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RISK BASED INSPECTION AND REPAIR OPTIMIZATION OF SHIP STRUCTURES CONSIDERING CORROSION EFFECTS

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ABSTRACT

A theoretical framework of risk based optimal inspection and repair planning is proposed for the ship structures subjected to corrosion deterioration. The planning problem is formulated as an optimization problem where the expected lifetime costs were minimized with a constraint on the minimum acceptable reliability index. The safety margins are established for the inspection events, the repair events and the failure events for ship structures. Moreover, the formulae are derived to calculate failure probabilities and repair probabilities. Based on them, a component subjected to pitting corrosion is investigated to illustrate the process of selecting the optimal inspection and repair strategy. Furthermore, some sensitivity studies were provided. The results show that the optimal inspection instants should take place before the reliability index reaches the minimum acceptable reliability index. The optimal target failure probability is 10^{-3}. In addition, a balance can be achieved between the risk cost and total expected inspection and repair costs by using the risk based optimal inspection and repair method, which is very effective in selecting the optimal inspection and repair strategy.

Nomenclature

a =is the measured corrosion size
a =year
A =coefficient representing material properties
a_{d} =minimum detectable corrosion size
B =coefficient representing the environmental characteristics
$\beta_{\min}$ =minimum annual acceptable reliability index
$\beta(T_{\mathrm{L}}, b, \mathbf{i}, d)$ =reliability index in the design lifetime T_{L}
b=design thickness of the plate
c=quality of coating
C=standard reference cost
$C_{\mathrm{F}}(b, \mathbf{i}, d)$ =expected failure cost

$C_{\mathrm{D}}(b,c)$ =initial cost

C_{F0} =cost associated with failure of the plate of ship structures,

$C_{\mathrm{I}}(b,\mathbf{i},d)$ =expected inspection cost

$C_{\mathrm{I0(i)}}(q)$ =inspection cost at the *i*th inspection

$C_{\mathrm{R}}(b,\mathbf{i},d)$ =expected repair cost

$C_{\mathrm{R0(i)}}(k)$ =repair cost at the *i*th repair

$C_{\mathrm{T}}(b,\mathbf{i},d)$ = total expected cost

d_{crit} = critical allowable thickness of corrosion wastage

$d(t)$ =wear of thickness due to corrosion

$d_{\max}$ =maximum allowable thickness of corrosion wastage

Δt =inspection interval

$H(t)$ =safety margin of inspection

k = repair quality

$M_{\mathrm{F}}(t)$ =safety margin of failure

$M_{\mathrm{R}}(t)$ =safety margin of repair

N =number of inspections

q =inspection quality

T_0 =any time after the plate putting in service

T_{i} =time of the *i*th inspection

T_{L} =design lifetime

τ_i =coating life

λ =scale parameter

1 Introduction

All materials and structures contain defects. Ship structures in particular are prone to manufacture and operation induced flaws duo to the scale, fabrication methods and operational environment of ship structures. To ensure the safety and reliability of ship structures during their service lifetime, inspections are essential and important to evaluating corrosion and fatigue damage and scheduling maintenance or repair. One difficulty associated with the inspection is its cost. It is well known that the cost of inspection for these damage categories represents an enormous financial burden for ship owners and operators. Another difficulty is the physical size of the inspection. With the introduction of large ships such as VLCCs, the task of conducting structural inspections has become increasingly challenging. Therefore, it is necessary to investigate the optimal inspection and repair planning to accomplish the most cost-effective inspection. Then, a balance between the risk costs and the inspection and repair costs can be achieved.

Several researchers studied the application of the risk based inspection in the inspections of ship structures. Ma et al.,[1] detailed the steps to be performed in conducting inspection of ship structures. Ma [2] presented the framework of a risk based inspection strategy for tankers. The risk based approach used two parameters, criticality and susceptibility, to rate the inspection priority so that structural details with higher risk can receive more attention. This approach, namely priority assessment, provides the basis for developing inspection strategies. Further details can be found in Ma et al.,[3] Generally speaking, Ma et al.,[1-3] mainly focused on a qualitative analysis of inspections for tankers. Landet et al.,[4] determined the target failure probability on the basis of cost optimal solution for an FPSO. Xu et al.,[5] presented the principles and strategies of in-service inspection programs for FPSO's. They presented some key problems to be resolved, including probability of detection updating, event updating and Bayesian estimation, existing in the risk based optimal inspection for FPSO hulls, which is very important to the development a systematic optimal inspection planning for ship structures. However, only a qualitative introduction was presented, and no detailed models and methods were given.

To resolve these problems, this paper proposes a risk based optimal inspection and repair method for ship structures. An optimization model for inspection and repair planning is developed in detail. Furthermore, this paper presents a simple method to solve the problem of inspection optimization. A numerical example is investigated to illustrate the effectiveness of the proposed method.

2 Optimization model of inspection and repair planning

2.1 Formulation of the optimization model

The decision problem of identifying the cost optimal inspection and repair plan can be solved within the framework of pre-posterior analysis from the classical decision theory. [6] The decision problem of inspection and repair can be represented in Fig.1.

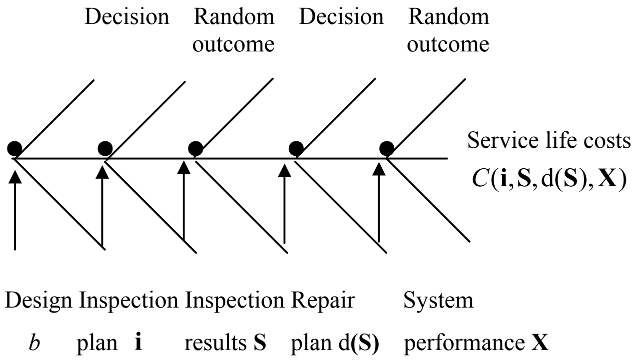

Fig.1. Decision tree for inspection and repair planning

defining the inspection and repair plan are the number of inspections N in the design lifetime T_L, the time intervals between the inspections $\Delta\mathbf{T}=(\Delta T_1,\Delta T_2,\cdots,\Delta T_N)$, the potential inspection methods or the inspection qualities $\mathbf{q}=(q_1,q_2,\cdots,q_N)$, the potential repair methods or the repair qualities $\mathbf{k}=(k_1,k_2,\cdots,k_N)$, the locations to be inspected at the inspection times $\mathbf{l}=(l(T_1),l(T_2),\cdots,l(T_N))$. For the sake of convenience, these parameters are collected in the inspection vector $\mathbf{i}=(N,\Delta\mathbf{T},\mathbf{q},\mathbf{k},\mathbf{l})$. For the initial inspection and repair plan established in the design phase, a design variable b representing the nominal design thickness of a plate is also included as an optimization parameter. The inspection results are uncertain due to the fact that they depend not only on the uncertain performance of the inspection itself but also on the uncertain state of degradation. The uncertain result of an inspection, i.e. the measured corrosion thickness studied herein, is modeled by the random vector $\mathbf{S}=(S(T_1),S(T_2),\cdots,S(T_N))$ in which the individual components refer to the results obtained from the inspections at the different locations. After an inspection is performed, a decision rule $d(\mathbf{S})$ is then applied to the inspection result to decide whether a repair of the plate should be performed or not. Finally, the different uncertain parameters modeling the state of nature such as load variables and material characteristics are collected in the vector $\mathbf{X}=(X_1,X_2,\cdots,X_N)$.

