a (linear) *predictor assumption*:

$$p(y, s_0) = \lambda_i y(s_i) \qquad \sum_{i=1}^{n} \lambda_i = 1 \quad (3)$$

The latter condition guarantees that the estimator is unbiased. Then it can be shown that the minimization of the mean-squared predicted error is equivalent to the minimization of the following expression:

$$C(0) + \lambda_i \lambda_j C(s_i - s_j) - 2\lambda_i C(s_0 - s_i) - 2m(\sum_{i=1}^{n} \lambda_i - 1) = 0 \quad (4)$$

Here $m$ is a Lagrange multiplier and $C(h)$ is a covariogram of the corresponding stochastic process. There are several forms of covariogram available in the literature[15]. Once the form of the covariagram is chosen, the next step is to find appropriate value of the free parameter(s). For a given covariogram, however, the solution is fairly straightforward. It is convenient to introduce the following notations: $n \times n$ matrix $R$, such that $R_{ij} = C(s_i - s_j)$; and $n$-dimensional vectors $c$ and $f$, such that $c_i = C(s_0 - s_i)$ and $f_i = 1$, respectively. Then one can directly express an unknown $n$-dimensional vector $\lambda$ and a scalar $m$:

$$\lambda^T = \left( c + f \frac{1 - f^T R^{-1} c}{f^T R^{-1} f} \right)^T R^{-1} \quad (5)$$

$$m = \frac{1 - f^T R^{-1} c}{f^T R^{-1} f} \quad (6)$$

In the present study, a Gaussian covariagram is chosen

$$C(h) = [exp(-\theta ||h||^2)] \quad (7)$$

where $||h||$ is $L_2$ metric in the $k$-space. The free parameter $\theta = 3$ was considered in the study, this value provided a good compromise between the accuracy (which requires this parameter to be small) and numerical stability (smaller $\theta$ lead to ill-conditioned matrices). Similar conclusions were reached in [6]. In addition, the tailoring of DoEs to the Kriging model was not considered in this study, instead, standard DoEs were used.

## SIMPLIFIED MODEL

A fully parametric model of a notional turbine nozzle was created using the APDL language in ANSYS with the goal of reflecting major salient features of a realistic gas turbine component. A generic nozzle is depicted. Parameters such as wall

