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MULTI-PULSE CHAOTIC MOTION FOR A NON-AUTONOMOUS BUCKLED PLATE BY USING THE EXTENDED MELNIKOV METHOD

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ABSTRACT

The multi-pulse Shilnikov orbits and chaotic dynamics for a parametrically excited, simply supported rectangular buckled thin plate are studied by using the extended Melnikov method. Based on von Karman type equation and the Galerkin's approach, two-degree-of- freedom nonlinear system is obtained for the rectangular thin plate. The extended Melnikov method is directly applied to the non-autonomous governing equations of the thin plate. The results obtained here show that the multi-pulse chaotic motions can occur in the thin plate.

KEYWORDS: multi-pulse; non-autonomous; buckled thin plate; Melnikov method

INTRODUCTION

With the use of thin plate in shuttles and large space stations, researches on nonlinear dynamics, bifurcation and chaos of thin plate has received considerable interests in the past decade. Abe et al. [1] used the method of multiple scales to analyze the two-mode responses of simply supported thin rectangular laminated plates subjected to harmonic excitation. The global bifurcations and Shilnikov type chaotic dynamics were investigated by Zhang et al. [2] and Zhang [3] for both parametrically and externally excited and parametrically excited simply supported rectangular thin plates. Awrejcewicz et al. [4] used the Bubnov-Galerkin with high-order approximations and finite difference methods to investigate the complex vibrations and bifurcations of a thin plate-strip excited transversally and axially. The dynamics of nonlinear polar orthotropic circular plates with simply supported boundary condition are investigated by Akour and Nayfeh [5], which considered Kirchhoff strain displacement relations for thin plates plus next higher-order nonlinear terms.

Although there are so many challenges and difficulties, certain progress has been achieved in developing analytical tools to study global bifurcations and chaotic dynamics for high-dimensional nonlinear systems and giving systematic applications to engineering problems in the past two decades. Based on studies given by Wiggins [6], Kovacic and Wiggins [7] developed a new global perturbation method which may be used to detect the Shilnikov type single-pulse homoclinic and heteroclinic orbits for four-dimensional autonomous ordinary differential equations. Based on studies in literature [6] and [7], Kaper and Kovacic [8] employed a modified Melnikov method to study the existence of several types of multi-bump homoclinic orbits to resonance bands for completely integral Hamiltonian systems with perturbations. Camassa et al. [9] extended the Melnikov method to investigate multi-pulse jumping of homoclinic and heteroclinic orbits in a class of perturbed Hamiltonian systems. Because of abstruseness in understanding and difficulties in proof and derivation, the method given in references [8,9] has seldom been used in engineering application. Zhang and Yao [10-12] studied Multi-pulse orbits in beam system and belt system by using energy-phase method developed by Haller and Wiggins [13].

In this paper, the multi-pulse Shilnikov type orbits and chaotic dynamics are analyzed for a parametrically excited simply supported rectangular thin plate. The global perturbation analysis is directly applied to the non-autonomous ordinary differential equations of motion for the thin plate by the method in reference [9] for the first time. The case of buckling for the rectangular thin plate is considered. The global dynamic analysis and numerical simulations indicate that the multi-pulse chaotic motion can occur in the rectangular thin plate.

FORMULATION

We begin with a brief outline of the theory developed in literature [8] and [9] that will be used in our analysis. The general systems under consideration have the following form

$$\dot{x} = JD_x H(x, I) + \varepsilon g^x(x, I, \gamma, \mu, \varepsilon) , \quad (1a)$$

$$\dot{I} = \varepsilon g^I(x, I, \gamma, \mu, \varepsilon) , \quad (1b)$$

$$\dot{\gamma} = \Omega(x, I) + \varepsilon g^\gamma(x, I, \gamma, \mu, \varepsilon) , \quad (1c)$$

where $(x, I, \gamma) \in \mathbf{R}^2 \times \mathbf{R} \times S^1$, $0 < \varepsilon << 1$, $\mu \in \mathbf{R}^p$ denotes parameters of the system, all functions are sufficiently differentiable on the domains of interest and

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

Let $\varepsilon = 0$, the unperturbed system is given as

$$\dot{x} = JD_x H(x, I) , \quad (2a)$$

$$\dot{I} = 0 , \quad (2b)$$

$$\dot{\gamma} = \Omega(x, I) . \quad (2c)$$

Function H is Hamiltonian of the unperturbed system (2). It is noticed that system (2) is an uncoupled two-degree-of-freedom nonlinear system, the variable I remains constant since $\dot{I} = 0$. There are two assumptions on equation (2):

Assumption 1. For every I with $I_1 < I < I_2$, equation (2a) possesses a hyperbolic equilibrium point $x = x_0(I)$ which varies continuously with I, and connected to itself by a homoclinic orbit $x^h(t, I)$.

