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A New Approach to Improve Material Models[1]

The inelastic behavior of materials is described most efficiently by unified models when their material functions are determined so that flow, hardening, creep etc. will be covered correctly. In this paper, the adaptation of a model is not confined to finding the optimal material parameters but is extended to the identification of the optimal shape of the material functions itself. Material functions given by series of simple shape functions defined in discrete sections which merge smoothly together lead to the best adaptation to experimental results. Furthermore, any remaining shortcomings of the model reveal deficiencies in the modelling of the microphysics of the material. Then by careful interpretation of the uncovered physical properties the original material model has to be amended leading to the derivation of even entirely new models. Thus, a powerful tool is presented here by which a unified model can be checked and improved.

1 Introduction

For the mathematical description of nonlinear material behavior, such as plasticity (time-independent), creep (time-dependent), and strain-rate sensitivity, material models (constitutive equations) have been proposed which treat time-dependent and time-independent strains uniformly as inelastic strains. The models vary in their physical assumptions as well as in their mathematical formulations of the material functions. In the case of physical assumptions, the models e.g., proposed by Bruhns (1988), Chaboche (1983), Nouailhas (1987), and Robinson (1985) assume that the inelastic strain rate depends on the overstress. The overstress is defined in the stress space as the distance between the total stress and the yield surface. The development of the yield surface is determined by internal variables which describe kinematic and isotropic hardening. For the mathematical formulations of the material functions, Bruhns and Chaboche propose terms in potential forms for the overstress only to define the inelastic strain-rate. Nouailhas and Robinson add a dependence on the isotropic hardening. Nouailhas introduces an exponential function into the equation for inelastic strain rates as a viscoplastic limit.

The parameters of a unified model are material constants. Therefore, these parameters have to be determined from experimental results by an adaptation procedure, e.g., by the evolution strategy (Braasch, 1993; Müller, 1989). However, even for very refined adaptations, often the remaining discrepancies between experiments and mathematical descriptions are not negligible. This is an indication that the chosen material model does not cover the physical behavior correctly. Consequently, the basic assumptions of the applied material model have to be modified or extended.

The adaptation and the improvement of a material model, which provides a satisfying fit to experimental results, can be achieved in three major steps:

- The constant value parameters of an a priori chosen model are determined by a minimum error algorithm in comparison to experimental results. At present, most applications of unified models do not go beyond this first step.
- A better fit is achieved by extending the adaptation to the mathematical functionals which express the dependences of the specific features of the chosen material model, such e.g., as sinh- or exp-functions.
- Then, if there are still non-negligible discrepancies between theory and experiment, alterations of the basic physical assumptions are necessary arriving so at an improved mathematical formulation of the model.

In this paper, a new concept is proposed for improving material models in all three steps given above. A main feature is the replacement of closed-form material functions by series of shape functions defined in discrete sections. The parameters of these shape functions comply with restrictions so that they merge with the adjacent functions without gaps or bends. By increasing the number of discrete sections, the material functions converge to the form which leads to the best performance of the a priori chosen model. If a further increase in the number of discrete sections does not result in a better adaptation to experimental results, then the best ever achievable shapes of the material functions are found. If the results of the model and the experimental data are sufficiently close, then the series of shape functions may be replaced by an appropriate closed form material function again. If, however, the discrepancies between model and experiments are not tolerable, then the physical assumptions of the model need amendments, e.g., more interactions between the internal variables and the inelastic strain rate have to be considered. An example how improvements of the physical assumptions lead to better results is given in Sections 5.3 and 5.4, a full length report is given by Braasch (1992).

[1] This article is dedicated to Professor F. G. Kollmann, Darmstadt, FRG, for the occasion of his 60th birthday.

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Contributed by the Materials Division for publication in the *Journal of Engineering Materials and Technology*. Manuscript received by the Materials Division January 25, 1993. Associate Technical Editor: P. Fallanshee.