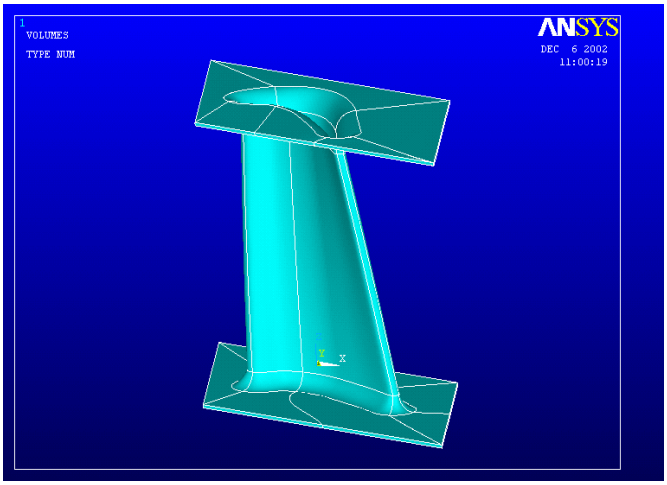


Figure 1. Solid Model

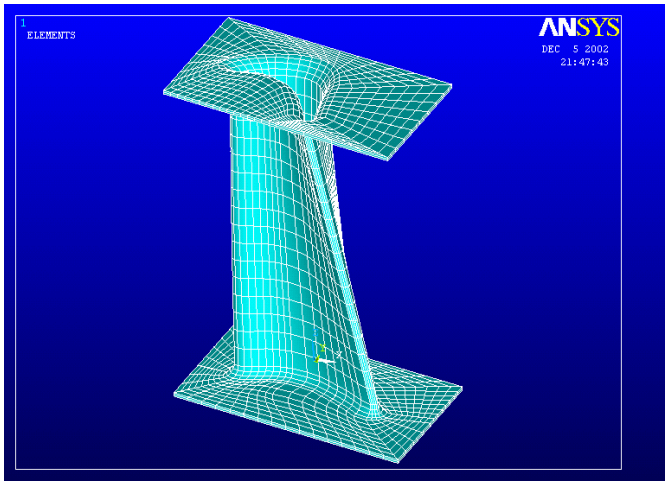


Figure 2. Mesh

thickness, fillet radii, and twist of the airfoil as well as all other dimensions can be automatically changed, including the airfoil shape. Fully mapped meshing was chosen in order to facilitate an orderly parametric comparison (including mesh size control). This presented a certain challenge, since it requires the continuity of the mesh flow. As shown in Fig.1, due to the twist of the airfoil, the cuts on the top and bottom platform are independently and parametrically adjusted in order to ensure a good mesh quality (see Fig.2). Thermal stresses are investigated. First a thermal analysis is conducted, where the temperature and entalpy of the gas flow,$T_g$ and $H_g$, as well as the same parameters for the cooling flow, $T_c$ and $H_c$, are mapped on the respective surfaces. These fields are calculated based on first principles as the valuses are adjusted for the presence of the thermal boundary coating (TBC). Fig. 3 provides a baseline temperature distribution. Once the metal temperature distribution is obtained, the results are used in the structural analysis. The corners of the platforms are appropriately constrained to eliminate extraneous boundary effects. An appropriate parametric pressure distribution is provided, and stresses and temperatures are recovered at the critical points.

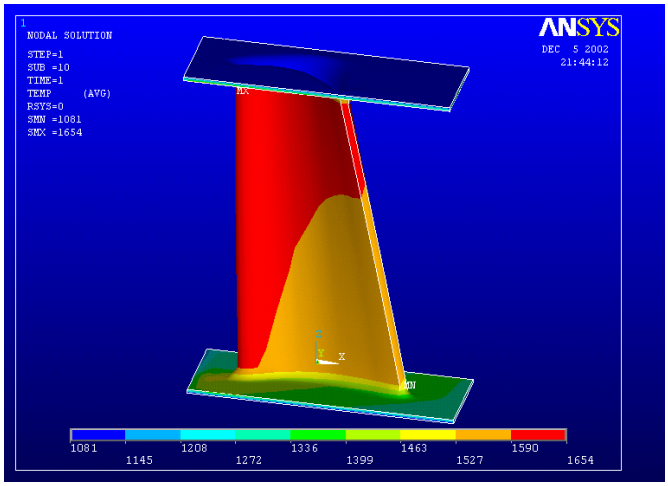


Figure 3. Sample Temperature Distribution

## RESULTS AND DISCUSSION

In what follows, English units are used. A model was considered where six parameters were varied independently: four scaling parameters $TEXT$, $HEXT$ (temperature and convection coefficients of the hot gas flow) as well as the corresponding parameters for the cooling flow $TINT$, $HINT$. These are multiplicative scalars, where unit values correspond to the baseline values: $T_{g,actual} = TEXT * T_{g,baseline} \ldots$ These scaling parameters are assumed to have normal distributions with the mean 1 and the standard deviation, $\sigma = 0.03$. In addition, two geometric parameters were varied uniformly: wall thickness, $WALLTH$, within the range $0.02in$ $0.04in$ and fillet radius within the range $0.1in$ and $0.2in$. The range of geometric parameters has been intentionally chosen to be wide in order to explore various practical situations. Traditionally, in design the ranges would be wider than those corresponding to modeling manufacturing tolerances. Eight responses were tracked to model various practical situations. First, there are two pairs of equivalent (Von Mises) stresses and temperatures: temperature $T_1$ and stress $\sigma_1$, which correspond to the node with the maximum temperature, and temperature $T_2$ and stress$\sigma_2$, which corresponds to the node with maximum stress. It must be noted that a hot spot can move from one node to another as parameters change. In order to isolate this phenomenon,

