

Applied Mathematical Sciences, Vol. 7, 2013, no. 2, 49 - 60

A New Bivariate Class of Life Distributions

Zohdy M. Nofal

Department of Statistics, Mathematics and Insurance
Faculty of Commerce, Benha University, Benha, Egypt
dr_znofal@hotmail.com

Abstract

Some concepts of multivariate aging for exchangeable random variables have been considered in Bassan and Spizzichino (1999) as special types of bivariate IFR, by comparing distributions of residual lifetimes of dependent components of different ages. Bassan et al. (2002) studied some properties of the IFR notion in the bivariate case. They introduced concepts of BDMRL aging and developed a treatment that parallels the one developed for BIFR. They analyzed a weak and a strong version, and discussed some of the differences between them. In the same spirit, we introduce and study a new family of life distribution. This class is bivariate increasing failure rate average (BIFRA) and its dual Bivariate decreasing failure rate average (BDFRA). We introduce concepts of BIFRA (BDFRA) aging and study the preservation properties of this class under reliability operations. Also, a shock model is introduced.

Keywords: Multivariate extension of IFRA, shock models, life distributions.

1. Introduction

Univariate concepts of aging like IFR (increasing failure rate), NBU (new better than used), DMRL (decreasing mean residual life) have played an important role in survival analysis, reliability theory, maintenance policies, operations research and many other areas of applied probability.

The notion of aging, for engineering systems has been characterized by several classes of life distributions in reliability. Various classes of life distributions and their duals have been introduced to describe several types of improvement that a company aging. The main classes introduced in the literature are based on IFR,

IFRA, DMRL, NBU, NBUE, NBAFR, HNBUE, DCCS, NBUA, NBUC, NBUCA, IFR(2), NBU(2), GHNBU, NBU_{Lt} and NBU_{mgf} concepts of aging, for definitions and some properties of these classes, see Bryson and Siddiqui (1990), Rolski (1975), Barlow and Proschan (1981), Loh (1984), Deshpande et al., (1986), Abouammoh et al. (1993), Ahmad et al. (2005), Hendi (1992), Cao and Wang (1991) and Klefsjo (1981), Li and Kochar (2001), Franco et al. (2001), and Li et al. (2001), among others.

A complex system usually consists of several components, which are working under the same environment, and hence their lifetimes are, generally, dependent. In the literature several attempts have been made to extend the concepts of univariate aging to the multivariate case. The most well known classes of life distributions based on multivariate aging property are multivariate increasing failure rate (MIFR), multivariate increasing failure rate average (MIFRA), multivariate new better than used (MNBU), multivariate decreasing mean residual life (MDMRL), multivariate new better than used in expectation (MNBUE), multivariate harmonic new better than used in expectation (MHNBU). Each of these classes has a corresponding dual class, see Bochanan and Singpurwalla (1977) and Basu, et al. (1983).

This paper introduces a new family of bivariate life distributions. This class is the bivariate increasing failure rate on average (BIFRA) and its dual (BDFRA). We study the preservation of BIFRA (BDFRA) under some reliability operations

- i)* Formation of coherent system,
- ii)* Convolution of life distributions, and
- iii)* Mixing of distributions.

A bivariate shock model is also considered.

This paper is organized as follows. In Section 2, we introduce the definitions of the BIFRA (BDFRA) property. In Section 3, we study preservation properties of BIFRA (BDFRA) under different reliability operation. Mixtures are considered in Section 4.

A bivariate shock model is discussed in Section 5.

2. The Bivariate IFRA Property.

In This section, we introduce some definitions of BIFRA (BDFRA) and other facts used throughout the paper.

