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Mathematical Modeling Vibration Protection System for the Motor of the Boat

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Abstract

The nonlinear mathematical model of the system to mount the motor in a niche of the outboard motor boat, which protects against external influences by means nonlinear shock absorbers and dampers, has been developed.
The mathematical model of the system is represented by three nonlinear differential equations of second order. The nonlinear shock absorbers and dampers, as a function of the generalized coordinates are represented by polynomials up to the fifth degree inclusive.
For approximate analytical solution of the nonlinear mathematical model is applied the modified method of polynomial transformations and the numerical method of Runge-Kutta.

Keywords: Vibration protection systems, Method of polynomial transformations, Nonlinear mathematical model, Outboard motor boat

1 Introduction

When driving on the water at the hull of the boat affect periodic forces, which are transmitted to the outboard motor. To protect the motor from the periodic influence the motor is mounted on the non-linear dampers and the shock absorbers [1-6]. The nonlinear mathematical model for mount the engine on niche of the boat has been developed. External forces, acting on the hull of the boat, is transmitted to the motor and represented as periodic functions. It is assumed that the system has no resonance. As result the mount of engine in a niche by the nonlinear shock absorbers is a significant reduction in engine vibration amplitudes on the all three axes. Nonlinear rubber shock absorbers [7-13] and dampers in the suspension of the motor are widely used on boats "Yamaha", "Tridin", "Eyrslot".

2 Mathematical model of the vibration protection system for motor of the boat

Consider mounting the motor of the boat in the niche by means of non-linear rubber shock absorbers and dampers that can be represented in the form of polynomials up to the fifth degree inclusive.

Assumed, the impact on the boat an external periodic force in three directions:

$$f_x = A_x \sin(\omega_1 t) + B_x \cos(\omega_1 t),\ f_y = A_y \sin(\omega_2 t) + B_y \cos(\omega_2 t),$$
$$f_z = A_z \sin(\omega_3 t) + B_z \cos(\omega_3 t),$$

where $A_x, B_x, A_y, B_y, A_z, B_z$ – amplitude, $\omega_1, \omega_2, \omega_3$ – the frequency of external periodic forces. For the equations of motion the dynamical system is applied the system of Lagrange equations:

$$\left\{ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q_x; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = Q_y; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = Q_z, \right.$$

where L - Lagrange function, Q_x - the force corresponding coordinate x.

Here x, y, z – the coordinates of the center of mass of the motor.

The generalized forces are equal:

$$Q_x = -b_1(\dot{x} - \dot{f}_x) - b_2(\dot{x} - \dot{f}_x)^2 - b_3(\dot{x} - \dot{f}_x)^3 - b_4(\dot{x} - \dot{f}_x)^4 - b_5(\dot{x} - \dot{f}_x)^5,$$
$$Q_y = -l_1(\dot{y} - \dot{f}_y) - l_2(\dot{y} - \dot{f}_y)^2 - l_3(\dot{y} - \dot{f}_y)^3 - l_4(\dot{y} - \dot{f}_y)^4 - l_5(\dot{y} - \dot{f}_y)^5$$

Here m - the weight of the motor, c_i, k_i – the spring constant, b_i, l_i – coefficients of friction the damping devices.

Substituting the expressions for the Lagrange equations, we obtain a system of three nonlinear differential equations of the second order:

$$\begin{cases} m\ddot{x} + b_1(\dot{x} - \dot{f}_x) + b_2(\dot{x} - \dot{f}_x)^2 + b_3(\dot{x} - \dot{f}_x)^3 + b_4(\dot{x} - \dot{f}_x)^4 + b_5(\dot{x} - \dot{f}_x)^5 + \\ + c_1(x - f_x) + c_2(x - f_x)^2 + c_3(x - f_x)^3 + c_4(x - f_x)^4 + c_5(x - f_x)^5 = 0 \\ m\ddot{y} + l_1(\dot{y} - \dot{f}_y) + l_2(\dot{y} - \dot{f}_y)^2 + l_3(\dot{y} - \dot{f}_y)^3 + l_4(\dot{y} - \dot{f}_y)^4 + l_5(\dot{y} - \dot{f}_y)^5 + p_1(y - f_y) = 0 \\ m\ddot{z} + k_1(z - f_z) + k_2(z - f_z)^2 + k_3(z - f_z)^3 + k_4(z - f_z)^4 + k_5(z - f_z)^5 = 0 \end{cases}$$