Assumption 2. For some $I = I_0$ with $I_1 < I_0 < I_2$, there exist conditions that $\Omega(x_0(I_0), I_0) = 0$ and $\frac{d\Omega(x_0(I), I)}{dI}(I_0) \neq 0$.

Assumption 1 indicates that in the four dimensional phase space, the set

$$M = \left\{(x, I, \gamma) \,\middle|\, x = x_0(I), I_1 \le I \le I_2 , 0 \le \gamma \le 2\pi\right\}, \quad (3)$$

is a two dimensional, normal hyperbolic invariant manifold. The manifold M has three-dimensional stable and unstable manifolds which are represented as $W^s(M)$ and $W^u(M)$, respectively. Moreover, the existence of homoclinic orbit of equation (2a) implies that $W^s(M)$ and $W^u(M)$ intersect non-transversally along a three- dimensional homoclinic manifold denoted by Γ :

$$\Gamma = \left\{(x, I, \gamma) \,\middle|\, x = x^h(t, I), I_1 < I < I_2, \gamma = \int_0^t D_I H(x^h(t, I), I) ds + \gamma_0\right\} \quad (4)$$

Because γ may represent the phase of the oscillations, with assumption 2 when $I = I_0$, which is called as a resonant I value, the phase shift $\Delta\gamma$ of the oscillations is defined as

$$\Delta\gamma = \gamma(+\infty, I_0) - \gamma(-\infty, I_0) . \quad (5)$$

$I =$ constant on the manifold M under assumption 2 denotes a circle of the singular fixed points, the phase shift $\Delta\gamma$ gives the shift in phase between the two endpoints of the heteroclinic trajectory along the circle of fixed points.

Now we analyze the dynamics on the perturbed system (1). In particular, the normal hyperbolic invariant manifold M along with its stable and unstable manifolds will persist in the perturbed system under arbitrary, sufficient small differentiable perturbations. It is noticed that hyperbolic singular points $x = x_0(I)$ may persist under small perturbations, in particular, $M \to M_\varepsilon$. Therefore, we obtain

$$M = M_\varepsilon = \{(x, I, \gamma) | x = x_0(I), I_1 \le I \le I_2 , 0 \le \gamma < 2\pi\}. \quad (6)$$

For what is to follow, some definitions are needed. First, the Melnikov function is given by the integral

$$M(I_0, \gamma_0, \mu) = \int_{-\infty}^{\infty} \left\langle n(p^h(t)), g(p^h(t) , \mu , 0)\right\rangle dt , \quad (7)$$

where

$$\mathbf{n} = (D_x H(x, I) , D_I H(x, I) - D_I H(x_0(I), I), 0) , \quad (8)$$

$$\mathbf{g} = (g^x(x, I, \gamma, \mu, 0), g^I(x, I, \gamma, \mu, 0) , g^\gamma(x, I, \gamma, \mu, 0)) , \quad (9)$$

$$p^h(t) = (x^h(t, I), I, \gamma^h(t, I) + \gamma_0) , \quad (10)$$

The vector n is the normal to the homoclinic manifold Γ. Second, the k pulse Melnikov function M_k, $k = 1, 2, \cdots$, is defined as

$$M_k(I_0, \gamma_0, \mu) = \sum_{j=0}^{k-1} M(I_0, \gamma_0 + j\Delta\gamma(I_0), \mu) , \quad (11)$$

where the amount of asymptotic phase changes is as follows:

$$\Delta\gamma(I_0) = \int_{-\infty}^{+\infty} \Omega(x^h(\tau, I_0), I_0) d\tau . \quad (12)$$

Theorem For some integer k , $\gamma = \bar{\gamma}_0$, $\mu = \bar{\mu}$ let the following conditions be satisfied:

1. The k pulse Melnikov function has a simple zero point in $\bar{\gamma}_0$, that is,

$$M_k(I_0,\bar\gamma_0,\bar\mu)=0\,,\quad D_\gamma M_k(I_0,\bar\gamma_0,\bar\mu)\neq 0\,. \qquad (13)$$

2. $M_i(I_0,\bar\gamma_0,\bar\mu)\neq 0 \quad i=1,\dots,k-1,k>1$.

