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FACTORIAL DESIGN OF EXPERIMENTS AND INTERPRETATION OF RESULTS - AN APPLICATION TO ICE LOADS ON A CONICAL STRUCTURE

Jenny Trumars
Norwegian University of Science and Technology (NTNU)
Department of Civil and Transport Engineering
Høgskoleringen 7a
NO-7491 Trondheim, Norway
Tel: +47 73 59 45 83
Email: jenny.trumars@ntnu.no

Sveinung Løset
Norwegian University of Science and Technology (NTNU)
Department of Civil and Transport Engineering
Høgskoleringen 7a
NO-7491 Trondheim, Norway
Tel: +47 73 59 46 29
Email: sveinung.loset@ntnu.no

ABSTRACT

The data for this study is taken from a publication by D. S. Sodhi et al. [1]. The experimental setup in their publication was made in a way which, after some manipulation, allowed it to be treated as a full two-level factorial design with four factors. The effects of the factors; sheet ice thickness, flexural strength, ice-structure friction and relative ice structure velocity are studied. For the mean horizontal response force the conclusion is that there are clear effects for the changes in friction, ice thickness and the interaction of these two factors, the other effects are not readily distinguishable from the noise. For the vertical force the only really clear effect on the force is the ice thickness, and possibly there is an effect from the flexural strength. The only effect on the frequency of the response is the velocity. Studying the results from the analysis it should be kept in mind that the experiment was not originally set up as a factorial design and that the results can be misleading, and the manipulation of the data has lead to the introduction of more noise. There is for example no randomization of the trials to eliminate unmeasured effects on the outcome and this study should be seen as a demonstration of a powerful methodology rather than a strict experimental design.

Keywords: Experiment, Ice Loads, Factorial Design

INTRODUCTION

In Sodhi et al. [1] the loads on an upward bending cone are studied in a laboratory experiment, where ice thickness, flexural strength, velocity and ice-structure dynamic coefficient of friction are varied; these will be the factors of the experiment. The set up by Sodhi et al. [1] makes it possible to use a factorial design approach to study the data. The factors are taken at two levels with a design scheme that only allows for the study of linear relationships between the two levels. In the setup all combinations of factors are used, giving $2^4 = 16$ combinations with four factors at two different levels. In order to apply the factorial design approach on the experiment by Sodhi et al. [1] the data has been manipulated and this significantly reduces the conclusions that can be drawn from the current study. The results from this study are not reliable and should not be used for *e.g.* design, this study is only intended as an introduction of a powerful method to design experiments and to interpret data from experiments.

The study is primarily intended as a discussion or demonstration of the factorial design method and it should be clear that the original experiment was not performed with this particular statistical method in mind. Thus the conclusions drawn from this analysis of the data should not be relied upon. In order to obtain trustworthy results the experiment will have to be redone with an appropriate experimental design including randomization or blocking of the trials in order to remedy the effect from unwanted and unmeasured influences. If randomization or other steps are not taken to reduce noise, it is not possible to be sure that the real effects from different factors are actually distinguished from the noise. This said, factorial design of experiments is a useful tool to investigate the factors having significant impact on the end result. The information obtained in an initial smaller experiment can be used in further experiments in order to reduce the number of input factors studied. Factorial design of experiments is used successfully for various industrial applications ranging from chemical process plants, medical research and the automotive industry.

METHODOLOGY

A very simple description of the factorial design approach is that all input factors are varied at the same time instead of changing one variable and keeping all other variables constant. By doing so main effects, *i.e.* the effect of change in one variable, of variables can be studied together with interaction effects where two or more variables act together.

To illustrate the method a 2^3 design is used as an example as it is easier to illustrate graphically, see Figure 1. The example is based on Chapter 10 in [2]. In the example three factors are used; ice thickness, velocity and friction. They are set to two levels: high (+) and low (–). The circle in each corner represents the result, or yield y_1 to y_8, for that particular combination of factors.

The *mean* of the outcome or yield is just the average of the results:

$$mean = \frac{1}{8}\left(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8\right) \qquad (1)$$

The *main effect* is the effect from only one variable and it is denoted with the symbol for that variable. For the effect on the result from only ice thickness it is obtained as the average of all results where the ice thickness is high subtracted with the average of all results were the ice thickness is low, this is called the main effect for ice thickness, h_i:

$$h_i = \frac{y_5 + y_6 + y_7 + y_8}{4} - \frac{y_1 + y_2 + y_3 + y_4}{4} = \bar{y}_+ - \bar{y}_- \qquad (2)$$

The *interaction effect* accounts for the effect of interaction of factors and for friction and ice thickness is denoted $\mu \times h_i$ and is calculated as:

$$\mu \times h_i = \frac{1}{4}\left(y_1 - y_2 + y_3 - y_4 - y_5 + y_6 - y_7 + y_8\right) \qquad (3)$$

To calculate the other effects in a 2^3 design a table of contrast coefficients can be used, see Table 1. All combinations of factors are listed using plus and minus signs to symbolize high or low levels of the factors, the possible combinations of trials are eight. The effect of each factor is calculated by adding all the results with the sign in the columns and dividing by the devisor, compare with the equations above.

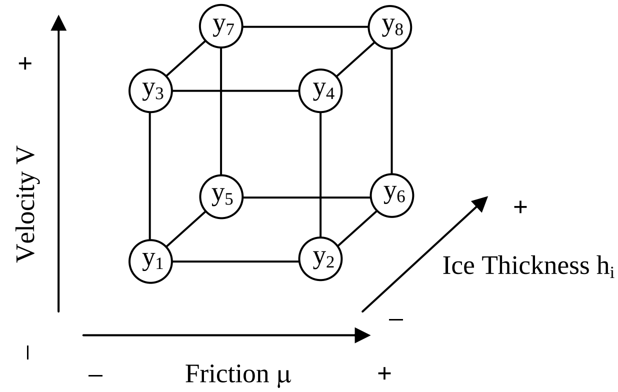

Figure 1: Graphic representation of a two-level three-factor factorial design, after [2].