Table 1 Summary of the experiment data for aluminum at $T=550$ K

	Tension test			Creep test		
Al 550-	Strain rate (s^{-1})	Strain (%)	Stress (MPa)	Stress (MPa)	Strain rate (s^{-1})	Strain (%)
07	$2\cdot10^{-3}$	0.718	7.01	7.01	$1.4\cdot10^{-5}$	20.72
				7.73	$2.2\cdot10^{-5}$	30.82
				8.39	$3.5\cdot10^{-5}$	45.72
09	$2\cdot10^{-3}$	1.885	8.87	8.87	$4.6\cdot10^{-5}$	22.98
				9.50	$6.8\cdot10^{-5}$	26.88
				10.10	$9.8\cdot10^{-5}$	31.88
				10.80	$1.4\cdot10^{-4}$	49.78
11	$2\cdot10^{-3}$	5.410	11.40	11.40	$1.5\cdot10^{-4}$	25.51
				12.00	$2.2\cdot10^{-4}$	35.41
				12.70	$3.4\cdot10^{-4}$	45.34
ZS	$2\cdot10^{-5}$	9.99	7.12			
	$2\cdot10^{-4}$	17.50	11.10			
	$2\cdot10^{-3}$	36.30	16.20			

2 Experimental Data

The proposed procedure is exemplarily demonstrated for the adaptation of an over-stress model to aluminum which exhibits creep under elevated temperatures. Here the material properties at a constant temperature of 550 K are considered. The experimental data of the following example were provided by Estrin (1990). A summary of the data is presented in Table 1. The tests Al 55007, Al 55009, and Al 55011 are used to evaluate the parameters of the model of Chaboche (1983). These tests are combinations of strain controlled loading phases and subsequent series of creep phases. The stresses are held constant until stationary strain rates are observed, then the stresses are increased.

For example, the test Al 55009 (Fig. 1) consists of one tension phase and four subsequent creep phases. First a constant strain rate of $\dot{\epsilon}=2\cdot10^{-3}\,s^{-1}$ is applied. At a strain of $\epsilon=1.885$ percent the stress of $\sigma=8.87$ MPa is observed. Then this stress is held constant until a stationary strain rate, $\dot{\epsilon}=4.6\cdot10^{-5}\,s^{-1}$, is observed. The total strain is $\epsilon=22.98$ percent at this point. Then the stress is increased to $\sigma=9.50$ MPa, followed once more by a creep phase until secondary creep is established, and so on.

The test AL 550ZS (Fig. 2) is a tension test with changing strain rates. The strain rates are increased from $\dot{\epsilon}=2\cdot10^{-5}\,s^{-1}$ in the first phase to $\dot{\epsilon}=2\cdot10^{-3}\,s^{-1}$ in the third phase. This test is not used to evaluate the parameters of the unified model. The purpose of these additional data is to investigate the capacity of the model and of the so far determined parameters to cover also those experiments which have not been used in the adaptation procedure.

3 The Constitutive Model Chosen for the Example

The constitutive equations taken as an example for this paper are those proposed by Chaboche (1983). The original model is based on the assumptions:

1. Inelastic strain rates occur only if the stresses are outside a yield surface. The inelastic strain rates depend on the overstress.
2. The development of the yield surface depends on the inelastic strain, however, not on the inelastic strain rate.

This leads to the following constitutive equations:

$$\dot{\epsilon}_{ij}^{in}=\frac{3}{2}\dot{p}\frac{s_{ij}-a_{ij}}{\sqrt{3I_2'}}, \quad (1)$$