The total expected costs are taken to be the sum of the initial/design, inspection, repair and failure costs. If the constraints related to the minimum acceptable reliability index and the low and up limit on the design thickness of the plate b are added, then the optimization problem can be formulated as

$$\begin{cases}\min\limits_{b,\mathbf{i},d} C_T(b,\mathbf{i},d)=C_D(b,c)+C_I(b,\mathbf{i},d)+C_R(b,\mathbf{i},d)\\ \qquad\qquad +C_F(b,\mathbf{i},d)\\ \quad s.t.\ \ \beta(T_L,b,\mathbf{i},\mathrm{d})\ge\beta_{\min}\\ \ b^{low}\le b\le b^{up}\end{cases}\tag{1}$$

where $C_T(b,\mathbf{i},d)$ is the total expected cost in the design lifetime T_L on the basis of the given design thickness of the plate. $C_D(b,c)$ is the initial cost, $C_I(b,\mathbf{i},d)$, $C_R(b,\mathbf{i},d)$ and $C_F(b,\mathbf{i},d)$ are the expected inspection, repair and failure costs, respectively. $\beta_{\min}$ is the minimum acceptable reliability index. $\beta(T_L,b,\mathbf{i},d)$ is the reliability index of the plate in the design lifetime T_L.

As seen from Eq.1, the initial cost $C_D(b,c)$ is a function of the design thickness of the plate b and the quality of the coating c, which is a most important factor. However, it should be noted that the initial cost does not vary with the different inspection and repair strategies. So the optimization problem of Eq.1 can be rewritten as

$$\begin{cases}\min\limits_{\mathbf{i},d} C_T(\mathbf{i},d)=C_I(\mathbf{i},d)+C_R(\mathbf{i},d)+C_F(\mathbf{i},d)\\ \ s.t.\ \ \beta(T_L,\mathbf{i},\mathrm{d})\ge\beta_{\min}\end{cases}\tag{2}$$

2.2 Modeling of expected costs

The expected costs are always defined as the product of the occurrence probability of an event and the costs. The expected inspection, repair and failure costs must be modeled as functions of the optimization variables indicated in Eq. 2. After analyzing literature, [7-9] the following formulae are presented to model the expected inspection, repair and failure costs.

The total expected inspection costs $C_{\mathrm{I}}(\mathbf{i},d)$ are modeled by

$$C_{\mathrm{I}}(\mathbf{i},d)=\sum_{i=1}^{N}C_{\mathrm{I0(i)}}(q)(1-P_{\mathrm{F}}(T_{\mathrm{i}}))\frac{1}{(1+\alpha)^{T_{\mathrm{i}}}} \qquad (3)$$

where $C_{\mathrm{I0(i)}}(q)$ is the inspection cost at the ith inspection as a function of the inspection method q . Generally, a more elaborated nondestructive inspection method has higher cost than that with lower detection quality, T_{i} is the time of the ith inspection (in years), $P_{\mathrm{F}}(T_{\mathrm{i}})$ is the probability of failure in the time interval $[0,T_{\mathrm{i}}]$, α is the real rate of interest which is adopted for expressing all the costs in present value.

The total expected repair costs $C_{\mathrm{R}}(\mathbf{i},d)$ are modeled by

$$C_{\mathrm{R}}(\mathbf{i},d)=\sum_{i=1}^{N}C_{\mathrm{R0(i)}}(k)P_{\mathrm{R}}(T_{\mathrm{i}})\frac{1}{(1+\alpha)^{T_{\mathrm{i}}}} \qquad (4)$$

where $C_{\mathrm{R0(i)}}(k)$ is the repair cost at the ith inspection as a function of the repair method k . Usually, a method which can achieve a better repair quality is more expensive. $P_{\mathrm{R}}(T_{\mathrm{i}})$ is the probability of performing a repair at the ith inspection when a failure has not occurred earlier.

The total expected failure costs $C_{\mathrm{F}}(\mathbf{i},d)$ are modeled by

$$\begin{aligned}C_{\mathrm{F}}(\mathbf{i},d)=\sum_{i=1}^{N}&(C_{\mathrm{F0}}P_{\mathrm{F}}(T_0)+C_{\mathrm{F0}}(P_{\mathrm{F}}(T_{\mathrm{i}})-P_{\mathrm{F}}(T_0))\frac{1}{(1+\alpha)^{T_{\mathrm{i-1}}}}\\&+C_{\mathrm{F0}}(P_{\mathrm{F}}(T_{\mathrm{i}})-P_{\mathrm{F}}(T_{\mathrm{i-1}}))\frac{1}{(1+\alpha)^{T_{\mathrm{i}}}})\end{aligned} \qquad (5)$$

where C_{F0} is the cost associated with failure of the plate of ship structures, $P_{\mathrm{F}}(T_0)$ is the probability of failure at the initial instant T_0, $P_{\mathrm{F}}(T_{\mathrm{i}})$ and $P_{\mathrm{F}}(T_{\mathrm{i-1}})$ are the probabilities of failure at T_{i} and $T_{\mathrm{i-1}}$, respectively.