2.1 Definition. A distance between two vectors $u=(0,u)$ and $v=(0,v)$ is defined by

$$d(\underline{u}, \underline{v}) = \|\underline{u}, \underline{v}\| = \sqrt{u^2 + v^2}$$

2.2 Definition.

A distribution $F(t_1, t_2)$ has BIFRA (BDFRA) if

- i) $\frac{1}{\sqrt{u^2+v^2}} \int_0^u \int_0^t r(x, y) dx dy$ is increasing (decreasing) in $t > 0$ for all fixed $u > 0$,
 - ii) $\frac{1}{\sqrt{u^2+v^2}} \int_0^u \int_0^t r(x, y) dx dy$ is increasing (decreasing) in $u > 0$ for all fixed $t > 0$,
- where

$r(x, y)$ is the bivariate failure rate defined by

$$\begin{aligned} r(x, y) &= \lim_{(\Delta x, \Delta y) \rightarrow (0,0)} \frac{P[x \leq X \leq x + \Delta x, y \leq Y \leq y + \Delta y | X \geq x, Y \geq y]}{\Delta x \Delta y} \\ &= \lim_{(\Delta x, \Delta y) \rightarrow (0,0)} \frac{P[x \leq X \leq x + \Delta x, y \leq Y \leq y + \Delta y]}{\Delta x \Delta y P[X \geq x, Y \geq y]} \\ &= \frac{1}{\bar{F}(x, y)} \frac{\partial^2}{\partial x \partial y} F(x, y) = \frac{f(x, y)}{\bar{F}(x, y)} \end{aligned} \quad (1)$$

Remark: It is obvious that BIFRA distribution F is characterized by

$$\begin{aligned} \bar{F}^{\frac{1}{\sqrt{t^2+u^2}}}(t, u) \downarrow \text{ on } [0, \infty) \times [0, \infty), \text{ while BDFRA distribution } F \text{ is characterized} \\ \text{by } \bar{F}^{\frac{1}{\sqrt{t^2+u^2}}}(t, u) \uparrow \text{ on } [0, \infty) \times [0, \infty). \text{ Hence } F \text{ is BIFRA (BDFRA) if and only if} \\ \bar{F}(\alpha t, \beta u) \geq (\leq) \bar{F}^{\sqrt{\alpha^2+\beta^2}}(t, u) \text{ for all } 0 < \alpha, \beta < 1, t \geq 0, u \geq 0 \end{aligned} \quad (2)$$

2.3 Definition.

A structure function $\phi(x_1, x_2, \dots, x_n)$ is monotonic if ϕ is increasing in each argument.

2.4 Definition.

A function $g(x, y)$ defined on $D \times D$ such that

$$i) \frac{1}{\sqrt{x^2+y^2}} g(x, y) \text{ is increasing in } x > 0 \text{ for fixed } y > 0, \quad (3a)$$

$$ii) \frac{1}{\sqrt{x^2+y^2}} g(x, y) \text{ is increasing in } y > 0 \text{ for fixed } x > 0, \quad (3b)$$

is called bivariate star-shaped, where $D = [0, \infty)$.

2.5 Definition.

Let F_1 and F_2 be bivariate distributions not necessarily confined to the positive axis. Then

$$F(t_1, t_2) = \int_{-\infty}^{\infty} F_1(t_1 - u, t_2 - v) dF_2(u, v), \quad (4)$$

is the convolution of F_1 and F_2 .

3. Preservation of BIFRA (BDFRA) under reliability operations.

We study the preservation of BIFRA (BDFRA) under the reliability operations

- i) Formation of coherent system.
- ii) Convolution of distributions.
- iii) Mixture of life distribution, see Barlow and Proschan (1981).

3.1 Theorem. Let $h(p)$ be the reliability function of a monotonic system. Then $h(p\alpha) \geq h\alpha(p)$ for $0 \leq \alpha \leq 1$, where $p\alpha = (p_1^\alpha, p_2^\alpha, \dots, p_n^\alpha)$.

Proof See Barlow and Proschan (1981, P. 85).

3.2 Theorem Suppose each of the independent components of a coherent system has a BIFRA life distribution. Then the system itself has a BIFRA life distribution.