$$\dot{a}_{ij}=h(\alpha)\dot{\epsilon}_{ij}^{in}-r(p)\alpha_{ij}, \quad (2)$$

$$\dot{K}=\Gamma(K)\dot{p}. \quad (3)$$

where

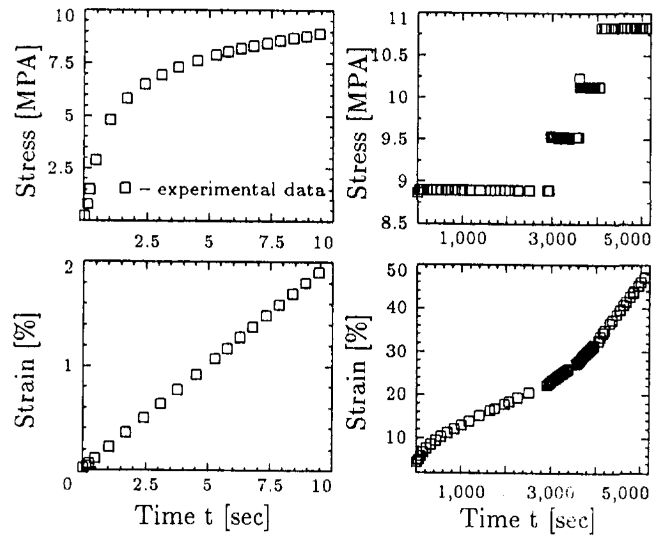

Fig. 1 Test Al 55009 as a combination of tension and creep phases

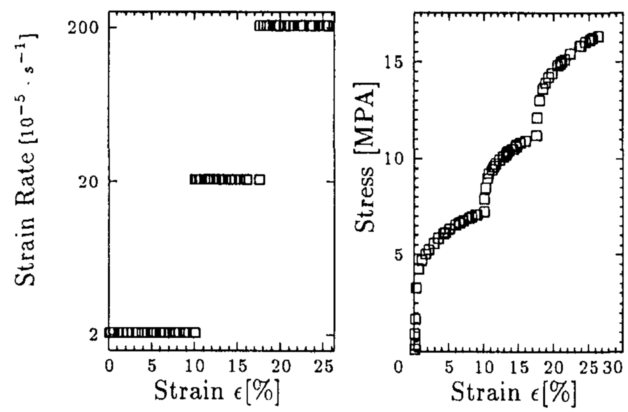

Fig. 2 Tensile test Al 550ZS with changing strain rates

$$I_2'=\frac{1}{2}(s_{ij}-a_{ij})(s_{ij}-a_{ij}), \quad (4)$$

$$\sigma_{ex}=\sqrt{3I_2'}-(K+k), \quad (5)$$

$$\langle x\rangle=\begin{cases}x & \text{if } x>0\\ 0 & \text{if } x\leq0,\end{cases} \quad (6)$$

with the material functions:

$$\dot{p}=\left\langle\frac{\sigma_{ex}}{D}\right\rangle^n, \quad (7)$$

$$h(\alpha)=C\frac{2}{3}a_\infty, \quad (8)$$

$$r(p)=Cp, \quad (9)$$

$$\Gamma(K)=b(Q-K). \quad (10)$$

D, n, k, C, a_∞, b, Q are material constants. s_{ij} is the deviatoric stress tensor. The state variable a_{ij} is the tensor of the back stress and governs kinematic hardening. The state variable K is a measure of isotropic hardening. Both, a_{ij} and K, follow a quasi-static theory.

The parameters of the model have to be adapted to experimental data. For this, methods applying evolution strategies have proven to be very efficient (Braasch, 1993; Müller, 1989).

4 Outline of the Proposed Concept

Based on physical assumptions, a unified model defines the material properties such as hardening or viscoplastic flow by material functions and by the internal variables of the model. For improvements or further developments of a chosen model, it is useful to assign the differences between theoretical and experimental results either to the shape of the material functions only or to the basic physical assumptions of the constitutive model.

In this paper, a concept is developed and verified by which both the material functions, as well as the constitutive model itself, can be improved to satisfy experimental results. It is proposed to compose the material functions by a series of simple shape functions defined in adjacent sections. Special constraints for the parameters of these shape functions ensure the continuity of the material function and of its derivative at the borders of each section. By increasing the number of discrete sections and by applying an adaptation algorithm for the numerical evaluation of the parameters, the material functions defined by this way converge to the best fit possible by the choice of these material functions. Having refrained so far from any modification of the basic physical assumptions of the model, remaining discrepancies are associated with the physical fundamentals of the model. Hence, in this case the constitutive equations themselves have to be modified.