2.3 Optimization of the inspection time

To further simplify the solution to the general optimization problem in Eq.2, a simplified method is proposed to determine the optimal time of inspection T_{i} on the basis of the minimization of the total expected cost. At the same time, the optimal inspection interval can be obtained. An optimal inspection time T_{i} must be obtained by minimizing the total expected costs with the constraint condition $T_{\mathrm{i}}\le T_{\mathrm{S}}$, where T_{S} refers to the time with a reliability index equal to the minimum acceptable reliability index $\beta_{\min}$. The expected inspection, repair and failure costs can be formulated as follows.

$$C_{\mathrm{I}}(T_{\mathrm{i}})=\frac{C_{\mathrm{i}}}{C}(1-P_{\mathrm{F}}(T_{\mathrm{i}}))/(1+\alpha)^{T_{\mathrm{i}}} \qquad (6)$$

$$C_{\mathrm{R}}(T_{\mathrm{i}})=\frac{C_{\mathrm{r}}}{C}P_{\mathrm{R}}(T_{\mathrm{i}})/(1+\alpha)^{T_{\mathrm{i}}} \qquad (7)$$

$$\begin{cases}C_{\mathrm{F}}(T_0)=\dfrac{C_{\mathrm{f}}}{C}P_{\mathrm{F}}(T_0)\\ C_{\mathrm{F}}(T_{\mathrm{i}})=\dfrac{C_{\mathrm{f}}}{C}((P_{\mathrm{F}}(T_{\mathrm{i}})-P_{\mathrm{F}}(T_0))/(1+\alpha)^{T_i}\\ C_{\mathrm{F}}(T_{\mathrm{S}})=\dfrac{C_{\mathrm{f}}}{C}(P_{\mathrm{F}}(T_{\mathrm{S}})-P_{\mathrm{F}}(T_{\mathrm{i}}))/(1+\alpha)^{T_{\mathrm{S}}})\end{cases} \qquad (8)$$

where $C_{\mathrm{I}}(T_{\mathrm{i}})$ and $C_{\mathrm{R}}(T_{\mathrm{i}})$ are the expected inspection and repair costs at T_{i} , respectively. $C_{\mathrm{F}}(T_0)$, $C_{\mathrm{F}}(T_{\mathrm{i}})$ and $C_{\mathrm{F}}(T_{\mathrm{S}})$ are the expected failure costs at T_0 , T_{i} and T_{S} , respectively. The time T_0 can be any time after the plate putting in service. C_{i} , C_{r} and C_{f} are the inspection , repair and failure costs, respectively. For the sake of simplicity, the non-dimensional values are used to represent various costs in this study. For this reason, a standard reference cost C is introduced, which does not influence the analytical results. Then the new inspection, repair and failure costs are defined as $C_{\mathrm{I0}}=C_{\mathrm{i}}/C$, $C_{\mathrm{R0}}=C_{\mathrm{r}}/C$ and $C_{\mathrm{F0}}=C_{\mathrm{f}}/C$. The optimal inspection time T_{i} is therefore determined as the optimal solution to the following minimization problem.

$$\begin{cases}\min\limits_{T_{\mathrm{i}}}C_{\mathrm{T}}(T_{\mathrm{i}})=\min\limits_{T_{\mathrm{i}}}(C_{\mathrm{I}}(T_{\mathrm{i}})+C_{\mathrm{R}}(T_{\mathrm{i}})+C_{\mathrm{F}}(T_0)\\ \qquad\qquad\qquad +C_{\mathrm{F}}(T_{\mathrm{i}})+C_{\mathrm{F}}(T_{\mathrm{S}}))\\ s.t.\ \ \beta(T_{\mathrm{S}})\ge\beta_{\min}\end{cases} \qquad (9)$$

2.4 Optimization of the inspection interval

The problem consists in the determination of an inspection interval Δt which induces a minimal expected cost during the design lifetime T_{L} of the component. For this reason, the number of inspections N is given in advance, and then the expected cost is evaluated. The procedure is performed for different number of inspections and the different expected costs are compared as well. The value N that provides the smallest cost gives to the optimal inspection period. A constraint related to the minimum acceptable reliability index $\beta_{\min}$ is added, the optimization problem can be formulated as

$$\begin{cases} \min\limits_{\Delta t} C_{\mathrm{T}}(\Delta t) = \min\limits_{\Delta t}(C_{\mathrm{I}}(\Delta t) + C_{\mathrm{R}}(\Delta t) + C_{\mathrm{F}}(\Delta t) \\ \qquad + C_{\mathrm{F}}(T_{\mathrm{L}})) \\ s.t. \quad \beta(\mathrm{T_L}) \geq \beta_{\min} \end{cases} \tag{10}$$

Since the optimal inspection interval Δt can be obtained from the sequential time of inspections, the solution of Eq.10 can be performed by solving Eq.9.

3 Safety margins

Corrosion results in loss of cross sectional thickness and could, for example, lead to reduction in strength, water-tightness. Performance functions for corroded components should be defined based on allowable values. Such functions could be based on increased corroded depth, thickness reduction, or even area or volume reduction. For demonstration purposes, it is assumed that the extent of thickness reduction is of primary interest.

3.1 Failure event margins

For the purposes of illustration, the corrosion model proposed by Paik et al., [10] is considered in this study. The wear of plate thickness due to corrosion may be generally expressed as a function of the time after the corrosion starts, namely

$$d(t) = A(t - \tau_i)^B \tag{11}$$

where $d(t)$ is the wear of thickness due to corrosion, τ_i is the coating life, A, B are coefficients.

The safety margin $M_{\mathrm{F}}(t)$ modeling failure at time t is formulated as

$$M_{\mathrm{F}}(t) = d_{\mathrm{crit}} - d(t) \tag{12}$$

where $d(t)$ is the wear of thickness due to corrosion, d_{crit} is the critical allowable thickness of corrosion wastage.

3.2 Inspection event margins

3.2.1 Probability of detection model

The probability of detection (PoD) expresses the probability of detecting a flaw of a given size. It is a common measure to evaluate the capability of an NDI technique. This paper assumes that visual inspection (VI) and ultrasonic inspection(UI) are applied to inspect hull structures. An exponential distribution is often used to model PoD[11], which is written as

$$PoD(a) = \begin{cases} 1 - \exp(-\dfrac{a - a_{\mathrm{d}}}{\lambda}) & a \geq a_{\mathrm{d}} \\ 0 & a < a_{\mathrm{d}} \end{cases} \tag{13}$$

where a is the measured corrosion size, λ is the scale parameter. a_{d} is the minimum detectable corrosion size below which the corrosion cannot be detected. The parameters in Eq (13) can be estimated through regression analysis of experimental data. For demonstration purposes, assuming that VI: $a_{\mathrm{d}} = 5.0\,\mathrm{mm}$, $\lambda = 1.0\,\mathrm{mm}$ [12]. The following parameters for UI method can be obtained by using regression analysis of the available data [13]: $a_{\mathrm{d}} = 1.0\,\mathrm{mm}$, $\lambda = 1.2\,\mathrm{mm}$. Both inspection methods are considered as possible inspection methods for the inspection and repair plan.