Then for all $I\to I_0$, $\mu\to\bar\mu$ the stable manifold and unstable manifold of perturbed system (1) intersect transversely.

EQUATIONS OF MOTION OF THE THIN PLATE

We consider the simply supported at the four edge rectangular thin plate which the edge lengths are a and b and thickness is h, respectively. The thin plate is subjected to in-plane excitation which can be expressed in form $p=p_0-p_1\cos\Omega t$. We establish a Cartesian coordinate system shown in Figure 1 such that coordinate Oxy is located in the middle surface of thin plate.

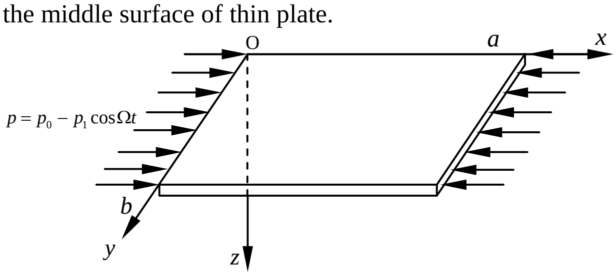


Figure 1. The model of a rectangular thin plate and the coordinate system

It is assumed that *u*, *v* and *w* represent the displacements of a point in the middle plane of the thin plate in the x, y and z directions, respectively. From van Karman type equations for the thin plate, we obtain the governing equations of motion for the rectangular thin plate as follows

$$D\nabla^4 w+\rho h\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2 w}{\partial y^2}\frac{\partial^2\phi}{\partial x^2}+2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\phi}{\partial x\partial y}+\mu\frac{\partial w}{\partial t}=0\,, \qquad (14)$$

$$\nabla^4\phi=\mathrm{E}h\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]. \qquad (15)$$

Where ρ is the density of thin plate, $D=\dfrac{Eh^3}{12\left(1-\nu^2\right)}$ is the bending rigidity, E is Young's modulus, ν is the Possion's ratio, ϕ is the stress function, and μ is the damping coefficient.

We assume that the boundary conditions of the simply supported at the four edge rectangular thin plate can be written as

$$\text{at}\quad x=0\quad\text{and}\quad x=a\,,\quad w=\frac{\partial^2 w}{\partial x^2}=0\,,$$

$$\text{at}\quad y=0\quad\text{and}\quad y=b\,,\quad w=\frac{\partial^2 w}{\partial y^2}=0\,. \qquad (16)$$

The boundary conditions satisfied by the stress function ϕ may be expressed in the following form

$$u=\int_0^a\left[\frac{1}{E}\left(\frac{\partial^2\phi}{\partial y^2}-\nu\frac{\partial^2\phi}{\partial x^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]dx=\delta_x\,, \qquad (17)$$

and

$$h\int_0^b\frac{\partial^2\phi}{\partial y^2}dy=p\,,\ \text{at}\quad x=0\quad\text{and}\quad x=a\,, \qquad (18)$$

$$v=\int_0^b\left[\frac{1}{E}\left(\frac{\partial^2\phi}{\partial x^2}-\nu\frac{\partial^2\phi}{\partial y^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]dx=0\,, \qquad (19)$$

and

$$\int_0^a\frac{\partial^2\phi}{\partial x^2}dx=0\,,\ \text{at}\quad y=0\quad\text{and}\quad y=b\,, \qquad (20)$$

where δ_x is the corresponding displacement in *x* direction at the boundary.

We mainly consider the nonlinear oscillations of the rectangular thin plate in the first two modes. Thus, we write the *w* in the following form

$$w(x,y,t)=u_1(t)\sin\frac{\pi x}{a}\sin\frac{3\pi y}{b}+u_2(t)\sin\frac{3\pi x}{a}\sin\frac{\pi y}{b}\,, \qquad (21)$$

where $u_\mathrm{i}(t)$ $(\mathrm{i}=1,2)$ are the amplitudes of two modes, respectively.