For the verification and the improvement of material models the following procedure is proposed:

1. For selecting a suitable unified model out of the possible alternatives one has to reflect first on the basic assumptions of the model, whether the main features of the experimental material behavior are included, and what types of material functionals and internal variables are assumed.
2. Then, for a specific set of constitutive equations, the material functions are replaced by simple shape functions defined in discrete sections, where the number of sections is still open for an adaptation procedure. This yields the most general form of the chosen material functionals.
3. The parameters of the model are evaluated by applying, e.g., the evolution strategy (Braasch, 1993; Müller, 1989).
4. With the parameters determined, the stress-strain-time paths are computed for those experiments which have been used for the parameter evaluation. An even stronger test is performed by also including those experimental data which have not been used so far for the adaptation procedure.
5. If the comparison with the experimental data shows that the model does not cover the observed material behavior adequately, then the basic assumptions of the model have to be modified. General rules for this cannot be given. However, careful interpretation of the worse covered parts of the experiments may give information for the necessary corrections.
6. For the model with modified physical assumptions, discretization, parameter evaluation, and comparison with experimental results are repeated, as in steps 2 to 4 until a satisfying coincidence between computed and experimental results is achieved.
7. For practical applications and the reduction of numerical work, it may be advisable to return to material functions in closed forms by replacing adequately the shape functions defined in discrete sections.

5 Application of the Procedure to a Chaboche Material Model adapted to Aluminum Tests

The procedure given in Section 4 is applied for the adaptation of the constitutive model as proposed by Chaboche (1993), Section 3, to aluminum tests, given in Section 2.

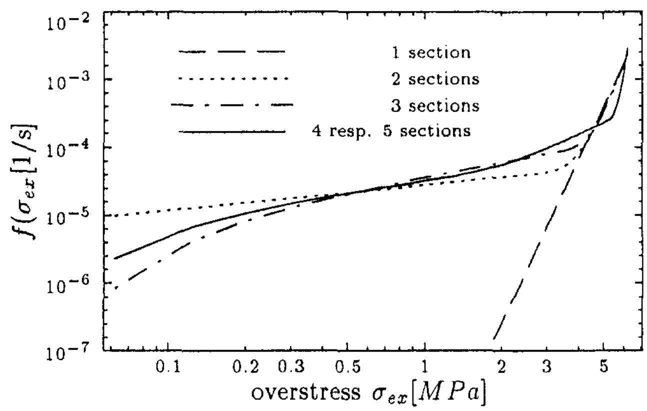

Fig. 3 Material functions with different numbers of discrete sections

5.1 Discretization of the Material Functions. In the original formulation of the Chaboche model, Eqs. (1) to (9), the viscoplastic strain rate is assumed to follow a closed-form function of an n-potential, Eq. (7), valid for the entire region of stresses and strains. Without changing the basic physical assumptions of this model the n-potential is replaced by a subdividing series of discrete sections of simple shape functions. The viscoplastic strain rate becomes:

$$\dot{p} = f(\sigma_{ex}) \tag{11}$$

$$f(\sigma_{ex}) = a_i \sigma_{ex}^{n_i} + b_i \tag{12}$$

$$\text{for } \sigma_{i-1} < \sigma_{ex} \le \sigma_i \tag{13}$$

$$\text{with } \sigma_0 = 0, \tag{14}$$

$$\text{where } a_i = \frac{n_{i-1}}{n_i} a_{i-1} \sigma_{ex}^{n_{i-1} - n_i}, \quad a_1 = D^{n_1}, \tag{15}$$

$$b_i = b_{i-1} + \left(1 - \frac{n_{i-1}}{n_i}\right) a_i \sigma_{ex}^{n_i}, \tag{16}$$

$$b_1 = 0. \tag{17}$$

Through this approach the number of the material parameters is extended, because the boundaries σ_i of the discrete sections and the exponents n_i of the shape functions are now additional parameters. The restriction $n_i > 0$ yields monotonously increasing functions. If Eq. (12) is defined for only one section, the new form of material function for viscoplastic strain rate is identical to the original one. By defining a series of sections, the new form has the capacity to approximate every possible material function such as sinh or exp as accurately as ever wanted. For actual computations, only a few sections are required to achieve convergence to an optimal shape of the material function. An example is shown in Fig. 3 where the convergence toward the best form of the functional is given for a successive subdivision into 1 to 5 sections. In this case, it is sufficient to subdivide the functional into four sections. Beyond that no improvements are achieved.