Let us consider the relative coordinates of the center mass the motor:
$\tilde{x} = x - f_x; \tilde{y} = y - f_y; \tilde{z} = z - f_z;$
To simplify further entries we omit the sign ~ for coordinates: $\tilde{x} \equiv x; \tilde{y} \equiv y; \tilde{z} \equiv z_z$.
In the relative coordinates the mathematical model can be written.

$$\begin{cases} m\ddot{x} + b_1\dot{x} + c_1 x + d_1 + b_2\dot{x}^2 + b_3\dot{x}^3 + b_4\dot{x}^4 + b_5\dot{x}^5 + \\ +c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 = A_1\sin(t\omega_1) + B_1\cos(t\omega_1) \\ m\ddot{y} + l_1\dot{y} + l_2\dot{y}^2 + l_3\dot{y}^3 + l_4\dot{y}^4 + l_5\dot{y}^5 + p_1 y + d_2 = A_2\sin(t\omega_2) + B_2\cos(t\omega_2) \\ m\ddot{z} + k_1 z + k_2z^2 + k_3z^3 + k_4z^4 + k_5z^5 + d_3 = A_3\sin(t\omega_3) + B_3\cos(t\omega_3) \end{cases}$$

Divide each equation on the weight of the motor. For simplicity, we write the system without renaming all the coefficients of the system:
$c_i \equiv c_i / m; k_i \equiv k_i / m; b_i \equiv b_i / m; l_i \equiv l_i / m.$

We write the nonlinear system of equations with the coefficients divided to unit weight of the motor.

$$\begin{cases} \ddot{x} + b_1\dot{x} + c_1 x + d_1 + b_2\dot{x}^2 + b_3\dot{x}^3 + b_4\dot{x}^4 + b_5\dot{x}^5 + \\ +c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 = A_1\sin(t\omega_1) + B_1\cos(t\omega_1) \\ \ddot{y} + l_1\dot{y} + l_2\dot{y}^2 + l_3\dot{y}^3 + l_4\dot{y}^4 + l_5\dot{y}^5 + p_1 y + d_2 = A_2\sin(t\omega_2) + B_2\cos(t\omega_2) \\ \ddot{z} + k_1 z + k_2z^2 + k_3z^3 + k_4z^4 + k_5z^5 + d_3 = A_3\sin(t\omega_3) + B_3\cos(t\omega_3) \end{cases}$$

3 The method of calculation the vibration protection system

For the calculation of nonlinear dynamic systems apply different methods [14-19]. Is applied known methods: the method of Van der Pol, harmonic balance method, averaging method, the method of small parameter, the method of Krylov-Bogolyubov, method of polynomial transformations, the Poincare perturbation method [20-25].
For the method of Van der Pol and for the method of averaging the truncated equation is considered. In the method of harmonic balance approximate solution takes into account only the basic frequency components. In the method of small parameter and perturbations the approximate solution is found, provided that the series converges.
For solved the nonlinear dynamic system was applied the method polynomial transformation [26].
The system of equations is written in matrix form:
$\dot{X}^* = RX^* + P$, where

$$X^* = [x, y, z, \dot{x}, \dot{y}, \dot{z},$$
$$\exp(i\omega_1 t), \exp(-i\omega_1 t), \exp(i\omega_2 t), \exp(-i\omega_2 t) \exp(i\omega_3 t), \exp(-i\omega_3 t)]^T$$

P - nonlinear vector the system.
As a result, the linear change of variables: $Y^* = AX^*$, we obtain the linear system with a diagonal matrix: $\dot{Y}^* = \Lambda Y^* + \widetilde{P}$.
By replacing the variables in accordance with the polynomial transformation method [26]: $y^*{}_S = z_S^* + \sum_{|\nu|=2}^{5} a_\nu^S \tilde{Z}^\nu$, up to terms of the fourth order we obtain an autonomous differential system:

$$\{\dot{z}_1^* = (\lambda_1 + q_1)z_1^*, z_4^* = \overline{z}_1^*, \dot{z}_2^* = (\lambda_2 + q_2)z_2^*, z_5^* = \overline{z}_2^*, \dot{z}_3^* = (\lambda_3 + q_3)z_3^*, z_6^* = \overline{z}_3^*.$$