Proof

Let $F(t_1, t_2)$ denote the system life distribution, while $F_i(t_1, t_2)$ denotes the life distribution of the i th components, $i = 1, 2, n$.

$$\bar{F}(\alpha t_1, \beta t_2) = h[\bar{F}_1(\alpha t_1, \beta t_2), \bar{F}_2(\alpha t_1, \beta t_2), \dots, \bar{F}_n(\alpha t_1, \beta t_2)] \quad (5)$$

Since F_i is BIFRA, then

$$\bar{F}_i(\alpha t_1, \beta t_2) \geq \bar{F}_i^{\sqrt{\alpha^2 + \beta^2}}(t_1, t_2), i = 1, 2, n.$$

Since h is increasing in each argument, it follows that

$$\bar{F}(\alpha t_1, \beta t_2) \geq h\left[\bar{F}_1^{\sqrt{\alpha^2 + \beta^2}}(t_1, t_2), \dots, \bar{F}_n^{\sqrt{\alpha^2 + \beta^2}}(t_1, t_2)\right]$$

But by Theorem 2, we have

$$h\left[\bar{F}_1^{\sqrt{\alpha^2 + \beta^2}}(t_1, t_2), \dots, \bar{F}_n^{\sqrt{\alpha^2 + \beta^2}}(t_1, t_2)\right] \geq h^{\sqrt{\alpha^2 + \beta^2}}[\bar{F}_1(t_1, t_2), \dots, \bar{F}_n(t_1, t_2)]$$

Combining the last two inequalities, and using (5), we conclude that

$$\bar{F}(\alpha t_1, \beta t_2) \geq \bar{F}^{\sqrt{\alpha^2 + \beta^2}}(t_1, t_2)$$

Thus the system life distribution $F(t_1, t_2)$ is BIFRA.

Next we present some interesting examples of BIFRA distributions.

Example1.

A bivariate exponential distribution (BVE), see Barlow and Proschan (1981, p. 129) has probability survival function as

$$\bar{F}(t_1, t_2) = e^{-[\lambda_1 t_1 + \lambda_2 t_2 + \lambda_{12} \max(t_1, t_2)]} \quad \lambda t_1, \lambda t_2 \geq 0.$$

We note that

$$\frac{1}{\sqrt{t_1^2 + t_2^2}} \log \bar{F}(t_1, t_2) = -\frac{1}{\sqrt{t_1^2 + t_2^2}} [\lambda_1 t_1 + \lambda_2 t_2 + \lambda_{12} \max(t_1, t_2)]$$

is decreasing in $t_1 > 0$ for fixed $t_2 > 0$ and is decreasing in $t_2 > 0$ for fixed $t_1 > 0$. Thus the BVE is BDFRA.

The following example shows that the BDFRA is not closed under formation of coherent structures.

Example2.

Let $F(t_1, t_2)$ be the life distribution of a parallel system of two non-independent components having respective life distribution.

$$\bar{F}_1(t) = 1 - e^{-(\lambda_1 + \lambda_{12})t}, \quad \bar{F}_2(t) = 1 - e^{-(\lambda_2 + \lambda_{12})t}$$

$$\bar{F}(t_1, t_2) = e^{-(\lambda_1 + \lambda_{12})t_1} + e^{-(\lambda_2 + \lambda_{12})t_2} - e^{-(\lambda_1 + \lambda_{12})t_1 - (\lambda_2 + \lambda_{12})t_2}.$$

It follows that

$$\begin{aligned} & \frac{1}{\sqrt{t_1^2 + t_2^2}} \log \bar{F}(t_1, t_2) = \\ & -\frac{1}{\sqrt{t_1^2 + t_2^2}} \log [e^{-(\lambda_1 + \lambda_{12})t_1} + e^{-(\lambda_2 + \lambda_{12})t_2} - e^{-(\lambda_1 + \lambda_{12})t_1 - (\lambda_2 + \lambda_{12})t_2}]. \end{aligned}$$

If this function is plotted, one can see that $\frac{1}{\sqrt{t_1^2 + t_2^2}} \log \bar{F}(t_1, t_2)$ is not decreasing in

t_2 on $[0, \infty)$, but is increasing in t_1 on $[t_2^*, \infty)$ for some t_2^* , which can be determined from the graph. We then conclude that the BDFRA is not closed under formation of coherent system.