Substituting equation (21) into equation (15), considering the boundary conditions (17)-(19) and integrating, we obtain the stress function as follows

$$\begin{aligned}\phi(x,y,t)&=\phi_{20}(t)\cos\frac{2\pi x}{a}+\phi_{02}(t)\cos\frac{2\pi y}{b}+\phi_{60}(t)\cos\frac{6\pi x}{a}\\&+\phi_{06}(t)\cos\frac{6\pi y}{b}+\phi_{22}(t)\cos\frac{2\pi x}{a}\cos\frac{2\pi y}{b}\\&+\phi_{24}(t)\cos\frac{2\pi x}{a}\cos\frac{4\pi y}{b}+\phi_{42}(t)\cos\frac{4\pi x}{a}\cos\frac{2\pi y}{b}\\&+\phi_{44}(t)\cos\frac{4\pi x}{a}\cos\frac{4\pi y}{a}-\frac{1}{2}py^2\,,\end{aligned} \qquad (22)$$

$$\text{where}\quad \phi_{20}(t)=\frac{9Eh}{32\lambda^2}u_1^2\,,\quad \phi_{02}(t)=\frac{9\lambda^2 Eh}{32}u_2^2\,,$$

$$\phi_{60}(t)=\frac{Eh}{288\lambda^2}u_2^2\,,\ \phi_{06}(t)=\frac{\lambda^2 Eh}{288}u_1^2\,,$$

$$\phi_{22}(t) = -\frac{\lambda^2 Eh}{(\lambda^2+1)^2}u_1u_2\,,\quad \phi_{24}(t) = \frac{25\lambda^2 Eh}{16(\lambda^2+4)^2}u_1u_2\,,$$

$$\phi_{42}(t) = \frac{25\lambda^2 Eh}{16(4\lambda^2+1)^2}u_1u_2\,,\quad \phi_{44}(t) = -\frac{\lambda^2 Eh}{16(\lambda^2+1)^2}u_1u_2\,,$$

$$\lambda = \frac{b}{a}\,. \tag{23}$$

In order to obtain the dimensionless equations, we introduce the transformations of the variables and parameters

$$\bar{x}_i = \frac{(ab)^{1/2}}{h^2}u_i \;\; (i=1,\,2)\,,\quad \bar{p} = \frac{b^2}{\pi^2 D}p\,,$$

$$\bar{\Omega} = \frac{ab}{\pi^2}\left(\frac{\rho h}{D}\right)^{1/2}\Omega\,,\quad \bar{t} = \frac{\pi^2}{ab}\left(\frac{D}{\rho h}\right)^{1/2}t\,,$$

$$\bar{\mu} = \frac{ab}{\pi^2 h^2}\left(\frac{12(1-\nu^2)}{\rho E}\right)^{1/2}\mu\,, \tag{24}$$

For simplicity, we drop overbars in the following analysis. By means of the Galerkin's method, substituting equations (21) and (22) into equation (14) and integrating, we obtain the governing equations of motion for the dimensionless as follows

$$\ddot{x}_1+\mu\dot{x}_1 - g_1x_1 + 2x_1 f_1\cos\Omega t + \alpha_1 x_1^3 + \alpha_2 x_1 x_2^2 = 0\,, \tag{25a}$$

$$\ddot{x}_2+\mu\dot{x}_2 - g_2x_2 + 2x_2 f_2\cos\Omega t + \beta_1 x_2^3 + \beta_2 x_1^2 x_2 = 0\,, \tag{25b}$$

where $$\alpha_1 = \frac{12(1-v^2)h^2}{ab}\frac{\lambda^4+81}{16\lambda^2}\,,$$

$$\beta_1 = \frac{3(1-v^2)h^2}{4ab}\left(81\lambda^2+\frac{1}{\lambda^2}\right),$$

$$\alpha_2 = \beta_2 = \frac{12(1-v^2)h^2}{ab}\left[\frac{17\lambda^2}{(1+\lambda^2)^2} + \frac{625\lambda^2}{16(4+\lambda^2)^2} + \frac{625\lambda^2}{16(1+4\lambda^2)^2}\right],$$

$$g_k = \left(h_k p_0 - (\omega_k^*)^2\right) \;\text{ and }\; h_k = \begin{cases}1\,, & k=1\,,\\ 9, & k=2\,,\end{cases}$$

$$p_1^* = (\omega_1^*)^2 = \frac{(9+\lambda^2)^2}{\lambda^2}\,,\quad p_2^* = (\omega_2^*)^2 = \frac{(9\lambda^2+1)^2}{\lambda^2}\,,$$

$$f_k = \frac{1}{2}h_k p_1\,,\quad k=1,\,2\,, \tag{26}$$

where $g_k\,(k=1,\,2)$ are two linear natural frequencies of the thin plate, $p_k^*\,(k=1,\,2)$ are the critical forces corresponding to two buckling modes at which thin plate loses the stability, $\omega_k^*\;\;(k=1,\,2)$ are the natural frequencies of the two buckling modes, and $f_k\,(k=1,\,2)$ are the amplitudes of parametric excitation. It is known that the buckling load is $p_{0c} = p_k^*$, in this paper, we restrict our attention to the case where the applied static load is larger than the buckling load, namely, $p_0 > p_{0c}/h_k$.