The solution of the autonomous system of equations can be written as: $z_3^* = \rho_1 \exp(q_1 t), z_2^* = \rho_2 \exp(q_2 t), z_3^* = \rho_3 \exp(q_3 t)$
The solution of the nonlinear dynamical system in the original variables *x*,*y*,*z* can be written as:

$$x(t) = -\left(F_1^2\left(7F_1^2\left(b_4\omega_1^4 + c_4\right) + 8b_2\omega_1^2 + 8c_2\right) + 8d_1\right)\left(\left(Q_1 - 8\omega_1^2\right)^2 + 64b_1^2\omega_1^2\right)/R_1 +$$
$$+8\sin\left(t\omega_1\right)\left(Q_1\left(5b_5B_1\omega_1^5F_1^4 + 6b_3B_1\omega_1^3F_1^2 + A_1Q_1 - 8A_1\omega_1^2 + 8b_1B_1\omega_1\right)\right)/R_1 +$$
$$+8\cos\left(t\omega_1\right)\left(Q_1\left(-5A_1b_5\omega_1^5F_1^4 - 6A_1b_3\omega_1^3F_1^2 - 8A_1b_1\omega_1 + B_1Q_1 - 8B_1\omega_1^2\right)\right)/R_1;$$
$$Q_1 \equiv F_1^2\left(5c_5F_1^2 + 6c_3\right) + 8c_1; F_1^2 \equiv A_1^2 + B_1^2; R_1 = 512c_1\left(b_1^2\omega_1^2 + \left(c_1 - \omega_1^2\right)^2\right)$$
$$y(t) = \sin\left(t\omega_2\right)\left(5B_2l_5\omega_2^5F_2^4 + 6B_2l_3\omega_2^3F_2^2 + 8A_2p_1 - 8A_2\omega_2^2 + 8B_2l_1\omega_2\right)/R_2 +$$
$$+\cos\left(t\omega_2\right)\left(-5A_2l_5\omega_2^5F_2^4 - 6A_2l_3\omega_2^3F_2^2 - 8A_2l_1\omega_2 + 8B_2p_1 - 8B_2\omega_2^2\right)/R_2 +$$
$$-\left(\omega_2^2F_2^2\left(7l_4\omega_2^2F_2^2 + 8l_2\right) + 8d_2\right)/(8p_1); F_2^2 \equiv A_2^2 + B_2^2, R_2 = 8\left(l_1^2\omega_2^2 + \left(p_1 - \omega_2^2\right)^2\right)$$
$$z(t) = -\left(F_3^2\left(7k_4F_3^2 + 8k_2\right) + 8d_1\right)\left(Q_3 - 8\omega_3^2\right)^2/R_3 +$$
$$+\sin\left(t\omega_3\right)8A_3Q_3\left(Q_3 - 8\omega_3^2\right)/R_3 + \cos\left(t\omega_3\right)8B_3Q_3\left(Q_3 - 8\omega_3^2\right)/R_3;$$
$$Q_3 \equiv F_3^2\left(5k_5F_3^2 + 6k_3\right) + 8k_1; F_3^2 \equiv A_3^2 + B_3^2; R_3 = 512k_1\left(k_1 - \omega_3^2\right)^2$$

We obtained the steady mode of motion with the following parameters of the dynamic system:

$$A_1 = 0.4; A_2 = 0.3; A_3 = 0.2; B_1 = 0.2; B_2 = 0.1; B_3 = 0.3; \omega_1 = 1; \omega_2 = 1.2; \omega_3 = 0.8;$$
$$m = 1; b_1 = 0.2; b_2 = 0.01; b_3 = 0.01; b_4 = 0.01; b_5 = 0.01; c_1 = 0.5; c_2 = 0.01; c_3 = 0.01;$$
$$c_4 = 0.01; c_5 = 0.01; d_1 = 0.02; d_2 = 0.01; d_3 = 0.03; p_1 = 0.3; k_1 = 0.04; k_2 = 0.01;$$
$$k_3 = 0.01; k_4 = 0.01; k_5 = 0.01; l_1 = 0.01; l_2 = 0.01; l_3 = 0.01; l_4 = 0.01; l_5 = 0.01;$$

The stationary mode of motion corresponding to the functions:

$$x = -0.550022\sin(t) - 0.624073\cos(t) - 0.0491024$$

$$y = -0.262094\sin(1.2t) - 0.0908144\cos(1.2t) - 0.0387381$$

$$z = -0.341722\sin(0.8t) - 0.512583\cos(0.8t) - 0.534267$$

We obtained the graphics of motion the center mass the motor (Figure 1).