3.3 Theorem.

If \bar{F}_1 and \bar{F}_2 are BIFRA, then their convolution is BIFRA.

Proof. For the convolution of \bar{F}_1, \bar{F}_2 we have

$$\bar{F}(\alpha t_1, \beta t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{F}_1(\alpha t_1 - u, \beta t_2 - v) d\bar{F}_2(u, v).$$

Replace u by αu and v by βv in the above integration to obtain

$$\bar{F}(\alpha t_1, \beta t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{F}_1(\alpha(t_1 - u), \beta(t_2 - v)) d\bar{F}_2(\alpha u, \beta v). \quad (6)$$

But $\bar{F}_i(x, y)$, $(i = 1, 2)$ is BIFRA, which implies that

$$\bar{F}_i(\alpha x, \beta y) \geq \bar{F}_i^{\sqrt{\alpha^2, \beta^2}}(x, y) \quad i = 1, 2, \quad 0 < \alpha, \beta < 1 \quad (7)$$

From (7) in (6), we have the following inequality

$$\begin{aligned} \bar{F}(\alpha t_1, \beta t_2) &\geq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{F}_1^{\sqrt{\alpha^2, \beta^2}}(t_1 - u, t_2 - v) d\bar{F}_2^{\sqrt{\alpha^2, \beta^2}}(u, v) \\ &\geq \bar{F}^{\sqrt{\alpha^2, \beta^2}}(t_1, t_2) \end{aligned}$$

This completes the proof.

Notice that, the convolution of BDFRA is not necessarily BDFRA. This is can be seen from the following.

Example3. Let F_1, F_2 be BVE, thus

$$\bar{F}_1(t_1, t_2) = e^{-[\lambda_1 t_1 + \lambda_2 t_2 + \lambda_{12} \max(t_1, t_2)]},$$

$$\bar{F}_2(t_1, t_2) = e^{-[\beta_1 t_1 + \beta_2 t_2 + \beta_{12} \max(t_1, t_2)]}, t_1 \geq 0, t_2 \geq 0.$$

The convolution of \bar{F}_1, \bar{F}_2 is given by

$$\bar{F}(t_1, t_2) = \int_0^{t_2} \int_0^{t_1} \bar{F}_1(t_1 - u, t_2 - v) d\bar{F}_1(u, v), \text{ if } t_1 < t_2,$$

$$\bar{F}(t_1, t_2) = \int_0^{t_2} \int_0^{t_1} \bar{F}_1(t_1 - u, t_2 - v) d\bar{F}_1(u, v), \text{ if } t_1 < t_2,$$

$$\bar{F}(t_1, t_2) = \int_0^{t_2} \int_0^{t_1} e^{-\lambda_1(t_1-u) - (\lambda_2 + \lambda_{12})(t_2-v)} d e^{-\beta_1 u - (\beta_2 + \beta_{12})v}$$

$$= \beta_1(\beta_2 + \beta_{12}) e^{-\lambda_1 t_1 - (\lambda_2 + \lambda_{12})t_2} \int_0^{t_2} \int_0^{t_1} e^{-(\beta_1 - \lambda_1)u} e^{-(\beta_2 + \beta_{12} - \lambda_2 - \lambda_{12})v} du dv$$

$$= \frac{\beta_1(\beta_2 + \beta_{12})}{(\beta_2 + \beta_{12} - \lambda_2 - \lambda_{12})(\beta_1 - \lambda_1)} \times$$

$$\left[e^{-\lambda_1 t_1 - (\lambda_2 + \lambda_{12})t_2} [1 - e^{-(\beta_1 - \lambda_1)t_1}] [1 - e^{-(\beta_2 + \beta_{12} - \lambda_2 - \lambda_{12})t_2}] \right] \quad (8)$$

$\lambda_1 = 0.2$, $\lambda_2 = 0.3$, $\lambda_{12} = 0.5$ and $\beta_1 = 0.3$, $\beta_2 = 0.4$, $\beta_{12} = 0.3$, then

$$\bar{F}(t_1, t_2) = 21e^{-0.2t_1 - 0.7t_2} [1 - e^{-0.1t_1}] [1 - e^{-0.1t_2}].$$

Thus

$$= \frac{1}{\sqrt{t_1^2 + t_2^2}} [\log(21) - 0.2t_1 - 0.7t_2 + \log(1 - e^{-0.1t_1})(1 - e^{-0.1t_2})].$$

Which when plotted shows that

$$\frac{1}{\sqrt{t_1^2 + t_2^2}} \log \bar{F}(t_1, t_2)$$

is increasing in t_2 . We conclude that the BDFRA is not closed under the convolution.