MULTI-PULSE ORBITS OF THE THIN PLATE

It is found from the aforementioned analysis that equations (25a) and (25b) are the governing equations of nonlinear oscillations of the rectangular thin plate for the first and the second modes, respectively. From the point view of engineering application, it is known that the motion of the second mode is faster than one of the first one. Therefore, we introduce the following coordinate transformation on equation (25):

$$\sqrt{\mu}u_1 = x_2\,,\; \sqrt{\mu}u_2 = \dot{x}_2\,,\; v_1 = x_1\,,\; \mu v_2 = \dot{x}_1\,,\; \phi = \Omega t\,. \tag{27a}$$

$$\mu \to \varepsilon\mu\,,\quad f_k \to \varepsilon f_k\,,\quad k=1,\,2\,. \tag{27b}$$

Then, the following equations can be obtained:

$$\dot{u}_1 = u_2\,, \tag{28a}$$

$$\dot{u}_2 = g_2u_1 - \tilde{\beta}_1u_1^3 - \alpha_2 v_1^2u_1 - \varepsilon\mu u_2 - 2\varepsilon u_1 f_2\cos\phi\,, \tag{28b}$$

$$\dot{v}_1 = \varepsilon\mu v_2\,, \tag{28c}$$

$$\dot{v}_2 = \tilde{g}_1v_1 - \tilde{\alpha}_1v_1^3 - \alpha_2u_1^2v_1 - \varepsilon\mu v_2 - 2\varepsilon v_1\tilde{f}_1\cos\phi\,, \tag{28d}$$

$$\dot{\phi} = \Omega \tag{28e}$$

where $\tilde{\beta}_1 = \mu\beta_1$, $\tilde{g}_1 = \frac{g_1}{\mu}$, $\tilde{\alpha}_1 = \frac{\alpha_1}{\mu}$, $\tilde{f}_1 = \frac{f_1}{\mu}$.

Let $\varepsilon = 0$ in equation (28), the unperturbed system has the following form

$$\dot{u}_1 = u_2\,, \tag{29a}$$

$$\dot{u}_2 = g_2u_1 - \tilde{\beta}_1u_1^3 - \alpha_2v_1^2u_1\,, \tag{29b}$$

$$\dot{v}_1 = 0\,, \tag{29c}$$

$$\dot{v}_2 = \tilde{g}_1v_1 - \tilde{\alpha}_1v_1^3 - \alpha_2u_1^2v_1\,, \tag{29d}$$

The Hamiltonian of unperturbed system (29) is obtained as

$$H = \frac{1}{2}u_2^2 - \frac{1}{2}g_2u_1^2 + \frac{1}{4}\tilde{\beta}_1u_1^4 + \frac{1}{2}\alpha_2u_1^2v_1^2 - \frac{1}{2}\tilde{g}_1v_1^2 + \frac{1}{4}\tilde{\alpha}_1v_1^4 \tag{30}$$

Now we want to find the hyperbolic fixed points of the equation (29a) and (29b) at which assumption 2 holds:

$$u_2 = 0\,, \tag{31a}$$

$$g_2u_1 - \tilde{\beta}_1u_1^3 - \alpha_2v_1^2u_1 = 0\,, \tag{31b}$$

$$\tilde{g}_1v_1 - \tilde{\alpha}_1v_1^3 - \alpha_2u_1^2v_1 = 0\,. \tag{31c}$$

The equation (19) can be solved as

$$(u_1^0\,,\,u_2^0) = (0\,,\,0)\,,\;\text{ where }\; v_1^0 = \pm\sqrt{\frac{\tilde{g}_1}{\tilde{\alpha}_1}}\,, \tag{32a}$$

$$(u_1, u_2) = (\pm\sqrt{\frac{g_2}{\beta_1}}, 0), \text{ where } v_1 = 0 . \quad (32b)$$

The singular fixed point of equation (29a) and (29b) $(u_1^0, u_2^0) = (0, 0)$ is a saddle, and fixed points $(u_1, u_2) = \left(\pm\sqrt{\frac{g_2}{\tilde{\beta}_1}}, 0\right)$ are centers, therefore equation (29a) and (29b) can exhibit the homoclinic bifurcations, and the homoclinic orbits which connect the saddle point $(u_1^0, u_2^0) = (0, 0)$ are obtained as

$$u_1^h = \pm\sqrt{\frac{2g_2}{\tilde{\beta}_1}} \sec h\sqrt{g_2}\, t , \quad (33a)$$

$$u_2^h = \pm\sqrt{\frac{2g_2}{\tilde{\beta}_1}} \sec h\sqrt{g_2}\, t \tanh\sqrt{g_2}\, t , \quad (33b)$$