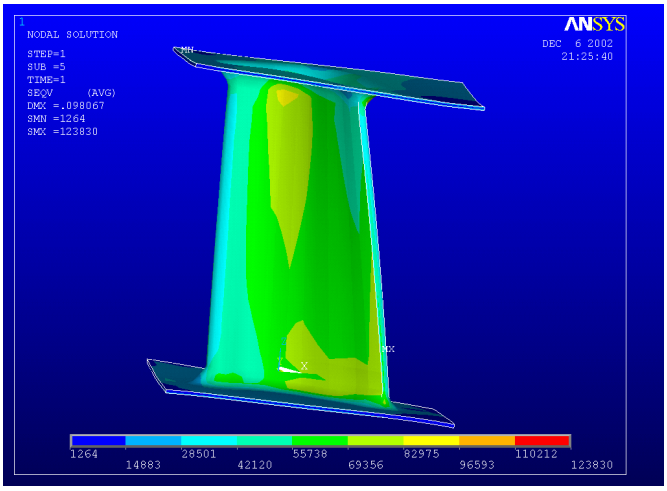


Figure 4. Sample Stress Distribution

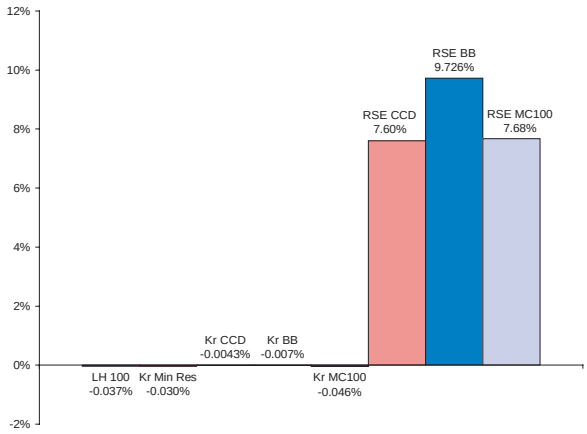


Figure 5. Predictions using various techniques for $T_1$ mean in percent difference compared to 800 LHS runs

four more responses are evaluated: $T_3, \sigma_3$ track the node that has a maximum temperature for the baseline, while $T_4, \sigma_4$ track the node that has maximum stress at the baseline. Since the meshing is mapped, even with the changes of geometric parameters, the nodes do not change their relative positions.

Let us look at the prediction of the mean of the $T_1$ distribution. In all of the following figures, a direct Latin Hypercube Sampling (LHS) with 800 points was used as a benchmark. A separate LHS with a 100 samples was conducted (denoted as LHS100). Several DoE were used: Central Composite Designs (CCD) - half-fraction design (45 sampling points), Box-Behnken design (49 sampling points, denoted as BB), Minimum resolution design (22 sampling points, denoted as Min. Res). For comparison purposes, a simple random design was studied as well. This design has 100 Monte Carlo points that are uniformly distributed in the design space (denoted as MC100). Once the results from the analysis were obtained, the same designs were used to fit either an RSE model or a kriging model. In order to assess the statistics of a meta-model, 10000 Monte Carlo runs was considered to be sufficient.

The relative difference to the benchmark for prediction of the mean and standard deviation for the temperature $T1$ is provided in Figs. 5 and6. It can be seem that mean is predicted very well by all the kriging models, while the RSE models yield an error rate of $7-10\%$. This translates into more than $120^\circ$F! More predictably, the RSEs underestimate the standard deviation by a factor of 10. This implies that higher-order variation has been smoothed out in RSE approximation.

A similar situation exists in predicting $\sigma_1$: Fig. 7 provides the results for 95% quantile prediction. Again, while the prediction is better for RSE's, the same pattern holds: kriging provides a significantly better fit.

However, predictions for $\sigma_2$ and especially for $T_2$ exhibit a more complex situation. Kriging still performs significantly better than RSE (Fig. 8 provides the results for 95% quantile prediction), however, the meta-modeling is not capable of capturing all the nonlinearity for a given number of samples. It is not surprising that Minimum Resolution Design performs particularly poorly. But even for CCD, the maximum actual versus predicted error for 800 samples from LHS is almost 10% for $\sigma_2$ (and more than 33% for the Box-Behnken design). The results are even worse for the temperature distribution. A closer look at the results reveal that as the independent parameters vary, the point of maximum stress migrates significantly: Fig. 9 depicts various locations for the CCD design. Clearly, such jumps lead to a very non-smooth behavior. As expected, tracking individual nodes leads to much better results (see Fig. 10). Obviously, in real-life situations, this necessitates the need for zoning: potentially different hot spots have to be treated separately. Finally, it might be instructive to compare the precision of a meta-model for various parameters: Figs. 10 and 11 provides some representative examples of the relative precision of various parameters: mean value, standard deviation, 95% and 5% quantiles as well as an averaged and maximum actual versus predicted errors Box-Behnken design and Kriging, while Figs. 12 and 13 provide the same parameters for Box-Behnken design and RSE.