This is relatively easy to do in matlab using a design matrix with plus and minus ones, and using it to obtain the signs for calculating the interaction effects.

Besides setting up the experiment varying all factors at once, the trials should be randomized to minimize the possible effect from trends due to for example temperature. By randomizing, the effect of the trend is spread out on all the trials and cannot be misinterpreted as an effect of the change in for example ice thickness from low to high values, however it may increase the noise in the data.

It is useful to look at the residuals after determining which effects to consider significant for the results. If the residuals lie close to a line in a normal probability plot it will confirm that all other effects than the significant ones are explained by random noise. If it is assumed that ice thickness, friction and their interaction are significant effects for the 2^3 design above, the residuals can be calculated as $y_n - \hat{y}_n$ where y_n is the test result for trial n = 1 to 8 and $\hat{y}_n$ is formulated as:

Table 1: Table of contrast coefficients for a full 2-level 3-factor factorial design.

Trial	Mean	μ	V	h_i	$\mu \times V$	$\mu \times h_i$	$V \times h_i$	$\mu \times V \times h_i$
1	+	–	–	–	+	+	+	–
2	+	+	–	–	–	–	+	+
3	+	–	+	–	–	+	–	+
4	+	+	+	–	+	–	–	–
5	+	–	–	+	+	–	–	+
6	+	+	–	+	–	+	–	–
7	+	–	+	+	–	–	+	–
8	+	+	+	+	+	+	+	+
divisor	8	4	4	4	4	4	4	4

$$\hat{y}_n = mean + \frac{\mu}{2} x_{n,1} + \frac{h_i}{2} x_{n,3} + \frac{\mu \times h_i}{2} x_{n,1} x_{n,3} \quad (4)$$

Where n = 1..8 and $x_{n,1}$ take the value -1 or +1 according to the location in the matrix marked by the thick border in Table 1. (For $x_{6,1}$ the value is +1 and for $x_{4,3}$ it is –1.) This method is useful if the number of significant effects is small compared to the total number of trials.

THE EXPERIMENT

The experiment is fully described in [1]. The structure model is a 45º upward bending cone with a waterline diameter of 1.5 m and a top diameter of 0.33 m, it is depicted in Figure 2. The objective of the study was to perform small-scale experiments in an ice tank to investigate ice sheet forces. The results from the experiment are time series for the horizontal and vertical force. Statistical data from the time series are presented as standard deviation, mean and maximum of the force together with the frequency of the response. The results are compared with a plastic limit analysis. A selection of the model test data is shown in Table 2, and with a scale factor of 40 it would in full scale correspond to ice thickness of 2 to 2.3 m, flexural strength from 0.8 to 1.6 MPa and ice velocities of 0.13 to 0.63 m/s. The model ice sheets were grown by seeding and freezing a 1% urea-in-water solution with an ambient temperature of –18ºC. The authors state that the resulting ice was columnar with two layers, a finely grained transition layer of about 10% of the total thickness and a coarse grained columnar bottom layer. Flexural strength, measured for upward bending, and characteristic length of the ice sheets were measured before and after the tests, the unconfined compressive strength was measures at the end of the tests. During the program five ice sheets were used, the first was used to find out an appropriate pushing length, and based on that 4.5 m was chosen as the test length for the trials with the next four ice sheets. A selection of test results is listed in Table 2, for the input parameters the characteristic length is omitted and for the forces only the mean values are listed besides the frequency. The experimental data and more details on the experiment are published in [1]. In total 28 runs were made in the original test and the authors ran four tests on the first ice sheet and six experiments on each of the remaining four ice sheets. Four input factors were varied during the experiment; ice thickness, flexural strength, ice drift velocity and the ice-structure friction coefficient. The ice-structure friction coefficient was set to 0.1 at the low level and 0.5 at the high level, where the high value is extremely high. The ice drift velocities used was at three constant levels 20, 60, 100 mm/s. The ice thickness was varied between five somewhat constant levels but there is some variation within each ice sheet.

The flexural strength has values at nine different levels. In order to demonstrate a full factorial design of four factors at two constant levels the choice was made to assign the data to two levels, low and high. The authors conclude that good agreement was obtained between the experimental results and a plastic limit analysis. Further ice velocity had little effect on the test results for the smooth cone (low friction). For the case with high friction, ice forces decreased with higher velocity.

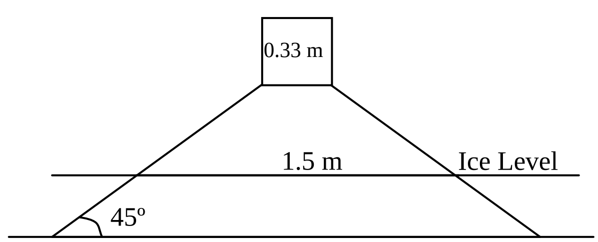

Figure 2: structure used in the experiment in [1].

SELECTED DATA

For the 2^4 design two constant levels for the four input parameters (flexural strength, ice thickness, ice-structure friction coefficient and velocity) are chosen. The factorial design approach has been applied to the experimental data without rescaling to full scale. In the data from [1] only two parameters have two constant levels, friction and ice velocity (where two out of three levels were selected). For the ice thickness and flexural strength the situation is more complex and in order to use the data the levels are simply just set to high (1) and low (–1) ignoring the variations at each high and low level. This is the second assumption which makes the results from the present study questionable other than as a demonstration of the methodology, the first being that the randomization is ignored. There are possibly ways to circumvent the variability within each high and low level but it is beyond the scope of this paper. Another consideration is that the test was run in blocks with six tests per ice sheet, this can