From the results obtained in paper [14], in four-dimensional phase space the set defined by

$$M = \left\{(u_1, u_2, v_1, v_2) \,\middle|\, u_1 = u_2 = 0, v_1 \in I, | v_2 | < H\right\}, \quad (34)$$

is a two-dimensional partial invariant manifold, it is known that partial invariant manifold M is normally hyperbolic. The manifold M has three-dimensional stable and unstable manifolds which are represented as $W^s(M)$ and $W^u(M)$, respectively. The existence of the homoclinic orbits of system (17) connected to singular point $(u_1^0, u_2^0) = (0, 0)$ indicates that $W^s(M)$ and $W^u(M)$ intersect along a three-dimensional homoclinic manifold denoted by Γ, which can be written as

$$\Gamma = \left\{(u_1, u_2, v_1, v_2) \,\middle|\, u_1^h, u_2^h, v_1 \in I, v_2 = \int_{-\infty}^{t} D_{v_1} H(u_1^h, u_2^h, \pm\sqrt{\frac{g_1}{\alpha_1}}) ds + v_2^0\right\} . \quad (35)$$

From equation (12), we can compute that

$$\Delta v_2 = \int_{-\infty}^{+\infty} \pm \alpha_2 \sqrt{\frac{\tilde{g}_1}{\tilde{\alpha}_1}} u_1^2 dt = \pm \frac{4\alpha_2}{\tilde{\beta}_1}\sqrt{\frac{\tilde{g}_1 g_2}{\tilde{\alpha}_1}} . \quad (36)$$

It is obvious that the unperturbed system (29) is a four dimensional equations, whereas the perturbed system (28) is a five dimensional system. When viewed in the full five-dimensional phase space $\mathbf{R}^4 \times S^1$, the partial normally hyperbolic invariant manifold M of formula (34) can be written as the following form

$$M(t) = \left\{(u_1, u_2, v_1, v_2, \phi) \,\middle|\, u_1 = u_2 = 0, v_1 \in I, \; | v_2 | < H, \phi = \Omega t + \phi_0\right\} \quad (37)$$

Based on the analysis in reference [15], we know that $M(t)$ along with its stable and unstable manifolds are invariant under small, sufficiently differentiable perturbations, moreover $M_\varepsilon(t)$, $W^s(M_\varepsilon(t))$ and $W^u(M_\varepsilon(t))$ are C^r ε-close to $M(t)$, $W^s(M(t))$ and $W^u(M(t))$.

Now we begin our analysis of the perturbed system. Consider the following cross-section of the phase space

$$\Sigma^{\phi_0} = \{(u, v, \phi) \,|\, \phi = \phi_0\} . \quad (38)$$

The geometrical explanation of the cross-section Σ^{ϕ_0} is shown as figure 2

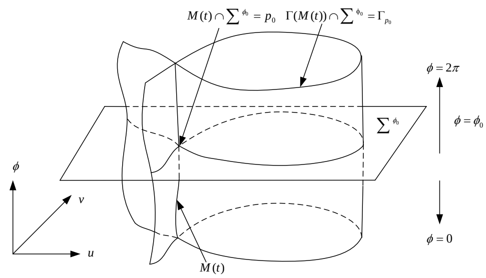


Figure 2. The geometrical structure of the cross section Σ^{ϕ_0}

We introduce the scale transformation near the resonant region as $v_1 = v_1^0 + \sqrt{\varepsilon}\bar{v}_1$, $\tau = \sqrt{\varepsilon}t$, and Taylor expand the equation (28c) and (28d) in $\sqrt{\varepsilon}$, to obtain the following equations, for simplicity, we drop over-bar

$$\dot{v}_1 = v_2 , \quad (39a)$$

$$\dot{v}_2 = (g_1 - 3{v_1^0}^2\alpha_1)v_1 - 3\sqrt{\varepsilon}v_1^0\alpha_1 v_1^2 - \sqrt{\varepsilon}\mu v_2 - 2\sqrt{\varepsilon}v_1^0 f_1 \cos\Omega t \quad (39b)$$

When $\varepsilon = 0$, unperturbed system of equation (39) can be written as

$$\dot{v}_1 = \mu v_2 , \quad (40a)$$

$$\dot{v}_2 = (\tilde{g}_1 - 3{v_1^0}^2\alpha_1)v_1 , \quad (40b)$$

The fixed point $(v_1, v_2) = (0, 0)$ is a center of equation (40), and is a saddle point of equation (39), from which we can conclude that homoclinic orbits of the system (39) are Shilnikov type orbits.