3.4 Theorem. Let $F(t_1, t_2)$ be BIFRA distribution, then its bivariate hazard (failure) function $\log F(t_1, t_2)$ is a bivariate star-shaped.

Proof The proof is a direct consequence of definitions 2.2 and 2.4.

4. Mixture of distributions.

An interesting fact in the literature and in applications of unidimensional aging notions, see, e.g., the review by paper by Shaked and Spizzichino (2001) and references therein. For a recent contribution, see Finkelstein and Esalova (2001) where it is stated that the mixture of distributions with some property of positive aging do not necessarily maintain the same property. Then one can be generally interested in finding conditions on the mixtures under which one-dimensional or multivariate aging properties are preserved.

We consider here conditionally dependent pairs of lifetimes. In this case, the joint law can be written as mixtures. It may be interesting to notice that bivariate dimensional aging properties may be preserved under mixtures, whereas the opposite may hold for one-dimensional properties.

4.1 Definition.

Given random variables T_1 and T_2 , we say that:

a) T_1 and T_2 are positively quadrant dependence (PQD) if

$$P[T_1 \leq t_1, T_2 \leq t_2] \geq P[T_1 \leq t_1]P[T_2 \leq t_2],$$

or $[T_1|T_2 > t_2] \geq_{st} T_1$, for all $t_2 > 0$

b) T_2 is stochastically increasing in T_1 if $P[T_2 > t_2 | T_1 = t_1]$ is increasing in T_1 for all T_2 .

We write it as $st(T_2 | T_1)$.

c) Let T_1, T_2 have joint probability density (or, in the discrete case, joint frequency function) $f(t_1, t_2)$, then $f(t_1, t_2)$ is totally positive of order 2 (TP_2) if

$$\begin{vmatrix} f(u_1, v_1) & f(u_1, v_2) \\ f(u_2, v_1) & f(u_2, v_2) \end{vmatrix} \geq 0, \text{ for all } u_1 < u_2, v_1 < v_2.$$

d) T_2 is right tail increasing in T_1 , $RTI(T_2 | T_1)$ if $P[T_2 > t_2 | T_1 > t_1]$ is increasing in t_1 for all t_2 .

e) Random variables $T = \{T_1, T_2, T_n\}$ are associated if

$$Cov[I(T), \Delta(T)] \geq 0,$$

for all pairs of increasing binary functions Γ, Δ .

4.2 Theorem.

$$TP_2(T_1, T_2) \Rightarrow S_I(T_1, T_2) \Rightarrow RTI(T_2 | T_1) \Rightarrow A(T_1, T_2) \Rightarrow PQD(T_1, T_2).$$

Proof See Barlow and Proschan (1981, p.143).

In the next theorem, we conjecture sufficient conditions under which the unconditional joint law of a pair of conditionally BIFRA random variables is BIFRA.

Let θ be a random variable taking values in a set L , H is its distribution and, when existing, h is its density. Given θ , (T_1, T_2) are conditionally distributed random variables with a common conditional survival function $\bar{G}(t_1, t_2 | \theta)$. The joint survival function is then given by

$$\bar{F}(t_1, t_2) = \int_L \bar{G}(t_1, t_2 | \theta) dH(\theta). \quad (9)$$

4.3 Theorem.

Let $(T_1, T_2 | \theta)$ be bivariate conditionally random variable with BIFRA, T_1, T_2 are conditionally PQD given that $\theta, T_i \uparrow$ st in $\theta, i=1, 2$. Then (T_1, T_2) is BIFRA.