3.2.2 Safety margins of inspection events

Three types of results of an inspection are considered in this study. The safety margins for three types of results from an inspection are illustrated as follows.

Case 1: Event of no corrosion detected.

In this case, the safety margin can be formulated as

$$H(t) = d(t) - a_{\mathrm{d}} \leq 0 \tag{14}$$

Case 2: Event of corrosion detected without size measurement.

In this case, the safety margin can be formulated as

$$H(t) = a_{\mathrm{d}} - d(t) \leq 0 \tag{15}$$

Case 3: Event of corrosion detected and size measured.

In this case, the safety margin can be formulated as

$$H(t) = d(t) - A = 0 \tag{16}$$

3.3 Repair event margins

Guedes Soares and Garbatov, [12] Sun and Bai [14] suggested that a repair of the plate should be made when the corrosion induced thickness reduction exceeds 25% of the original plate thickness. For demonstration purposes, it is assumed that the extent of thickness reduction is of primary interest. In order to consider the measurement error associated with the inspection method, the following safety margin of a repair event is formulated as

$$M_{\mathrm{R}}(t) = d_{\max} + \varepsilon - d(t) \le 0 \qquad (17)$$

where $d_{\max}$ is the maximum allowable thickness of corrosion wastage, ε is a zero mean stochastic variable modeling the inspection method's measurement error.

4 Modeling of failure and repair probabilities

The failure and repair probabilities can be expressed in terms of intersections of failure and repair events formulated through the event margins presented in Section 3. Since the outcome of the next inspection is unknown, all possible outcomes must be taken into account in modeling of failure and repair probabilities.Fig.2 shows the event tree for inspection and repair planning of a component, where the possible outcomes up to four inspections are shown. No repair is indicated by 0 and repair by 1. As seen from Fig.2, after the first inspection, there are two possible outcomes, namely repair and no repair. After the second inspection, there are four possible outcomes. Similarly, after N inspections in the design lifetime, the total number of possible outcomes is 2^N.

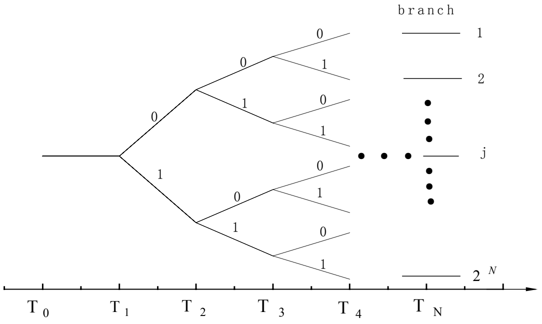

Fig.2. Event tree for inspection and repair planning

In the following part, failure and repair probabilities are formulated. These probabilities correspond to those necessary conditions for establishing the inspection and repair plan in the design phase. It should be noted that the formulated failure probabilities are the probabilities in the time interval from [0, t].

4.1 Formulae of failure probabilities

The probability of failure in the time interval from installation of the plate to the first inspection, i.e. for $0 < t \le T_1$, is formulated as

$$P_{\mathrm{F}}(t) = P(M_{\mathrm{F}}(t) \le 0) \qquad (18)$$

where $M_{\mathrm{F}}(t)$ is the safety margin modeling failure at *t* given by Eq.12.

In the time interval from the first to the second inspection, i.e. for $T_1 < t \le T_2$, the probability of failure is formulated as

$$P_{\mathrm{F}}(t) = P_{\mathrm{F}}(T_1) + P(M_{\mathrm{F}}(T_1) > 0 \cap H(T_1) > 0 \cap M_{\mathrm{F}}{}^{0}(t) \le 0)$$

$$+ P(M_{\mathrm{F}}(T_1) > 0 \cap H(T_1) \le 0 \cap M_{\mathrm{F}}{}^{1}(t) \le 0) \quad (19)$$

where $M_{\mathrm{F}}(T_1)$ is the safety margin modeling failure at T_1. $H(T_1)$ is the event margin modeling repair of the plate at T_1 given by Eqs.14~16. $M_{\mathrm{F}}{}^{0}(t)$ and $M_{\mathrm{F}}{}^{1}(t)$ are the safety margins modeling failure at $t > T_1$ corresponding to no repair and repair at the first inspection. Similar expressions can be obtained for the failure probabilities at $t > T_2$.

4.2 Formulae of repair probabilities

Two repair strategies are considered in this study. Repair strategy 1 corresponds to the repair of all corrosion damages, while repair strategy 2 leads to only the repair of corrosion damages exceeding a fixed depth. The repair probabilities can be modeled in a similar way as the failure probabilities. For illustrative purposes, only the formulae of repair probabilities for the strategy 1 are formulated as follows.

The probability of repair at the first inspection is formulated as

$$P_{\mathrm{R}}(T_1) = (H(T_1) \le 0) \qquad (20)$$

The probability of repair at the second inspection is formulated as

$$P_{\mathrm{R}}(T_2) = (M_{\mathrm{F}}(T_1) > 0 \cap H(T_1) > 0 \cap H^{0}(T_2) \le 0)$$

$$+ (M_{\mathrm{F}}(T_1) > 0 \cap H(T_1) \le 0 \cap H^{1}(T_2) \le 0) \quad (21)$$

where $H^{0}(T_2)$ and $H^{1}(T_2)$ are the inspection event margins modeling repair at T_2 corresponding to no repair and repair of the plate at the first inspection, respectively. Similar expressions can be obtained for the repair probabilities at T_3, T_4 etc. Using the similar method, one can obtain the formulae of repair probabilities for the repair strategy 2. In order to shorten the length of this paper, the corresponding results are not given herein.

5 Numerical Example

Corrosion is one of the main corrosion mechanisms in ship structures. Which affects the integrity of the ship structure by reducing the effectiveness of the stiffening element, i.e., having it trip, and promoting a loss of water tightness. Furthermore, it can lead to leaks resulting in environmental risk. Therefore, in the following example, a ship structural component subjected to pitting corrosion is considered. Based on the available data, [11,14,15] all the deterministic and random parameters used in this example are listed in Table 1. For illustrative purpose, a normal distribution [15] of A is used herein.