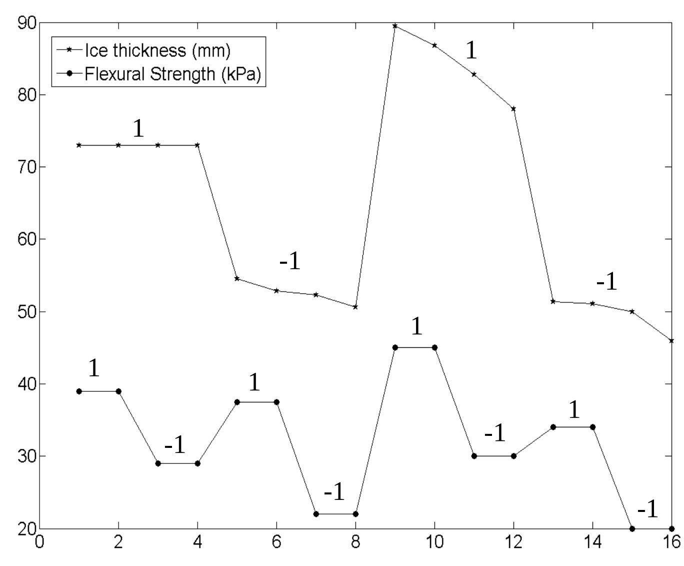

Figure 3: Manipulation of the test results from [1] by setting clearly unequal levels to high, +1, and low, –1, levels.

Table 2: A selection of the original test results from [1].*

Ice Thickness (mm)	Flexural Strength (kPa)	Velocity (mm/s)	Friction Coeff. (–)	Mean Horizontal Force (N)	Mean Vertical Force (N)	Frequency (Hz)
64	*28*	*20*	*0.1*	*442*	*411*	*0.092*
64	*28*	*20*	*0.1*	*428*	*683*	*0.11*
64	*28*	*100*	*0.1*	*411*	*707*	*0.43*
64	*28*	*100*	*0.1*	*423*	*755*	*0.41*
73	**39**	**20**	**0.1**	**717**	**1230**	**0.13**
73	*39*	*60*	*0.1*	*640*	*922*	*0.33*
73	**39**	**100**	**0.1**	**651**	**1180**	**0.33**
73	**29**	**20**	**0.1**	**507**	**782**	**0.14**
73	*29*	*60*	*0.1*	*547*	*913*	*0.39*
73	**29**	**100**	**0.1**	**511**	**903**	**0.48**
54.5	**37.5**	**20**	**0.1**	**307**	**549**	**0.15**
54	*37.5*	*60*	*0.1*	*321*	*578*	*0.27*
52.9	**37.5**	**100**	**0.1**	**345**	**603**	**0.31**
52.3	**22**	**20**	**0.1**	**290**	**514**	**0.097**
51.8	*22*	*60*	*0.1*	*294*	*517*	*0.37*
50.6	**22**	**100**	**0.1**	**294**	**530**	**0.33**
89.5	**45**	**20**	**0.5**	**2906**	**1424**	**0.087**
87.7	*45*	*60*	*0.5*	*2707*	*1538*	*0.156*
86.8	**45**	**100**	**0.5**	**2413**	**1505**	**0.31**
82.8	**30**	**20**	**0.5**	**2460**	**1086**	**0.068**
84	*30*	*60*	*0.5*	*2199*	*1154*	*0.23*
78	**30**	**100**	**0.5**	**2027**	**1173**	**0.47**
51.4	**34**	**20**	**0.5**	**936**	**497**	**0.058**
51.2	*34*	*60*	*0.5*	*885*	*566*	*0.23*
51.1	**34**	**100**	**0.5**	**914**	**641**	**0.43**
50	**20**	**20**	**0.5**	**893**	**489**	**0.078**
49.4	*20*	*60*	*0.5*	*727*	*611*	*0.25*
46	**20**	**100**	**0.5**	**820**	**534**	**0.45**

*The bold face data is selected for the factorial design analysis and out of 28 trials 16 are selected.

also be accounted for in the experimental design so that the main effects are not confounded with other effects. The second manipulation is to remove data which we do not need to make a full factorial design for four variables at two levels. In order to study all combinations of input variables, $4^2 = 16$ trials are needed, and consequently selected from the original data reducing it from the original 28 trials. The selected data is marked as bold in Table 2 and the high and low levels for flexural strength and ice thickness are illustrated in Figure 3.

RESULTS

By just looking at all the results from the original experiment in Figure 4 it can be seen that high velocity gives high values for the frequency of the load, and this seems to be the main contributor to the frequency of the force. This is related to the breaking length of the ice and if it was studied as an outcome of the experiment it would be influenced by the velocity. There is also evidence that there is an effect on the resulting force from increased ice thickness, flexural strength and friction after test number 16. So there is a strong possibility that all of these factors act together to give higher forces but it is more difficult to see which of these factors that have the strongest effect on the resulting force or if they act together. See Figure 4.