We are now at the point where we can compute the Melnikov function. According to equation (7), the Melnikov function of the first pulse can be computed as

$$M = \int_{-\infty}^{+\infty} [-\mu u_2^2 - 2 f_2 u_1 u_2 \cos(\Omega(t + t_0) + \phi_0) - \alpha_2 \mu v_2 {v_1^0} u_1^2] dt$$
$$= -\frac{4\mu\sqrt{g_2}}{3\tilde{\beta}_1} \pm \frac{2 f_2 \Omega^2 \pi}{\tilde{\beta}_1\sqrt{g_2}} \sin(\Omega t_0 + \phi_0) \csc h \frac{\Omega\pi}{2\sqrt{g_2}} + \frac{4\alpha_2\mu}{\tilde{\beta}_1}\sqrt{\frac{\tilde{g}_1 g_2}{\tilde{\alpha}_1}} v_2^0 \quad (41)$$

The k-pulse Melnikov function can be computed according to equation (11)

$$M_k = -\frac{4\mu\sqrt{g_2}}{3\tilde{\beta}_1} k \pm \frac{2 f_2 \Omega^2 \pi}{\tilde{\beta}_1\sqrt{g_2}} k \sin(\Omega t_0 + \phi_0) \csc h \frac{\Omega\pi}{2\sqrt{g_2}}$$
$$+ \frac{4\alpha_2\mu}{\tilde{\beta}_1}\sqrt{\frac{\tilde{g}_1 g_2}{\tilde{\alpha}_1}} v_2^0 k + k(k-1)\frac{2\alpha_2\mu}{\tilde{\beta}_1}\sqrt{\frac{\tilde{g}_1 g_2}{\tilde{\alpha}_1}} . \quad (42)$$

If k-pulse Melnikov function M_k has simple points, that is:

$$\pm\frac{2 f_2\Omega^2\pi}{\beta_1\sqrt{g_2}}\sin(\Omega t_0+\phi_0)\csc h\frac{\Omega\pi}{2\sqrt{g_2}}+\frac{4\alpha_2}{\beta_1}\sqrt{\frac{\tilde{g}_1 g_2}{\alpha_1}}v_2^0$$

$$+(k-1)\frac{2\alpha_2}{\beta_1}\sqrt{\frac{\tilde{g}_1 g_2}{\alpha_1}}-\frac{4\mu\sqrt{g_2}}{3\beta}=0\,, \qquad (43)$$

and

$$D_{v_2^0}M_k=\frac{4\alpha_2\mu}{\tilde{\beta}_1}\sqrt{\frac{\tilde{g}_1 g_2}{\tilde{\alpha}_1}}k\neq 0\,. \qquad (44)$$

Then, equation (43) can be written as following:

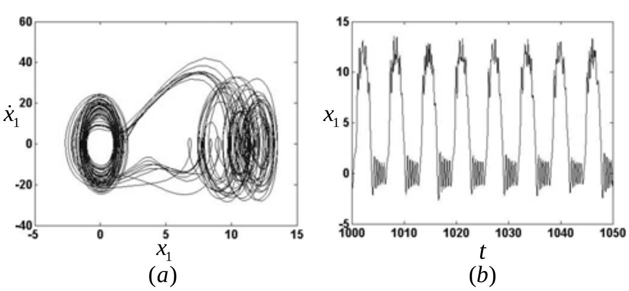
$$\frac{2\sqrt{\tilde{\alpha}_1}}{3\alpha_2\sqrt{\tilde{g}_1}}\mp\frac{f_2\Omega^2\pi\sqrt{\tilde{\alpha}_1}}{\mu g_2\alpha_2\sqrt{\tilde{g}_1}}\sin(\Omega t_0+\phi_0)\csc h\frac{\Omega\pi}{2\sqrt{g_2}}-2v_2^0\,. \qquad (45)$$

We may choose proper parameters in equation (45), such that the value of the following formula (46)

$$\pm\frac{f_2\Omega^2\pi}{g_2\sqrt{\tilde{g}_1}\alpha_2}\sin(\Omega t_0+\phi_0)\csc h\frac{\Omega\pi}{2\sqrt{g_2}}+\frac{2\mu}{3\sqrt{\alpha_2\tilde{g}_1}}-\frac{2v_2^0\sqrt{\alpha_2}}{\sqrt{\alpha_1}}\,, \qquad (46)$$

is a nonnegative integer. Thus, system (28) has *k*-pulse Shilnikov homoclinic orbits according to the theorem.