## CONCLUSIONS

A simplified model of a first-stage turbine stator vane is presented. The goals are to investigate the fidelity of each method as applied to probabilistic life assessment and to provide guidelines on the required number of function calls in real-life applications. The following conclusions can be made based on the statistical study of the model:

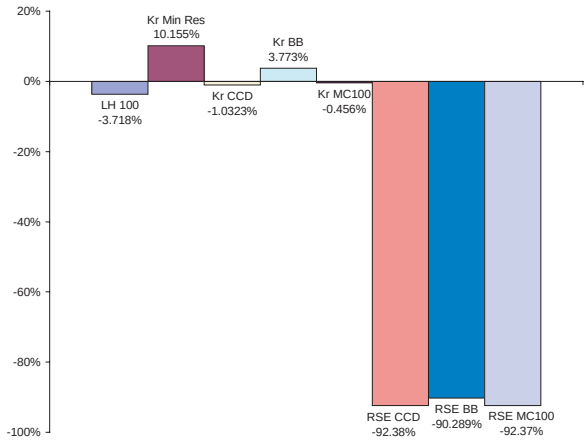


Figure 6. Predictions using various techniques for $T_1$ standard deviation in percent difference compared to 800 LHS runs

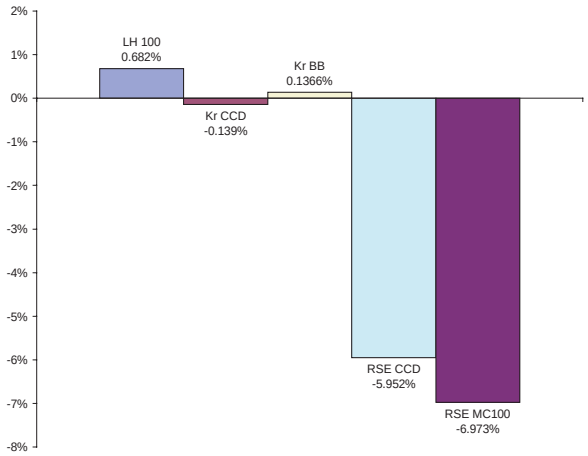


Figure 7. Predictions using various techniques for $\sigma_1$ 95% quantile in percent difference compared to 800 LHS runs

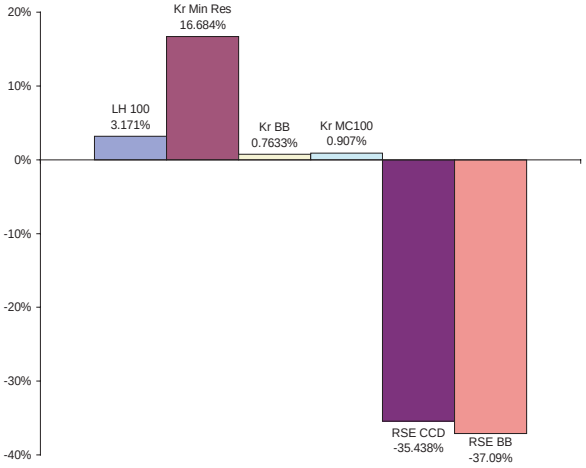


Figure 8. Predictions using various techniques for $\sigma_2$ 95% quantile in percent difference compared to 800 LHS runs

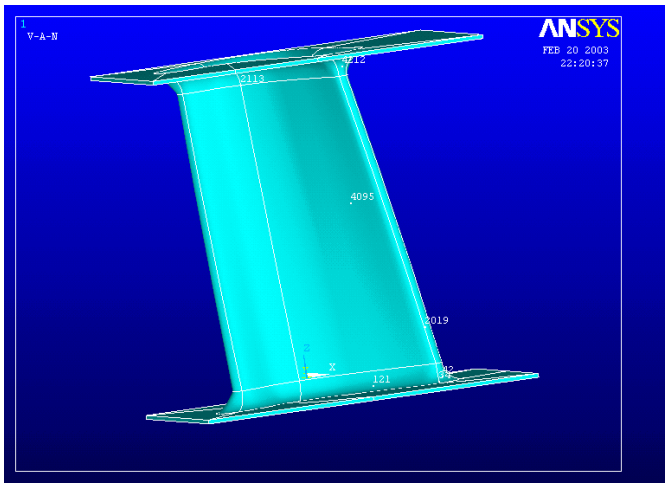


Figure 9. Various locations of maximum stress nodes for CCD design

1. tracking global properties requires much more complex modeling due to the fact that a node with maximum values can drift as parameters change, thus causing highly nonlinear behavior. Separating the structure into different zones with nonoverlapping hot spots helps to remedy this situation.
2. Complex FE models result in highly nonlinear variations of the responses. This implies that Response Surface Equations should be used with extreme caution (and only when the ranges of the independent variables are very small).
3. Kriging, on the other hand, appears to be a viable candidate for meta-modeling. It consistently provides a higher precision than RSE for the same Designs of Experiments, and successfully competes in all statistical measures with direct sampling, while maintaining the flexibility of meta-modeling
4. The particular choice of DoE matters much less than the fitting method: CCD performed marginally better than Box-Behnken. Further studies are recommended to investigate DoE that are specifically tailored to kriging.