Table1. Statistical characteristics of basic parameters

Variables	Dimension	Distribution	u / mm	σ / mm	δ
A	mm/a	Normal	2. 10	0.021	0.01
B		Deterministic	1		
τ_i	a	Deterministic	3		
d_{crit}	mm	Normal	40	8	0.20
a_{d} (VI)	mm	Normal	5.00	0.5	0.10
a_{d} (UI)	mm	Normal	1.00	0.1	0.10
$d_{\max}$	mm	Deterministic	10		
ε	mm	Normal	0	0.5	

The time-dependent reliability index and failure probability can be obtained by using the FORM/SORM methods. The results are plotted in Fig.3 and Fig.4, respectively. In this study, it is assumed in advance that the target annual failure probability P_{fmin} is 10^{-3} and the corresponding minimum acceptable reliability index $\beta_{\min}$ is 3.09. The design lifetime for ship structures T_{L} is 20a. From Figs. 3 and 4, it can be seen that a minimum acceptable reliability index $\beta_{\min} = 3.09$ is reached at the end of a ten years period. Therefore, the first inspection must be performed before the instant $t = 10\text{a}$.

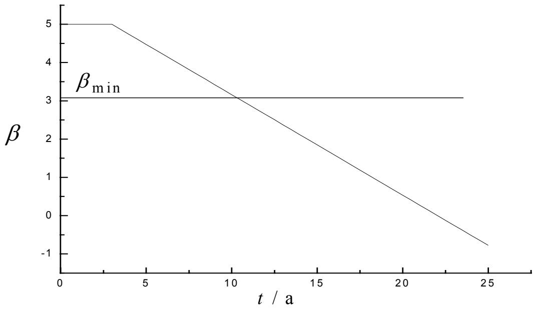

Fig. 3. Time dependent reliability index as a function of time

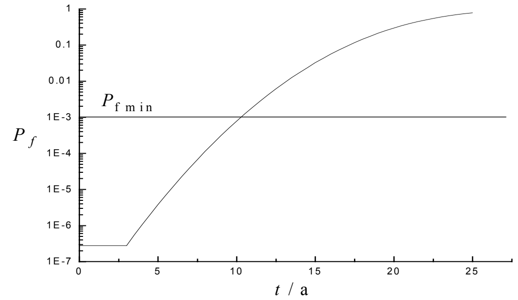

Fig. 4. Time dependent probability of failure as a function of time

According to the available data, [16-17] the values of various costs used in this example are given in Table 2. As an illustration, this paper assumes that three types of repair methods are considered in this study. At the same time, it is assumed that after an inspection performed at T_i , the reliability index will increase if a repair is performed while the reliability index will remain the same if no repair is performed. Besides, further assuming that the increases of reliability indexes are 1.0,1.5 and 2.0 for the three repair methods used in this study, respectively.

Table 2. The values of various costs

C_{I0}		C_{R0}			C_{F0}	α
Inspection method		Repair method				
VI	UI	1	2	3		
10	20	10	20	30	100000	0.04

5.1 *Analysis of results*

Using the proposed method the optimal inspection and repair plan can be obtained in different conditions. The Hohenbichler method [18] is used to calculate the probability of intersection events. For shortening the length of this paper, only the results of the inspection and repair plan are given for the repair strategy 1, the first inspection method

VI and the first repair method. The corresponding results are plotted in Figs 5~8. If there are not any exceptional explains, all the latter results are the results of this case, too.

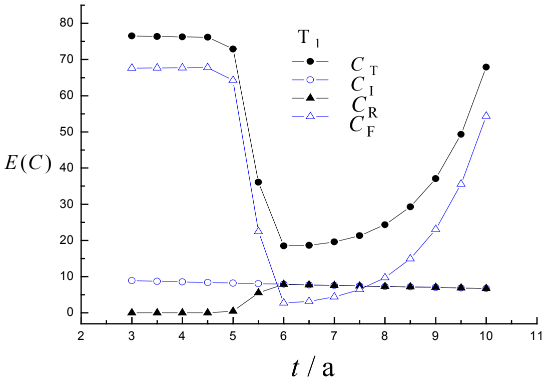

Fig. 5. The variation of the expected cost with inspection instants

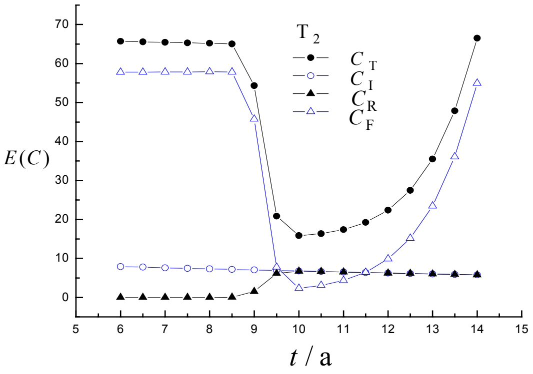

Fig. 6. The variation of the expected cost with inspection instants

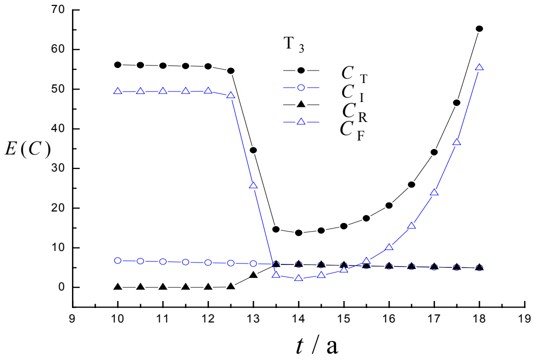

Fig. 7. The variation of the expected cost with inspection instants

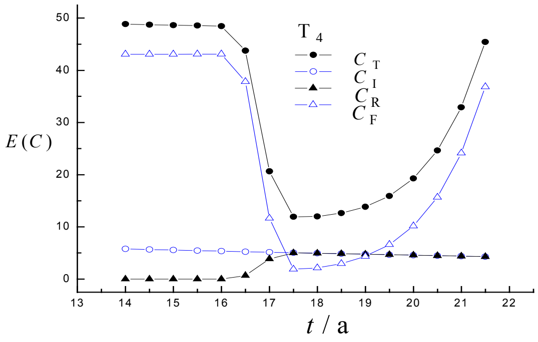

Fig. 8. The variation of the expected cost with inspection instants

Figs. 5~8 show the total expected costs corresponding to the first, second, third and fourth inspections, respectively. It is seen that the optimal inspection instants are $T_1 = 6\text{a}$, $T_2 = 10\text{a}$, $T_3 = 14\text{a}$ and $T_4 = 17.5\text{a}$, respectively. Since the design lifetime of ship structures is usually 20a,three inspections should be performed to meet the demand of a minimum acceptable reliability index $\beta_{\min}$. It is also seen that the minimum value of C_T is not sensitive to the different optimal inspection instants. Furthermore, the variation laws of $C_\text{T}, C_\text{I}, C_\text{R}$ and C_F with the inspection instant t remain the same. The variation law of C_T is approximately the same as that of C_F. The reason is that a very large C_F dominates the variation law of C_T. Since the real rate of interest α has more significant influence on C_I than that of the occurrence probability of an inspection event, C_I decreases as the time increases. Similarly, C_R increases and then decreases with the time due to the effect of the real rate of interest α .