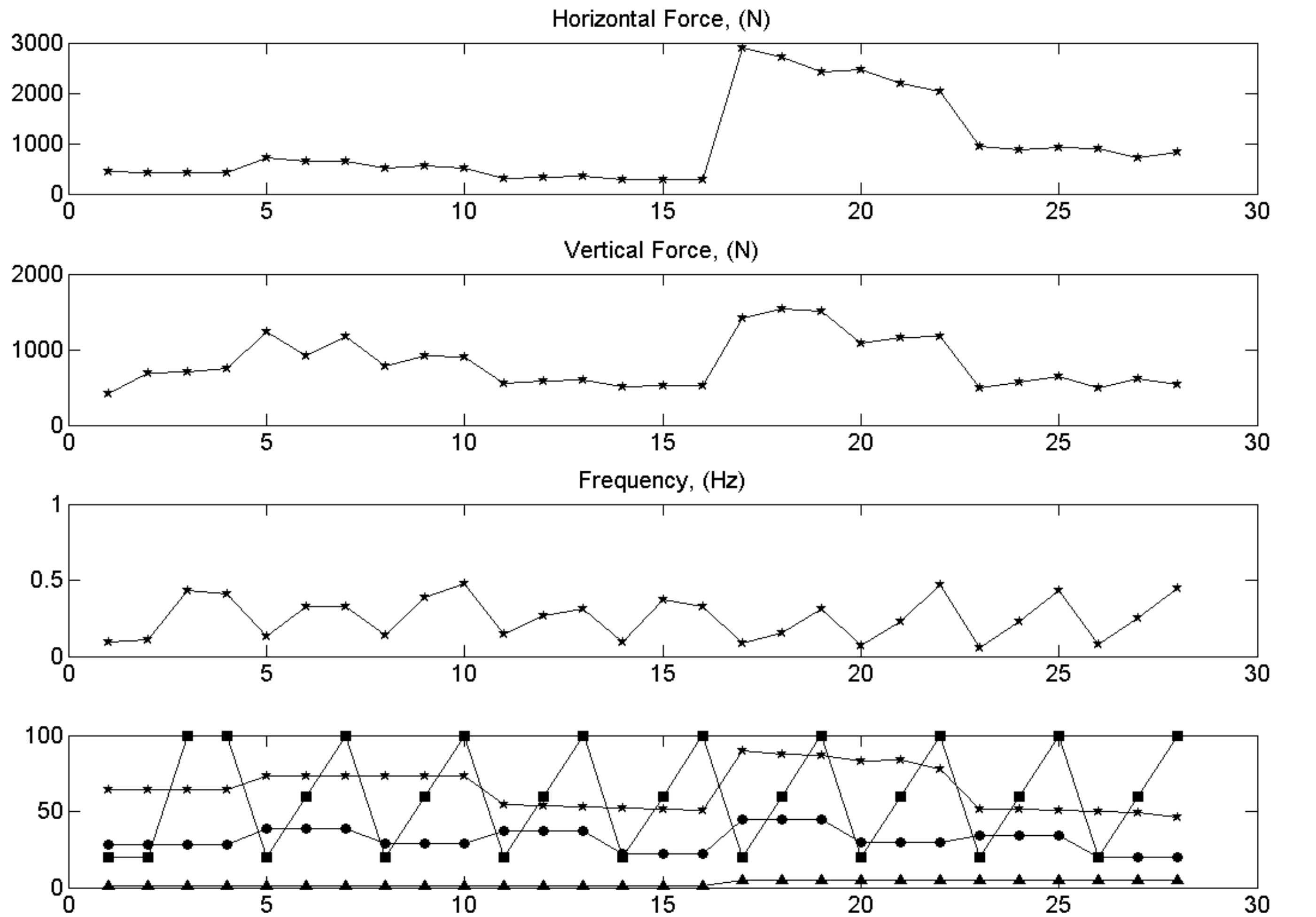

Figure 4: Plot of the all the experimental data with the input factors at the bottom. The numbers on the x-axis is the trial numbers in sequential order. Note the relationship between velocity and frequency, and the change in both horizontal and vertical force from trial 16 to 17.

Table 3: Calculated effects

Effects	Horizontal Force (N)	Vertical Force (N)	Frequency (Hz)
mean	1062	853	0.245
h_i	**924**	**616**	0.014
σ_f	173	202	-0.039
V	-130	62	**0.288**
μ	**1218**	132	-0.002
$h_i \times \sigma_f$	122	147	-0.037
$h_i \times V$	-117	-3	0.004
$h_i \times \mu$	**637**	141	-0.034
$\sigma_f \times V$	-6	-5	-0.049
$\sigma_f \times \mu$	69	-6	-0.007
$V \times \mu$	-125	27	0.055
$h_i \times \sigma_f \times V$	-27	-39	-0.031
$h_i \times \sigma_f \times \mu$	52	-8	0.012
$h_i \times V \times \mu$	-91	-3	-0.033
$\sigma_f \times V \times \mu$	3	28	0.004
$h_i \times \sigma_f \times V \times \mu$	-1	13	-0.014

MAIN AND INTERACTION EFFECTS

The results for main and interaction effects are listed in Table 3. The effect of a step in μ from 0.1 to 0.5 gives an increase of 1218 N for the horizontal force, the effect of a change in ice thickness from low to high is 924 N. If the effects are compared with the mean of each result, forces and frequency, it can be seen that some effects are of comparable size as the mean value, marked with bold in Table 3. These effects may have significant impact on the result. From Figure 4 it is clear that the frequency and velocity are closely related, this is confirmed in Table 3 were the velocity effect on frequency is much higher than all other effects. For the horizontal force ice thickness, friction and the interaction between them are important. The influence of ice thickness on the vertical force is also dominant and possibly there is an effect from the flexural strength. In the next section the t-distribution and the normal distribution probability plot will be used to try to distinguish the effects from noise.

ESTIMATION OF SIGNIFICANT EFFECTS

If the assumption is made that all interaction effects for three variables or higher are noise, the variance or noise can be estimated from these. The estimation of the variance is done by taking the sum of the squared the effects from three- and four-factor interactions and dividing by the degrees of freedom. The standard error is then the square root of the estimated variance. The effects can then be plotted together with a t-distribution formulated as in equation 5. Where $\nu = 5$ is the degrees of freedom, σ is the estimated standard error and $\mu = 0$ is the location of the center of the distribution along the x-axis. Duplication of the test for the same input variables can also give the standard error of the results, by analyzing the results for the duplicated tests.

$$f\left(x\middle|\sigma,\nu,\mu\right)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left[\frac{\nu+\left(\frac{x-\mu}{\sigma}\right)^{2}}{\nu}\right]^{-\left(\frac{\nu+1}{2}\right)} \tag{5}$$