5.2 Parameter Adaptation. The parameters of the constitutive model are evaluated from the tests Al 55007, Al 55009, and AL 55011 simultaneously by applying the evolution strategy and by minimizing weighted absolute values of the differences between model and experimental data. This is done for both models: the original Chaboche model with closed-form functionals as given by Eqs. (1) to (9) and for the discrete form as in Eqs. (10) to (13).

Figures 4 to 6 show the computed results for both models in comparison with the experimental data of the tests. In these and all the following figures the same symbols for the line drawings are used as shown in the Fig. 3. The results obtained

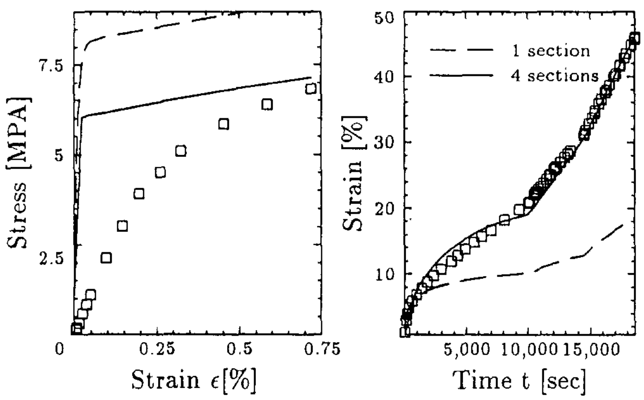

Fig. 4 Computed results and test Al 55007

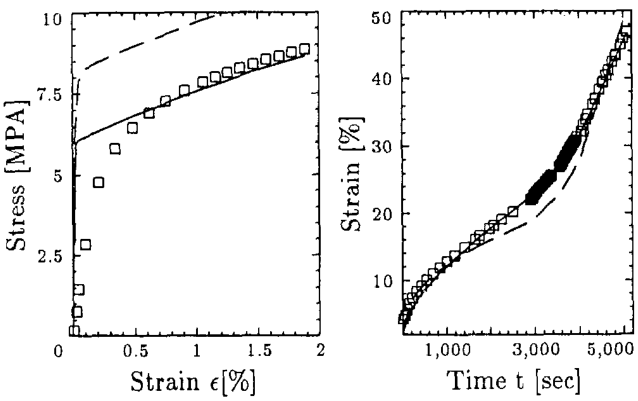

Fig. 5 Computed results and test Al 55009

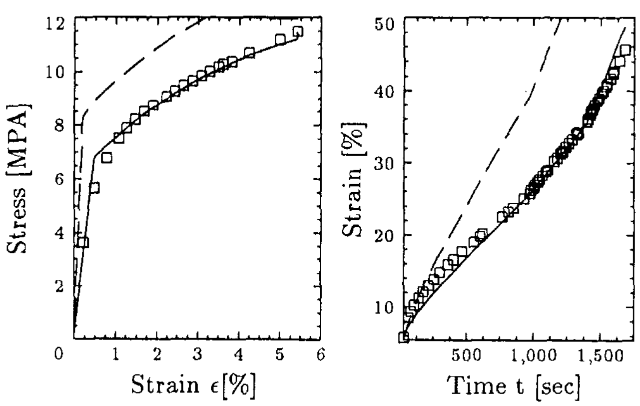

Fig. 6 Computed results and test Al 55011

for the closed-form material functions of Eq. (7)—corresponding to the curves for 1 section—deviate too far for being acceptable, although optimal parameters have been evaluated.