Table 3. Comparison of results for different number of inspections

	Number of inspections		
	2	3	4
Time of inspection	6 10	6 10 14	6 10 14 17.5
Inspection cost	14.66	20.43	25.47
Repair cost	14.62	20.39	25.39
Failure cost	260.38	14.47	8.16
Total cost	289.66	56.29	59.02
Total failure probability	5.59E-03	2.20E-04	2.84E-06

Table 3 shows the results of different number of inspections. It can be seen that the total failure probability decreases gradually with the increase of the number of inspections. Furthermore, the total failure probability for three and four inspections meets the demand of the target failure probability. While the total failure probability for two inspections does not meet the demand of the target failure probability. By comparing the total expected costs, it is seen that the optimal inspection and repair plan is to perform three inspections in the design lifetime. Fig.9 shows the comparison of the risk costs, the inspection and repair costs among the above three inspection and repair strategies. It can be found that the risk costs decrease while the inspection and repair costs increase with the increase of the number of inspections. For this reason, it is necessary to optimize the inspection and repair plan so that a balance is achieved between the risk costs and the inspection and repair costs.

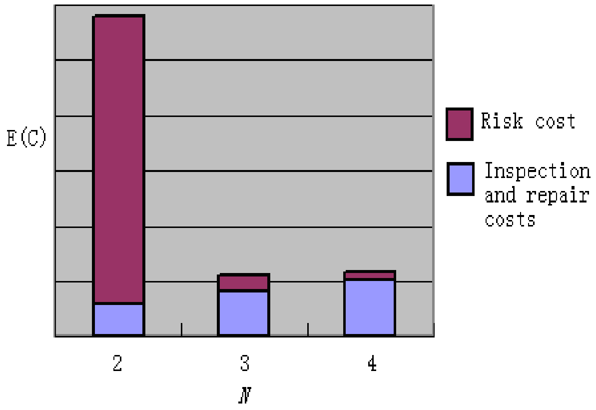

Fig. 9. Comparison of risk cost for different inspection strategies

Table 4 presents the comparison between the different inspection and repair strategies for the first inspection method. It is seen that different inspection and repair strategies significantly influence the number of inspections N, the inspection instant T and the inspection interval Δt. It is further seen that the total expected cost is minimum for the pair of the repair strategy 1 and repair method 2, which is the optimal inspection and repair strategy. The total expected costs of the repair strategy 2 are higher than those of the repair strategy 1. The reason is that compared with the repair strategy 1, the failure costs have significant increases for the repair strategy 2. This indicates that all the detected corrosion damages should be repaired after an inspection performed.

To consider the effects of different repair methods on the optimal inspection and repair strategy, three different costs of repair are taken into account herein for three different repair methods. The corresponding results are shown in Figs.10 and 11.

Table 4. Comparison among different inspection and repair strategies

	Repair strategy 1			Repair strategy 2		
Number of inspections	Time of inspection			Time of inspection		
	Method 1	Method 2	Method 3	Method 1	Method 2	Method 3
1	6	6	6.5	8.5	8.5	8.5
2	10	12	14	12	14	16
3	14		16			
Inspection cost	20.43	14.15	13.52	18.75	12.94	12.50
Repair cost	20.39	28.22	40.57	18.39	25.60	37.31
Failure cost	15.47	11.97	30.01	48.17	53.27	25.51
Total cost	56.29	54.34	57.10	85.31	91.81	75.32
Total failure probability	2.20E-04	2.20E-04	2.84E-06	2.20E-04	2.20E-04	2.84E-06

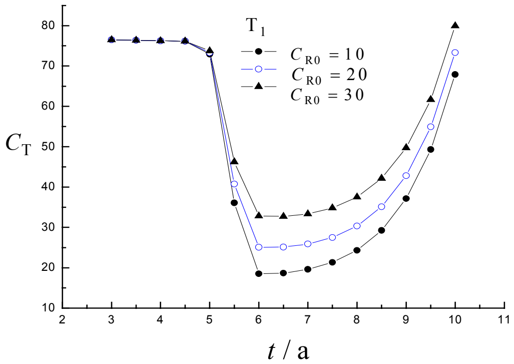

Fig. 10. Comparison of inspection instant between different repair methods

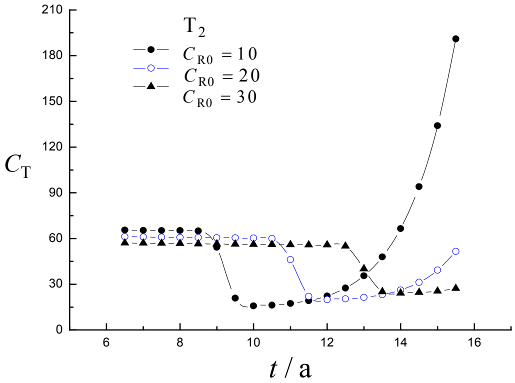

Fig. 11. Comparison of inspection instant between different repair methods

It can be seen from Fig. 10 that the first optimal inspection instant is not sensitive to the variation of the repair costs. Furthermore, the total expected costs increase as the repair costs increase, which further leads to different minimum total expected costs. However, different repair costs do not influence the variation law of the total expected costs. From Fig. 11, it can be seen that the second optimal inspection instant is sensitive to the variation of the repair costs. It is also seen that different repair costs do not influence the variation law of the total expected costs. Furthermore, the minimum total expected costs remain the same. When $t \le 9$a and $t \ge 13.5$a, the total expected costs decrease with the increase of time. These conclusions are very different from the results for the first inspection instant. The reason for this is that a repair is performed after the first inspection. The higher the repair costs are, the smaller the failure probability is. This further leads to the decrease of the risk costs, which can adequately compensate the increase of the repair costs.