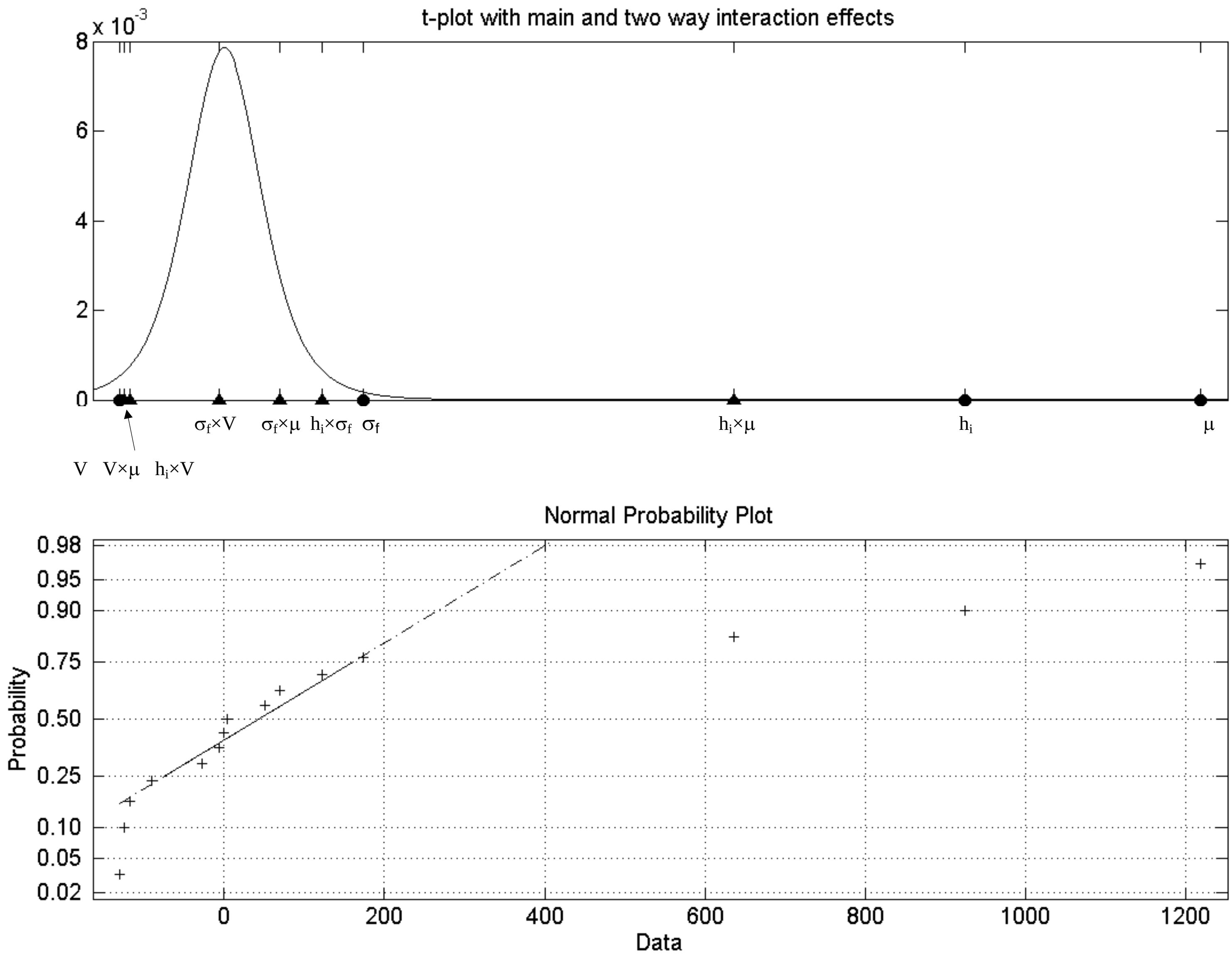

Figure 5: Horizontal force, t-distribution and normal probability plot of the effects. Clearly the main effects of friction and ice thickness and their interaction seem to be significant.

All effects that fall outside the body of the t-distribution can then be considered to be significant effects distinguishable from the noise. This can be confirmed or refuted by plotting all effects in a normal probability distribution plot. Plotting all effects, main and interaction effects, in a normal probability plot reveals outliers as deviations from a straight line in the plot. The distribution plots for the horizontal force can be seen in Figure 5, the significant effects are friction, level ice thickness and the interaction between them. Both distribution plots confirm the initial results in Table 3. For the vertical force the effect of ice thickness is clearly significant but the t-distribution plot suggests that the main effect from flexural strength also is important. Further there is a cluster consisting of the main effect of friction and the interaction effects $h_i \times \mu$ and $h_i \times \sigma_f$, at the tail of the t-distribution. It is of value to note that the center of the t-distribution was placed at zero and that this is not so for the normal distribution plot. this may cause some effects to look like outliers in the t-distribution plot and not in the normal probability plot. For the vertical force and the effect from flexural strength is has to be said that the flexural strength in this analysis is tainted by a lot of noise as the test data was manipulated, and set to a high and low level. It is known that flexural strength has a real effect on the vertical force, but from the results in Figure 6 nothing conclusive can be said, the noise in the selected levels for the experiments is probably too large. For the frequency response there is a clear effect from velocity.