5. Shock Model.

Univariate shock models relate the continuous survival probability of a device subject to shocks occurring randomly in time to the (discrete) probability of surviving any specified number of shocks.

Suppose that a device is subject to shocks occurring randomly in time according to the counting process $\{N(t), t \geq 0\}$. Let the device have probability \bar{P}_k of surviving k shocks, $k = 0, 1, 2, \dots$, where $1 = \bar{P}_0 \geq \bar{P}_1 \geq \dots$ the probability $\bar{H}(t)$ that the device survives beyond t is given by

$$\bar{H}(t) = \sum_{k=0}^{\infty} P[N(t) = k] \bar{P}_k \quad (10)$$

Such shock models have been studied by Esary et al. (1973) when $N(t)$ is a homogeneous Poisson process and by A-Hameed and Proschan (1973,1975) when $N(t)$ is a non-homogeneous Poisson process. In all these cases the authors prove that $\bar{H}(t)$ is IFR, IFRA, DMRL, NBU, or NBUE under suitable conditions on $N(t)$ if \bar{P}_k has the corresponding discrete property. Klefsjo (1981) has considered (10) for the HNBUE class. Abouammoh et al. (1988) has studied some shock models for NBUFR and NBAFR classes. Abouammoh and Hendi (1991) considered shock models for NBURFR and NBARFR. Abouammoh and Ahmed (1990) have proved similar results for the GHNBU class under homogeneous and non-homogeneous Poisson shocks.

The main object of this section is to establish different results of bivariate shock models for newly introduced classes in Bassan and Spizzichino (1999) and Bassan et al. (2002).

First we consider a bivariate exponential distribution for the life lengths of two non-independent components. Suppose three independent sources of shocks are present in environment and also not necessarily fatal. Rather a shock from source 1 causes the failure of component 1 with probability q_1 ; it occurs at a random time U_1 , where $P[U_1 > t] = \exp(-\lambda_1 t)$. A shock from source 2 causes the failure of component 2 with probability q_2 ; it occurs at a random time U_2 , where $P[U_2 > t] = \exp(-\lambda_2 t)$. Finally, a shock from source 3 causes the failure of:

a) Both components, with probability q_{11} , b) component 1 only, with probability q_{10} , c) component 2 only, with probability q_{01} , d) neither components, with probability q_{00} ; it occurs at a random time U_{12} . Thus the life length T_1 of

Component 1 satisfies $T_1 = \min(U_1, U_{12})$, while the life length T_2 of component 2 satisfies $T_2 = \min(U_2, U_{12})$.

Then the joint survival probability for (T_1, T_2) is

$$P[T_1 > t_1, T_2 > t_2] = \left\{ \sum_{k=0}^{\infty} e^{-\lambda_1 t_1} \frac{(\lambda_1 t_1)^k}{k!} (1 - q_1)^k \right\} \left\{ \sum_{l=0}^{\infty} e^{-\lambda_2 t_2} \frac{(\lambda_2 t_2)^l}{l!} (1 - q_2)^l \right\} \times \left\{ \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e^{-\lambda_1 t_1} \frac{(\lambda_{12} t_1)^m}{m!} \left[e^{-\lambda_{12}(t_2 - t_1)} \frac{(\lambda_{12}(t_2 - t_1))^n}{n!} (q_{01} - q_{10})^n \right] \right\},$$

when $0 \leq t_1 < t_2$. We obtain

$$\bar{F}(t_1, t_2) = \exp\{-[\lambda_1^* t_1 + \lambda_2^* t_2 + \lambda_{12}^* \max(t_1, t_2)]\}, \quad (11)$$

Where $\lambda_1^* = \lambda_1 q_1 + \lambda_{12} q_{10}$, $\lambda_2^* = \lambda_2 q_2 + \lambda_{12} q_{01}$, $\lambda_{12}^* = \lambda_{12} q_{11}$

5.1 Theorem. The Nonfatal shock model in (11) has BDFRA

Proof

The proof of this Theorem can be obtained using similar arguments to that used in example 1.