The new approach, which replaces the material functions by a series of shape functions, Eq. (12), improves the model considerably. The predictions fit very well with the experimental data in the creep phases. However, the σ-ϵ-curves computed even by the better model of discrete functions show sharp bends (Figs. 4 and 5) not observed by the experimental data. In spite of the achieved optimal shape of the material function $\dot{\epsilon}^{in}=f(\sigma_{ex})$, the stress-strain-curves still show unacceptable differences between model predictions and experimental data. Since the optimal shape of the material function has already been achieved, curve fitting would not correct the model. One has to reconsider the physical dependences between the inelastic strain-rate and the internal variables of the model.

5.3 Modification of Original Chaboche Model. The model considered in Sections 3 and 5.1 predicts a fast development of the overstresses in the tension phases. This is why sharp bends for the σ-ϵ-curves are computed. In order to obtain a model with a slower increase of the overstresses, the first assumption of the model (Section 3) is altered to:

1a. The inelastic strain rates should depend on the overstress and additionally also on isotropic hardening.

According to the second step in Section 4, the material function for the inelastic strain rate is modified by substituting it by two series of shape functions f and g:

$$\dot{p}=f(\sigma_{ex})\cdot g(K) \tag{18}$$

$$g(K)=c_j\left(\frac{K}{Q}\right)^{l_j}+d_j \tag{19}$$

$$\text{for } K_{j-1}<K\leq K_j \tag{20}$$

$$K_j=\left(\frac{j}{J}\right)^2 Q \tag{21}$$

$$c_j=\frac{l_{j-1}}{l_j}c_{j-1}K_j^{l_j-l_{j-1}},\quad c_1=1 \tag{22}$$

$$d_j=d_{j-1}+\left(1-\frac{l_{j-1}}{l_j}\right)c_jK_j^{-1},\quad d_1=0 \tag{23}$$

where $f(\sigma_{ex})$ is unchanged. The l_j are parameters such as the n_i of the shape functions for $f(\sigma_{ex})$. The entire range of K, $(0\ldots Q)$, is subdivided into J sections. The size of the sections increases quadraticly.

Two different approaches are investigated. For one of them all material functions are kept in closed form. For the other one the discretization is applied by subdividing $f(\sigma_{ex})$ into four and $g(K)$ into two sections. This is needed to achieve sufficient convergence to the optimal shape of these material functions. When the loading enters the region where first inelastic strain rates occur, then there are only small values of the isotropic hardening given by the variable K. Therefore, the functional $g(K)$ is large which is equivalent to small values of the overstress:

$$\log f(\sigma_{ex})=\log\dot{\epsilon}^{in}-\log g(K) \tag{24}$$

When for further loading the hardening variable K increases gradually, then the values of the overstress are increasing gradually, too, i.e., the sharp bends for low strains in Figs. 4 to 6 disappear. Yet, out of all alternatives, only the model of optimal material functions found by the series of shape functions describes the creep phases observed in the tests appropriately (Figs. 7–9).

The so far developed expressions for the model are now tested with respect to additional experimental data from other types of loading history which did not enter the parametric adaptation. The results are shown in Fig. 10. The numerical evaluation corresponding to the ZS-Test is not in agreement with the experimental results. Therefore, the basic physical assumptions have to be improved further on.

5.4 Second Modification of the Material Model. For a second improvement of the physical properties the assumption 2 (Section 3) is additionally modified to:

2a. Hardening depends on inelastic strain as well as on inelastic strain rate.

Hereby, the assumption of a quasi static hardening behavior is no longer preserved. The 2a assumption is realized by adding a material function for static recovery onto the equation of evolution of the kinematic hardening variable α:

$$\dot{\alpha}_{ij}=h(\alpha)\dot{\epsilon}_{ij}^{in}-(r(p)+r_s(\alpha))\alpha_{ij} \tag{25}$$