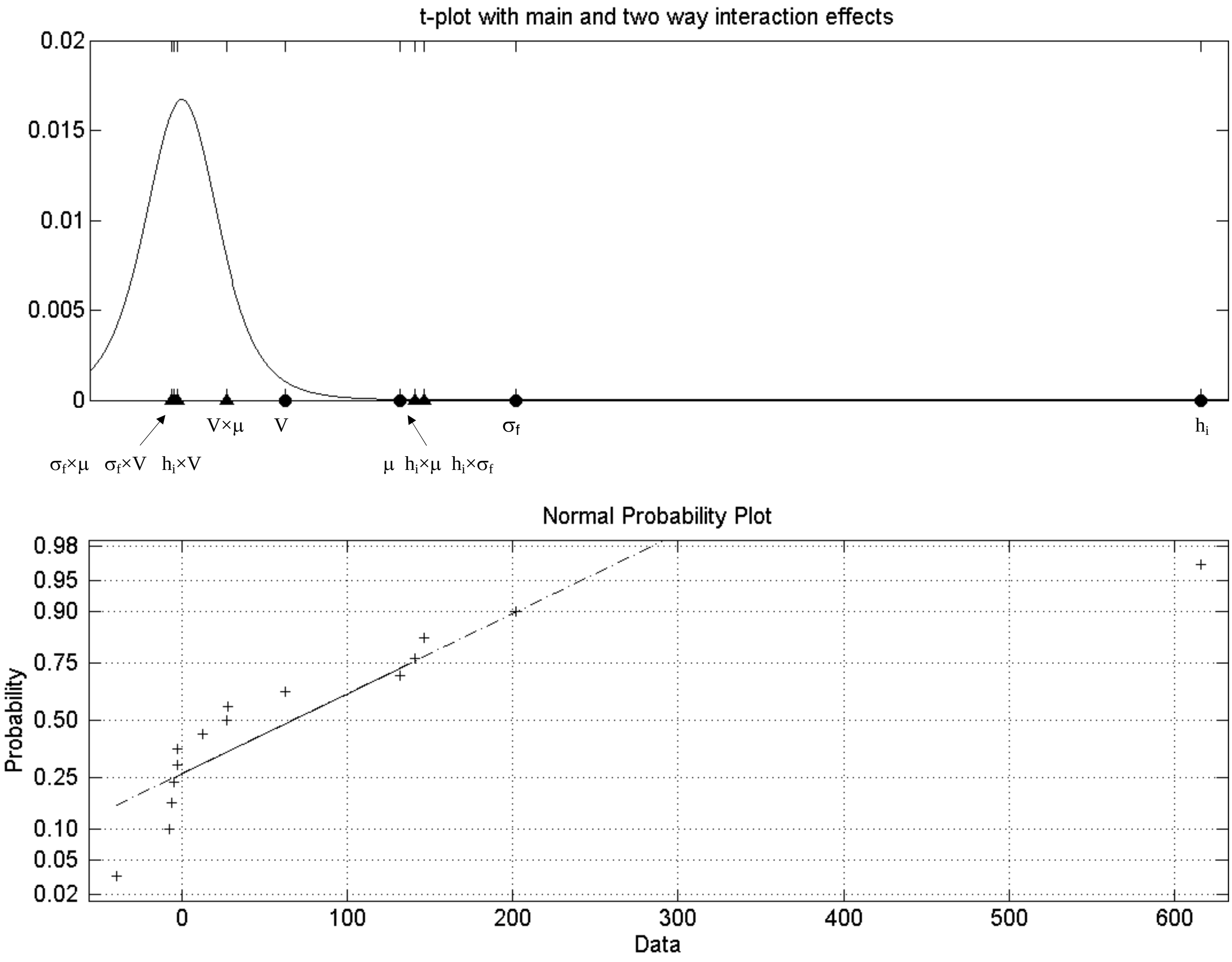

Figure 6: Vertical force, t-distribution and normal probability plot of the effects. The conclusion is that the effect of ice thickness is significant, and that there is a possibility that flexural strength is also significant.

RESIDUALS

Plotting the residuals is a diagnostic check for the conclusions drawn from the t- and normal-distribution plots. If the residuals fall nicely on a straight line in a normal probability plot the conclusions are most likely correct. In Figure 8 the solid lines and stars represent the initial conclusion about the results above and the dot-dashed lines the inclusion of more effects as significant in the calculation of the residuals. Deviations from the normal probability distribution for the residuals can indicate that the experiment was disturbed by some unmeasured effect, it was not set up properly or that there is not enough contrast between the levels of the factors. The following is a discussion of the residuals and the results.

Horizontal Force

For the horizontal force the main effects for ice thickness, friction and their interaction were considered significant. In the first plot in Figure 8 the residuals for that case are plotted (solid line and stars) and it can be seen that there are two values that distinctly deviate from the normal distribution. The inclusion of more effects as significant does not improve the result and the residuals are now positioned further away from the zero mean. For this issue to be resolved the experiment will have to be run again with randomization and two constant levels of flexural strength and ice thickness.