If $q_1 = q_2 = q_{11} = 1$, the nonfatal shock model in (11) reached to the fatal shock model, see Barlow and Proschan (1981).

References

1. A.M. Abouammoh and A.N. Ahmed, The class of better failure rate at a specific age, Dirasat, Ser. B. 17(1990), 27-37.
2. A.M. Abouammoh and A.N. Ahmed and A.M. Barry, Shock models and testing for the renewal remaining life classes, Microelectron & Reliability, 33(1993), 729-740.
3. A.M. Abouammoh and M.I. Hendi, Shock models with renewal failure rate properties, Statistische Hefete, 32(1991), 19-34.
4. A.P. Bau, N. Ebrahimi and B. Klefsjo, Multivariate harmonic new better than used in expectation distributions, Secand. J. Statist, 10(1983), 19-25.

5. B. Bassan and F. Spizzichino, stochastic comparison for residual lifetimes and Bayesian notions of multivariate aging, *Advances in Applied Probability*, 31(1999), 1078-1094.
6. B. Bassan and F. Spizzichino and S. Kochar, some bivariate notions of IFR and DMRL and related properties. to appear(2002).
7. B. Klefsjo, HNBUE survival under some shock models, *Scand. J. Statist.*, 8(1981), 39–47.
8. I. Ahmad, M. Kayid and X. Li, The NBUT class of life distributions, *IEEE Trans. Reliability*, Sep, vol. 54, No.3(2005), pp. 396–401.
9. I. Rolski, Mean residual life. *Bull. Intern. Statist. Inst.*, 46(1975), 266-270.
10. J. Cao and Y. Wang, The NBUC and NWUC classes of life distribution. *J. Appl. Prob.*, 28(1991), 473-479.
11. J.D Esary, A.W. Marshall and F. Proschan, Shock models and wear processes, *Ann. Prob.*, 1(1973), 627-649.
12. J.V. Deshpande, S.C. Kochar and H. Singh, Aspects of positive aging, *J. Appl. Probab.*, 23(1986), 748–758.
13. M. S. Finkelstein and V. Esaulova, Modeling a failure rate for a mixture of distribution functions, *Probability in Engineering and Informational Sciences*, 15(2001), 383- 400.
14. M.C. Bryson and M.M. Siddiqui, Some criteria for aging, *J. Amer. Statist. Assoc.*, 64(1990), 1472–1483.
15. M. Franco, J.M. Ruiz and M.C. Ruiz, On closure of the IFR(2) and NBU(2) classes, *Journal of Applied Probability*, 38(2001), 235-241.
16. M. I. Hendi, On decreasing cumulative conditional survival classes of life distributions, *Pak.J. Statist.*, 8(3) B(1992), 1-8.
17. M.S. A-Hameed and F. Proschan, Nonstationary shock models, *Stoc., Process. Appl.*, 1(1973), 383-404.

18. M.S. A-Hameed and F. Proschan, Shock models with underlying birth process. *J. Appl. Prob.*, 12(1975), 18-28.
19. M. Shaked and F. Spizzichino, Mixtures and monotonicity of failure rate functions, in *Advances in Reliability, Handbook of Statistics*, Vol. 20 Elsevier Science B.V.(2001).
20. R. E. Barlow and F. Proschan, *Statistical Theory of Reliability and Life Testing. To Begin With*: Silver Spring, Maryland (1981).
21. W.Y. Loh, A new generalization of the class NBU distributions. *IEEE Trans. Reliability*, R-33(1984), 419-422.
22. W.B. Buchanan and N.D. Singpurwella, Some Stochastic characterization of multivariate survival, *Theory and Applications of Reliability*, Vol. I(1977), 329-48.
23. X.H. Li and S.C. Kochar, Some new results involving the NBU(2) lass of life distributions, *J. Appl. Probab.*,38(2001), 242–247.
24. X. Li, Z. Li and B. Jing, Some results about the NBUC class of life distributions, *Statist. Probab. Lett.*, 46(2000), 229–237.

Received: September, 2012