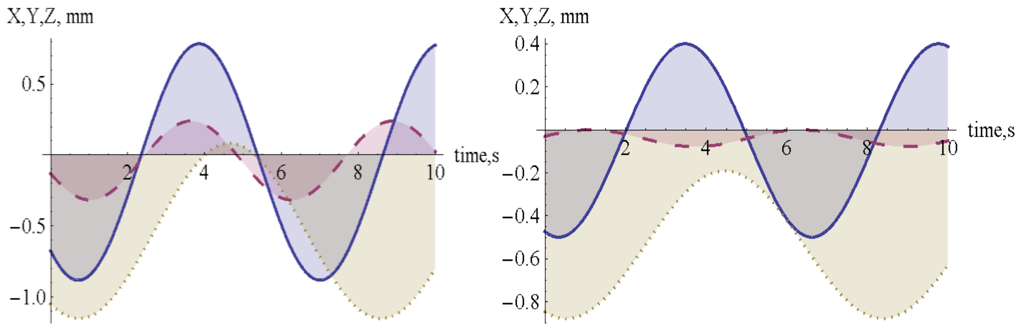

Fig.1. The relative and absolute movement of the center mass the motor.

Legends: —— x, – – – y, $\cdots\cdots$ z

The analytical solution is compared with the numerical solution by the Runge-Kutta method. Error of solving for the polynomial transformation method does not exceed 2%.

Figure 2 shows the dependence of the error (ε in percent) on the parameter c_1 and on the time, when $c_1 = 0.5$ the error does not exceed 0.05%.

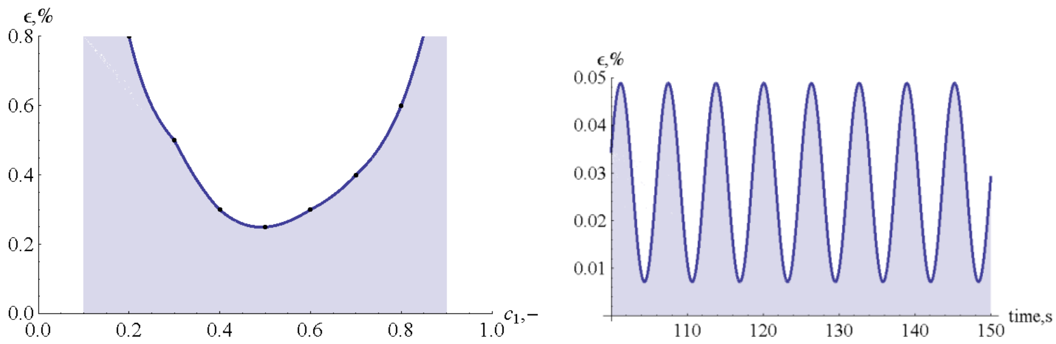

Fig.2. The dependence the error on the c_1 and on the time when $c_1 = 0.5$

Figure 3 shows the dependence the error (ε in percent) on the parameter b_1 and on the time, when $b_1 = 0.3$ the error does not exceed 0.25%.

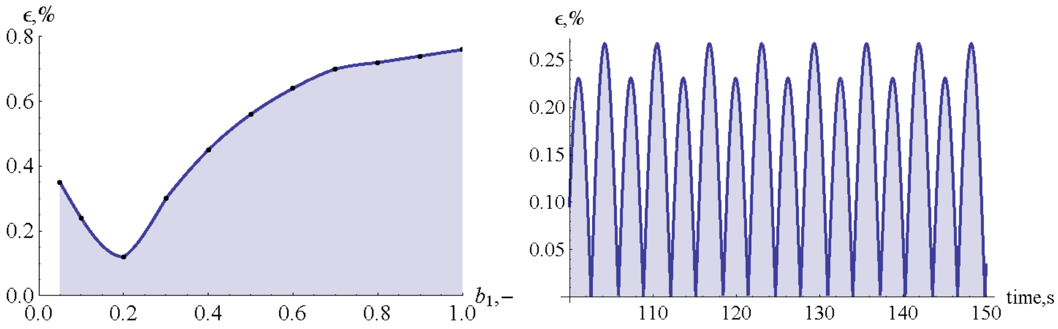

Fig. 3. The dependence the error on the b_1 and on the time when $b_1 = 0.3$

We obtained graph the maximum amplitude of the dynamic oscillation system depending on the frequency ω_1 (Figure 4)