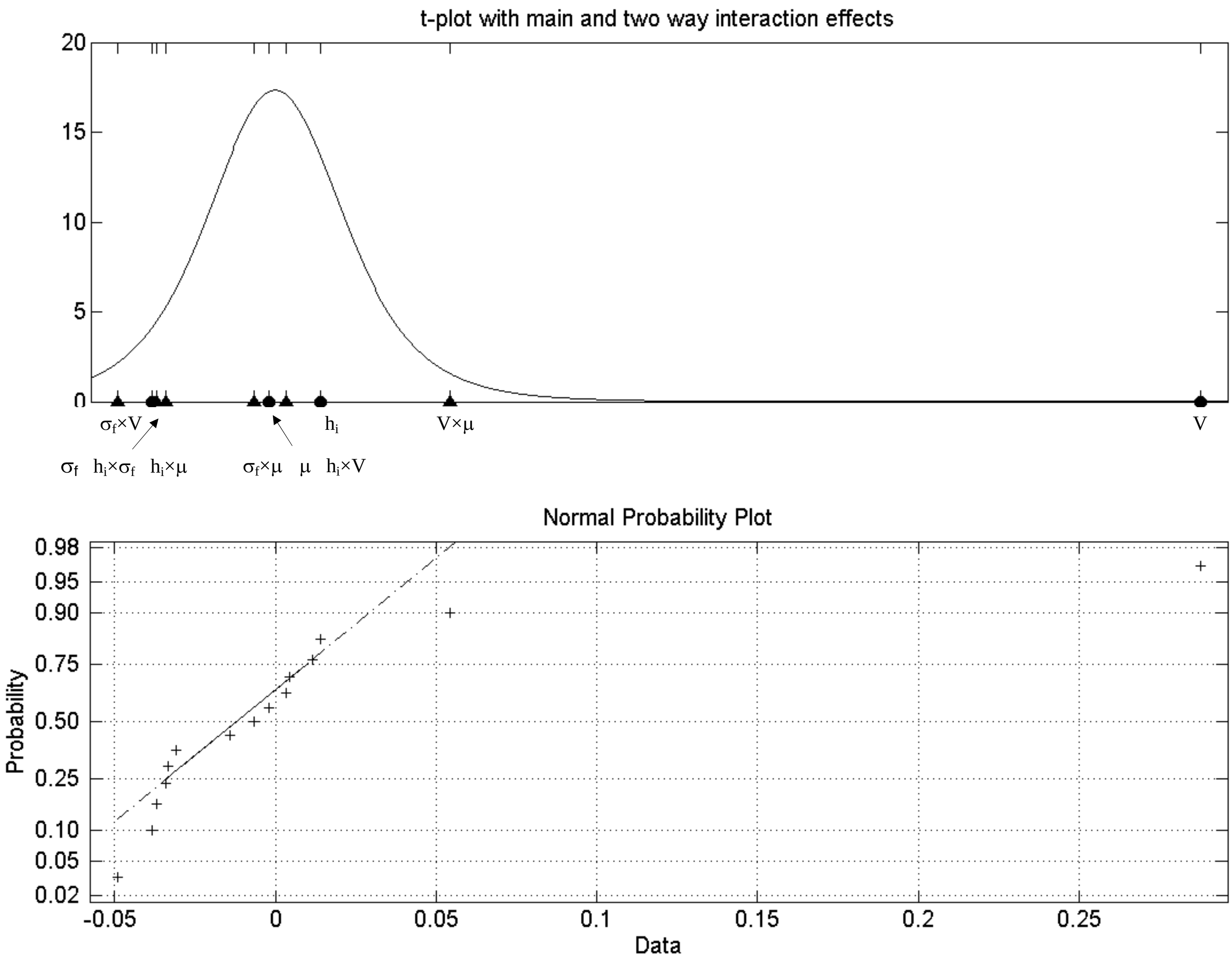

Figure 7: Frequency, t-distribution and normal probability plot of the effects. Velocity stands out as a clearly significant effect.

experiment at two constant levels of flexural strength.

Vertical force

For the vertical force the initial thought is that the ice thickness is certainly significant and then there was some doubt about the flexural strength. The solid line and stars in plot two in Figure 8 show the residuals for ice thickness as the only significant effect. For this case there are four residuals deviating from the normal probability distribution. However if the main effect for flexural strength is included the situation improves (dot-dash line and dots) and the suspicion that the flexural strength is important is confirmed. But the improvement is not that big so it would be good to repeat the experiment at two constant levels of flexural strength.

Frequency

It can be seen in the first figure of the results that there is a strong correlation between velocity and frequency of the load, see Figure 4. In the distribution plots of the effects this is confirmed, but studying the normal distribution plot in Figure 7 it can be suspected that the flexural strength and the interaction effect of velocity and flexural strength also have some impact on the results. Further the interaction effect between velocity and friction also deviates from the normal distribution. A residual plot where only velocity is considered as a significant (solid line and stars) in plot three in Figure 8, gives good result