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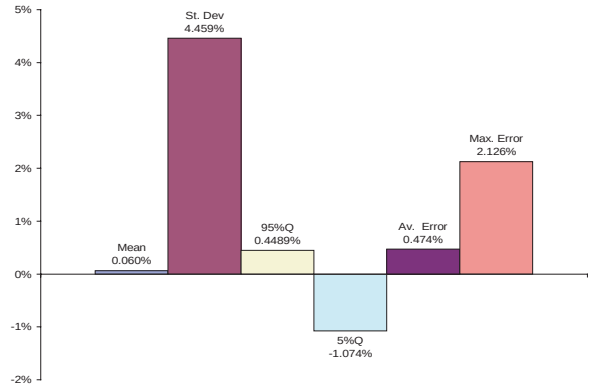


Figure 10. Various prediction characteristics for $\sigma_4$ using Kriging and Box-Behnken design

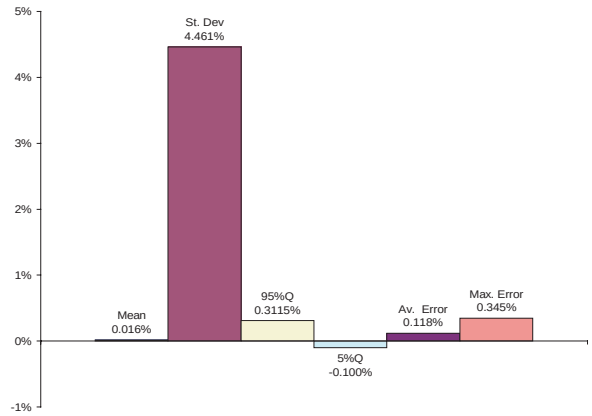


Figure 11. Various prediction characteristics for $T_4$ using Kriging and Box-Behnken design

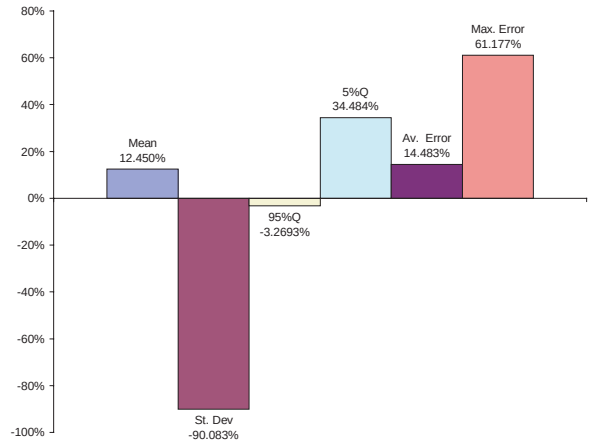


Figure 12. Various prediction characteristics for $\sigma_4$ using RSE and Box-Behnken design

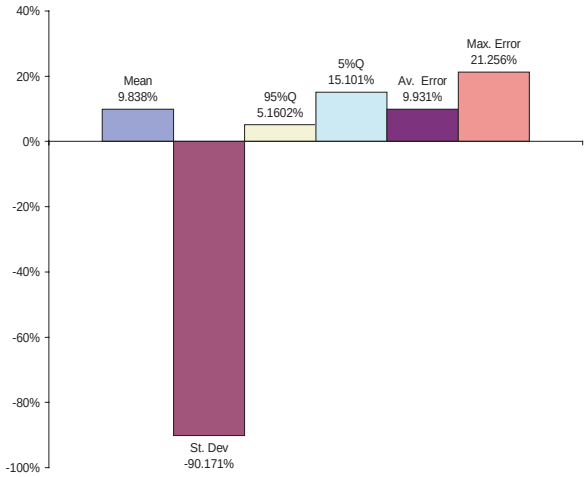


Figure 13. Various prediction characteristics for $T_4$ using RSE and Box-Behnken design

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