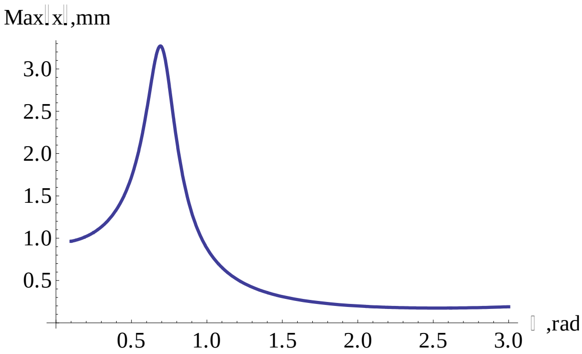

Fig.4. The dependence maximum amplitude of the oscillation on the frequency ω_1

We performed the analysis the effect of nonlinear parameters of shock absorbers and dampers to the maximum vibration amplitude (Figure 5).

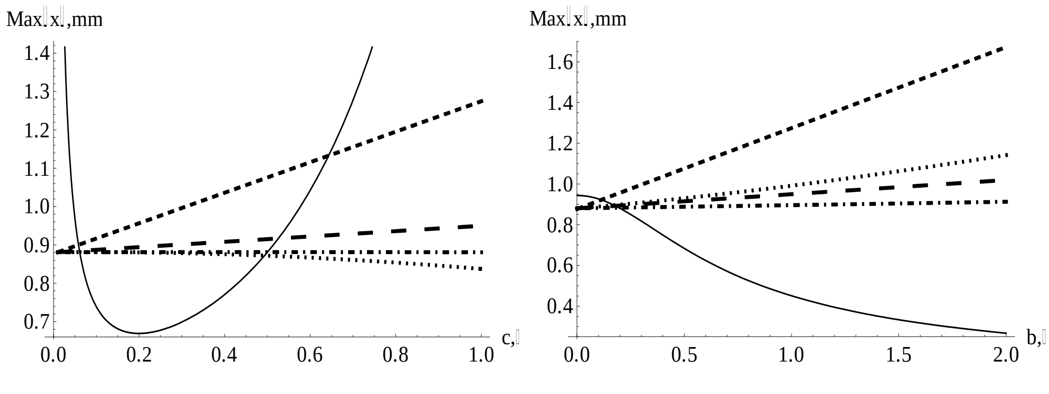

Fig.5. The dependence the maximum amplitude of the vibrations on the parameters of the shock absorber **c** and **b**.

Legends: $\text{——} c_1, \text{– – –} c_2, \cdots\cdots c_3, \text{– – –} c_4, \text{–·–·–} c_5$, $\text{——} b_1, \text{– – –} b_2, \cdots\cdots b_3, \text{– – –} b_4, \text{–·–·–} b_5$

From the figure 5, it follows that for reduce the amplitude of oscillations is expedient to increase the non-linear parameters of shock absorbers c_3, c_5 , to reduce the non-linear parameters c_2, c_4 , the linear parameters $c_1 \approx 0.2$. For reduce the amplitude of the vibrations is necessary to increase parameter dampers b_1 and to reduce the nonlinear parameters b_2, b_3, b_4 .

4 Conclusion

Thus, the approach in the proposed work, combining analytical and numerical methods, allows to complete analysis of the nonlinear dynamic system and to gets the basic characteristics the motion of dynamical system. Evaluation the accuracy of analytical solution of the nonlinear dynamic system shows an error of less than two percent. The effectiveness of nonlinear dynamical system essentially depends on the type of external influence and the selected parameters. When considering the parameters dynamic system for shock absorbers mounted in the motor niche is necessary to increase the nonlinear parameters c_3, c_5 , to reduce parameters c_2, c_4 and linear parameters $c_1 \approx 0.2$. For parameters of dampers should be to increase the parameters b_1 and to reduce the nonlinear parameters: b_2, b_3, b_4 .

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