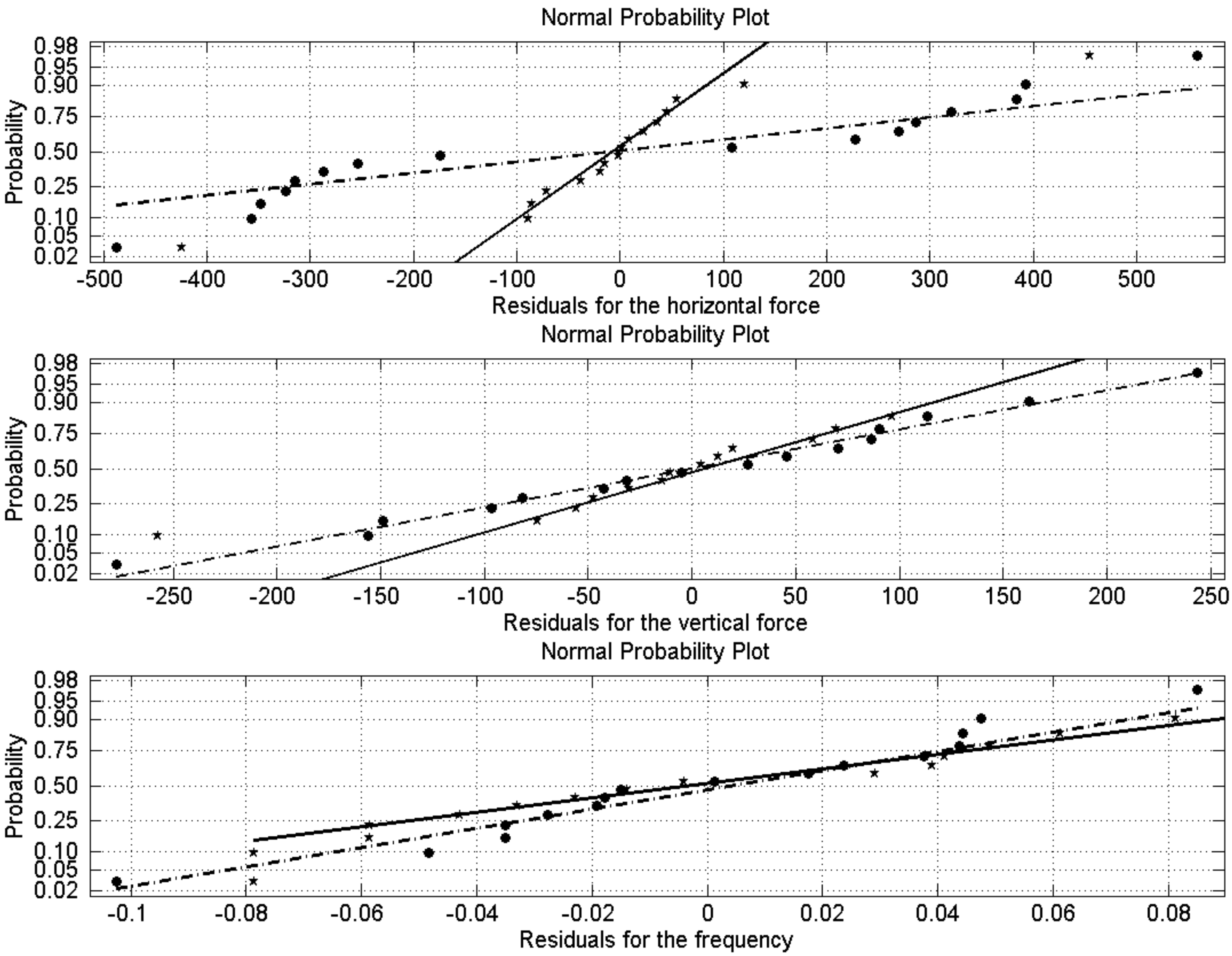

Figure 8: Plot of residuals as a diagnostic check of the conclusions drawn from the t-distribution and normal probability plots above. Stars represent the residuals for the first estimation of significant effects and dots a second estimation made as a comparison.

with almost all residuals on the line in the normal probability plot. The inclusion of flexural strength and the interaction effect of velocity and flexural strength as significant effects (dot-dash line and dots) give little change to the residual plot. There is a possibility that flexural strength is important for the prediction of frequency as well as the velocity, but it is not clear from this analysis.

DISCUSSION

If the original experiment by Sodhi [1] had been set up as a factorial design from the beginning, what could we have seen from this analysis? We would know that for these levels of input factors the effect on the horizontal force comes from ice thickness, ice structure friction and their interaction, we would also know that the residuals are not fully normal distributed so that there is something affecting the results of the experiment. For the vertical force we can conclude that the effect of ice thickness is strong and that there is a strong possibility that flexural strength is important too. For further and more detailed studies it would make sense to select these variables as factors in a new factorial experimental design. The velocity does not seem to be that important, at least not for this range of input levels and this experiment. The frequency of the load seems to be almost completely determined by the velocity. However, there are indications that the flexural strength and the interaction effect of velocity and flexural strength have some impact on the results. Future studies can be designed to resolve these effects. The breaking length of the ice is sometimes used as input to some models for calculating the load, but it is a result of the input – velocity, ice mechanical properties, friction and the structure design. If the purpose is to build a mathematical model of the frequency of the load, the velocity would be an important factor to consider, but as the relationship is probably not linear an experimental design has to be done for more than two levels of velocity. Looking at the data in Table 2 it can be seen that the increase in frequency with increasing velocity becomes weaker for higher velocities.

It should also be noted that this is only a model test in a test tank and that the result will have to be compared with and confirmed by full scale measurements. The ice used in this experiment is probably quite homogenous and has not been affected by the dynamic motion that influences real sea ice. It is very possible that in real conditions other factors such as cracks and varying thickness or local weaknesses are of high importance.

Possibilities

There are many possibilities with a good experimental set up and design of experiments. It is possible to map important parameters with relatively little work, in this case 16 instead of 28 trials. Fractional experimental designs can be used initially to explore the problem and decide which factors are relevant for further studies. The methodology can be used both for experiments conducted to gain a fundamental understanding of the physics of structure and ice interaction, and as a tool when performing experiments to explore structure designs. For the latter the slope or other structure design considerations can be used as factors in the experimental design.

Drawbacks

It is important to set up the experiment so that unwanted and unmeasured effects do not taint the results. A way to conduct the experiment is to randomize the tests to avoid noise from unwanted and or unmeasured effects, *e.g.* temperature (which highly affects flexural strength). However, the randomization might make the test more time consuming, changing the set up of a test can take a lot of effort. There is also the possibility that in an experiment important factors of study are omitted, or not a part of the design of the experiment. The design used in this study does not capture non-linear behavior between the high and low levels of the experiment, in order to do that a design with a central point has to be used.

Future research

It would be very interesting to apply this methodology from the start of the experimental design and to actually look at properly randomized experiments with only two set input levels, were noise from unmeasured effects on the result has been evened out over the experiment. Another interesting application would be to try to use the method to map important parameters in order to focus on measuring the parameters that will have the most effect on the load, reducing the time measuring parameters that have little effect on the load or which effects are drowned in the noise. Locating the factors of significance for the load might enable a reduction of input data to two-dimensional space instead of four or more factors. This can be utilized to find out which factors to spent time on measuring in the field or future experiments. If the effect of some factors is drowned in noise, or is not significant for the load for some conditions, perhaps it is better to spend efforts elsewhere.

Another experimental approach is to look at response surfaces to capture nonlinear behavior in the experiment. There is also the possibility to fit analytical methods for calculating the load to the experimental data, or constructing empirical models from the test results.

Another application is to combine the factorial design approach with computationally heavy methods to reduce the effort spent and to localize areas of interest. This will be an experiment in a numerical test tank instead.

REFERENCES

[1] Sodhi D. S.; Morris C. F.; Cox G. F. N., 1985, "Sheet Ice Forces on a Conical Structure: An Experimental Study", p. 643-655, *Proc. POAC 85: The 8th International Conference on Port and Ocean Engineering under Arctic Conditions*, Horsholm, Denmark.

[2] Box G. E. P., Hunter W. G., Hunter J. S., 1978, *"Statistics for Experimenters: An introduction to Design, Data Analysis, and Model Building"*, Chapter 10, John Wiley & Sons, ISBN 0-471-09315-7