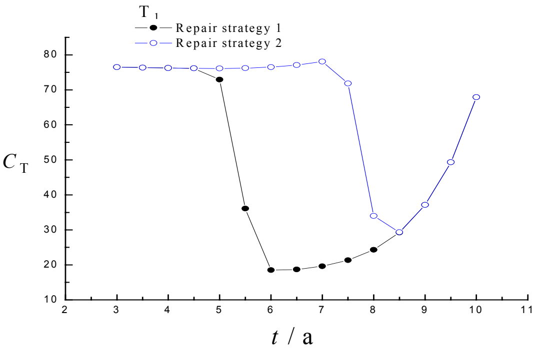

Fig. 12. Comparison of inspection instant between different repair strategies

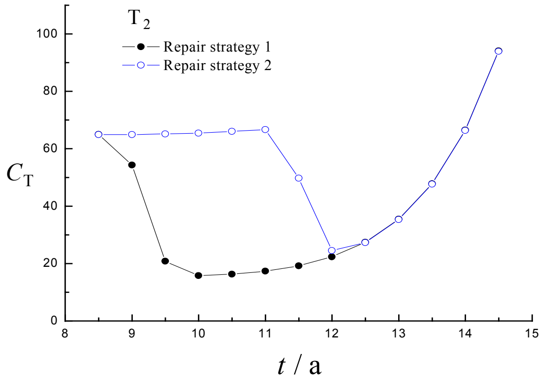

Fig. 13. Comparison of inspection instant between different repair strategies

Figs.12 and 13 give the results for two different repair strategies studied in this paper. It is seen that different repair strategies have significant influence on the optimal inspection instant, while the optimal inspection interval is 4a for both repair strategies. It is also seen that the minimum total expected costs are sensitive to the repair strategy.

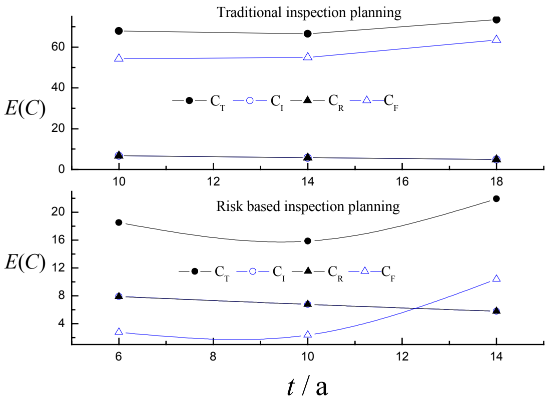

Fig. 14. Comparison of inspection instant between different inspection planning

To compare with the traditional inspection method, Fig.14 gives the results for both the proposed method and the traditional inspection method. For the traditional inspection strategy, an inspection and repair must be performed when $\beta(t)=\beta_{\min}$. So the first, second and third inspection instants are 10a,14a and 18a, respectively . While the first, second and third inspection instants are 6a,10a and 14a by using the proposed method, respectively . It can be seen that there exists a significant difference between these two inspections planning, while the inspection intervals and the number of inspections are all the same. It is further seen that 60% costs can be reduced by using the risk based inspection method due to the difference of the number of inspections. This indicates that the risk based inspection planning gains an advantage over the traditional inspection planning.

5.2 Sensitivity Analysis

5.2.1 Sensitivity of the minimum detectable corrosion size

To investigate the effect of the variation of the minimum detectable corrosion size on the total expected costs, the coefficients of variation of 0.1,0.3 and 0.5 are adopted for the minimum detectable corrosion size. Figs. 15 and 16 give the total expected costs with the inspection instants for the VI and the UI methods, respectively.

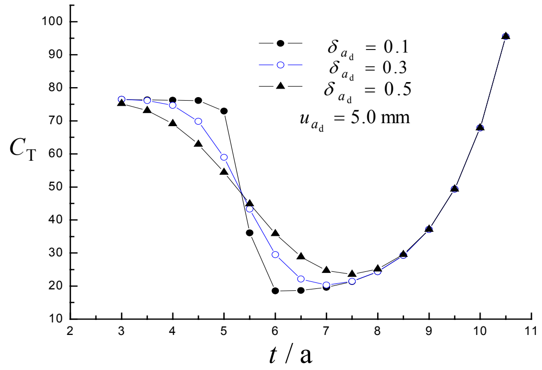

Fig.15. Optimal inspection instants for different minimum detectable sizes of corrosion

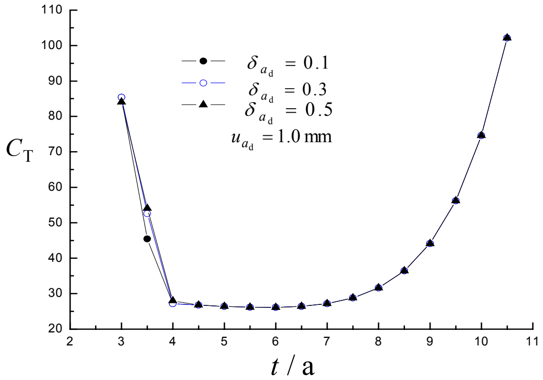

Fig.16. Optimal inspection instants for different minimum detectable sizes of corrosion

From Fig.15 it can be seen that different values of δ_{a_d} have significant influence on the optimal inspection instant. It can be further seen that different values of δ_{a_d} have significant influence on the total expected costs with t ranging from 3a to 8.5a. While this influence disappears when t exceeds 8.5a . From Fig.16 it can be seen that different values of δ_{a_d} have no influence on the optimal inspection instant. It can be also seen that different values of δ_{a_d} have significant influence on the total expected costs with t ranging from 3a to 4.5a. While this influence disappears when t exceeds 4.5a.

5.2.2 Sensitivity of the real rate of interest

Tilly [19] suggested that the real rate of interest is 0.02 in Switzerland and 0.1 in the United States, and the majority of the developed countries fix the real rate of interest between 0.06 and 0.08. To cover the above mentioned range, four values of $\alpha=0$, $\alpha=0.01$, $\alpha=0.04$ and $\alpha=0.08$ are used in this study. Where $\alpha=0$ denotes that the effect of the real rate of interest is not considered. The results are given in Fig. 17.