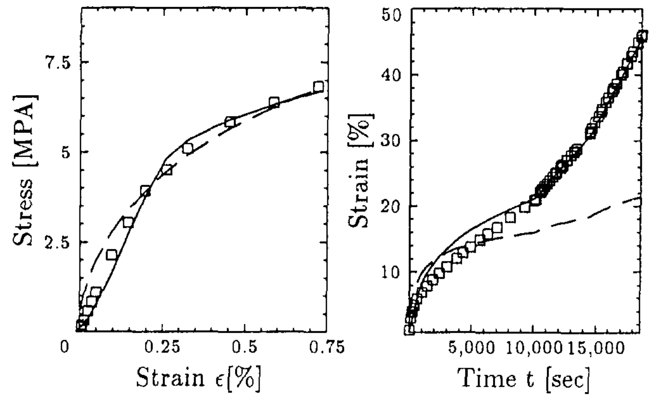

Fig. 7 Computed results and test Al 55007

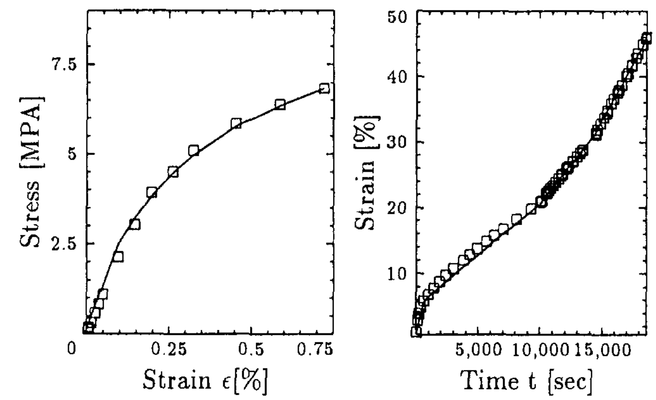

Fig. 11 Computed results and test Al 55007

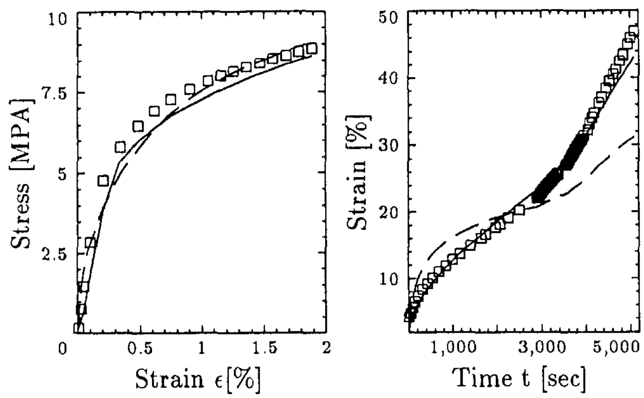

Fig. 8 Computed results and test Al 55009

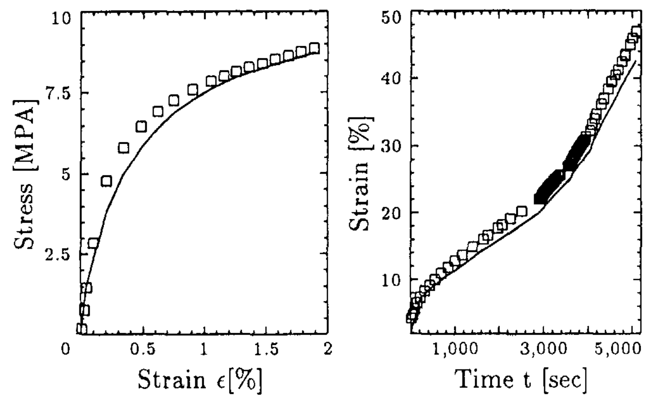

Fig. 12 Computed results and test Al 55009

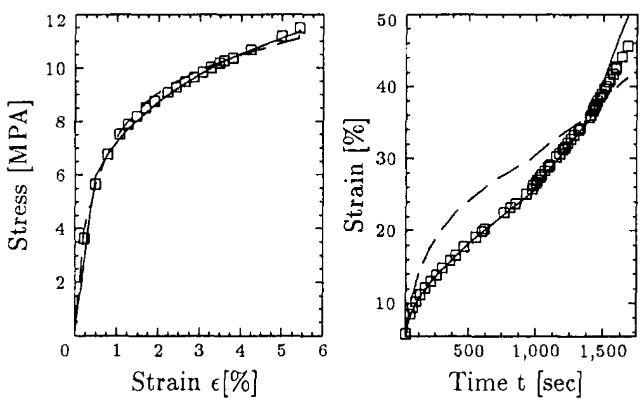

Fig. 9 Computed results and test Al 55011

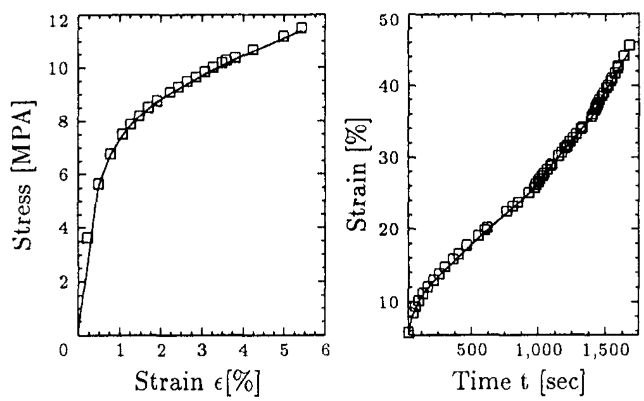

Fig. 13 Computed results and test Al 55011

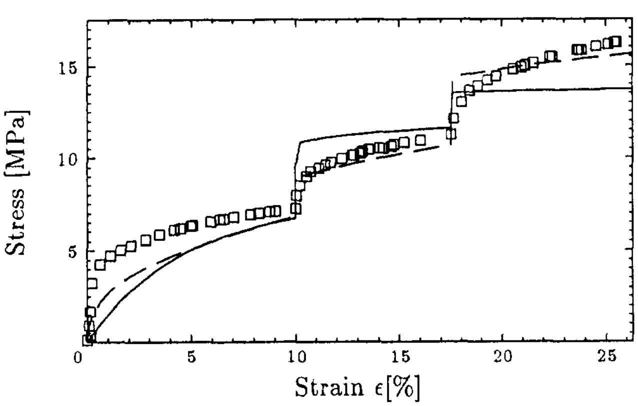

Fig. 10 Prediction of test Al 550ZS

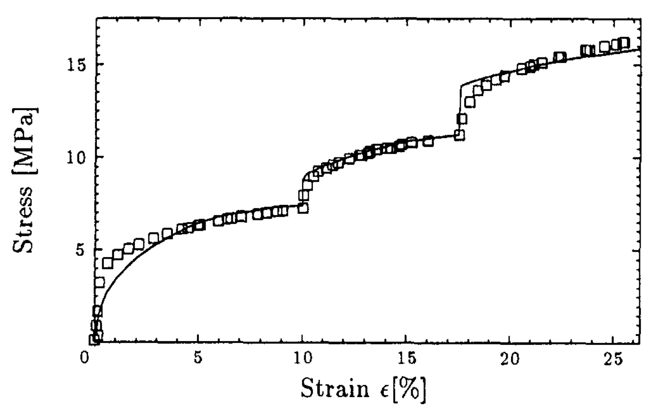

Fig. 14 Prediction of test Al 550ZS

with the new material function

$$r_s = \gamma \left\{ \sqrt{3I_2'(\alpha)} \right\}^{m-1}. \tag{26}$$

This formulation of the model was already tested for other materials (Chaboche, 1989). Although the material constants are evaluated only with regard to the tests Al 55007, Al 55009, and Al 55011. Figures 11 to 13, the model finally derived predicts the observed behavior of the material in the test Al 550ZS fairly well (Fig. 14).

The aluminum tests considered here are taken as an example. The general approach can be applied for any material and any stress-strain time behavior. This is the main advantage of the shape functional approach: It can be proven whether insufficient shapes of the material functions or neglected interactions between the state variables of the model lead to observed discrepancies between theory and experiments.

Acknowledgments

This work has been supported by the Deutsche Forschungsgemeinschaft through SFB 319. The contribution of Y. Estrin by providing the experimental data is gratefully acknowledged.

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