Now numerical simulations are used to predict the chaotic motion of the parametrically excited, simply supported rectangular thin plate. The fourth-order Runge-Kutta algorithm is employed to explore the existence of the chaotic motions of the thin plate. Figure 3 demonstrates the existence of the multi-pulse chaotic motion of equation (25) with two buckling modes. The chosen parameters and the initial conditions are $g_1=0.82$, $g_2=0.76$, $\mu=0.52$, $\alpha_1=1.36$, $\alpha_2=0.88$, $\beta_1=2.56$, $f_1=112$, $f_2=109$, $\Omega=1$ and $(x_1,\dot{x}_1,x_2,\dot{x}_2)=(0.1, 0.21, 0.5, 0.16)$, respectively. Picture (a) represents phase portrait on plane $(x_1,\dot{x}_1)$; (b) waveform on plane (t,x_1); (c) phase portrait on plane $(x_2,\dot{x}_2)$; (d) waveform on plane (t,x_2); (e) phase portraits in three-dimensional space $(x_2,\dot{x}_2,x_1)$.



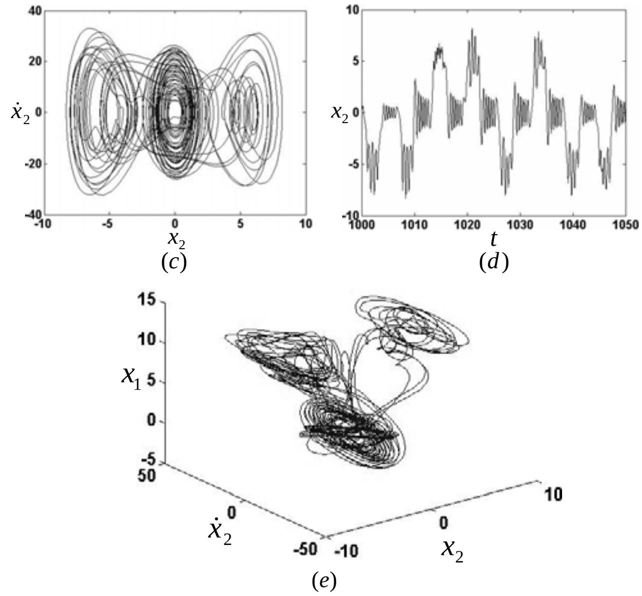


Figure 3. Multi-pulse chaotic motion of the plate

CONCLUSIONS

Engineering researchers have taken great pains researching on the nonlinear oscillations, bifurcations and chaos of the buckled rectangular thin plates in the case of large deformation. In this paper, the multi-pulse Shilnikov orbits and chaotic dynamics are analyzed on the non-autonomous buckled rectangular thin plate by using the extended Melnikov method for the first time, which cannot be analyzed using the method of multiple scales.

Most of the studies in literature on using the global perturbation method to analyze the global and chaotic dynamics are focused on autonomous differential equations. For example, in papers [2,3] the non-autonomous ordinary differential equations of the thin plate with two-degree-of-freedom were derived by von Karman-type equation and Galerkin's approach. Then, the method of multiple scales was used to transfer non-autonomous governing equation of motion to the autonomous averaged equation. Based on the averaged equation, the theory of normal form and the global perturbation method were employed to study the global and chaotic dynamics. The extended Melnikov method in paper [9] is also used to dealt with four-dimensional autonomous ordinary differential equations, while in this paper, the method in paper [9] is generalized to resolve non-autonomous ordinary differential equations by introducing the cross section Σ^{ϕ_0}.

Furthermore, the extended Melnikov method is focused on the perturbed Hamiltonian systems, where the variable $\gamma\in[0,2\pi)$ is bounded. In this paper, the system is in Cartesian coordinate; therefore, the geometrical structure of the normal hyperbolic invariant manifold M may be different.

We deal with this deficiency by introducing the concept of part invariant manifold in virtue of the reference. The geometrical structure of the manifold $M(t)$ and the cross section Σ^{ϕ_0} need further deep research.

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