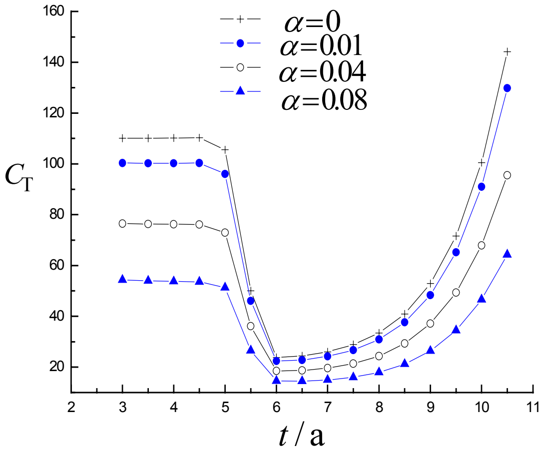

Fig.17. Optimal inspection instants for different real rates of interest

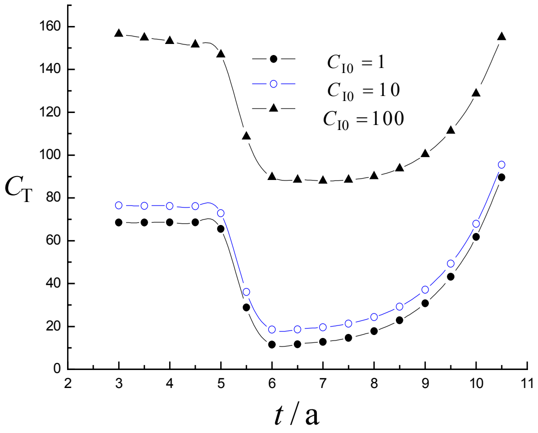

Fig.18. Optimal inspection instants for different inspection costs

From Fig.17 it is seen that different real rates of interest have no influence on the optimal inspection instant. It is also seen that different real rates of interest significantly change the total expected costs. Furthermore, the total expected costs decrease with the increase of the real rate of interest. However, the different real rates of interest do not influence the variation law of the total expected costs.

5.2.3 Sensitivity of the inspection costs

Three values of $C_{I0}=1$, $C_{I0}=10$ and $C_{I0}=100$ are adopted for investigating the effect of the variation of the inspection costs on the total expected costs. The results are given in Fig. 18. It is seen that the division of the inspection costs by ten has no influence on the optimal inspection instant. While the multiplication of the inspection cost by ten significantly influence the optimal inspection instant. It is also seen that the division of the inspection cost by ten does not significantly influence the total expected costs. While the multiplication of the inspection cost by ten significantly influence the total expected costs. When the inspection costs varies from 10 to 100, the total expected costs approximately increase by five times. In addition, it should be noted that different inspection costs do not influence the variation law of the total expected costs.

5.2.4 Sensitivity of the repair costs

Three values of $C_{R0}=1$, $C_{R0}=10$ and $C_{R0}=100$ are adopted for investigating the effect of the variation of the repair costs on the total expected costs. The results are given in Fig. 19. It is seen that the division of the repair costs by ten has no influence on the optimal inspection instant. While the multiplication of the repair costs by ten significantly influence the optimal inspection instant. It is also seen that the division of the repair costs by ten does not significantly influence the total expected costs . While the multiplication of the repair costs by ten significantly influence the total expected costs. When the repair costs varies from 10 to 100, the total expected costs approximately increase by five times. Besides, it should be noted that different repair costs do not influence the variation law of the total expected costs.

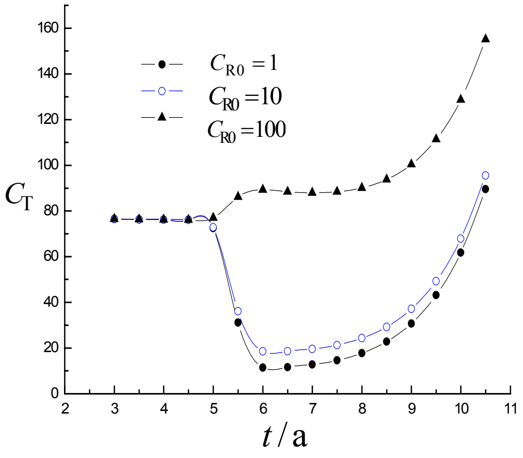

Fig.19. Optimal inspection instants for different repair costs

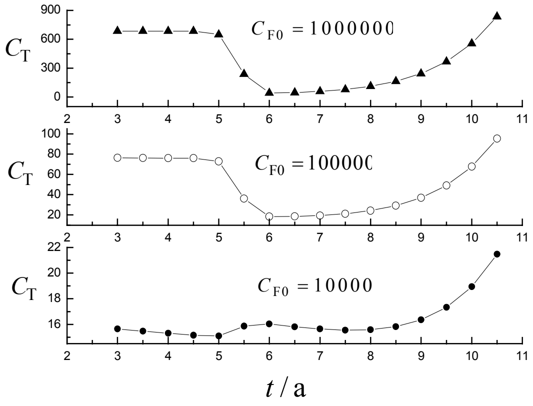

Fig. 20. Optimal inspection instants for different failure costs

5.2.5 Sensitivity of the failure costs

Three values of $C_{F0} = 1$, $C_{F0} = 10$ and $C_{F0} = 100$ are adopted for investigating the effect of the variation of the failure costs on the total expected costs. The results are given in Fig. 20. It is seen that the multiplication of the failure costs by ten has no influence on the optimal inspection instant. While the division of the failure cost by ten significantly influence the optimal inspection instant. It is also seen that the multiplication of the failure costs by ten does not significantly influence on the total expected costs . While the division of the failure costs by ten significantly influence the total expected costs. Besides, it should be noted that the minimum total expected costs are not sensitive to the variation of the failure costs. Furthermore, different failure costs do not influence the variation law of the total expected costs.

5.2.6 Sensitivity of the repair criteria

Generally, the repair criteria are determined according to the corresponding codes. For illustrative purposes, the maximum allowable thickness of the corrosion wastage of 10mm is adopted herein. To consider the effect of the uncertainty of the maximum allowable thickness of corrosion wastage on the total expected costs, three values of $d_{\max} = 8\text{mm}$, $d_{\max} = 10\text{mm}$ and $d_{\max} = 12\text{mm}$ are investigated. The results are given in Fig. 21. It is seen that different $d_{\max}$ have a significantly influence on the optimal inspection instant and the minimum total expected costs. Furthermore, the minimum total expected costs increase with the increase of $d_{\max}$. It is also seen that different $d_{\max}$ have a significant influence on the total expected costs with t ranging from 6.5a to 9a. While this influence nearly disappears when the value of t is less than 6.5a or more than 9a.

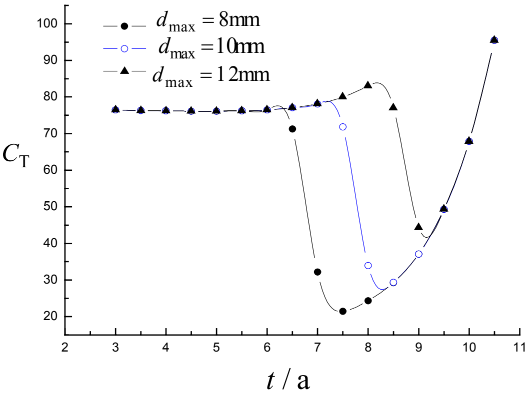

Fig.21. Optimal inspection instants for different repair criteria

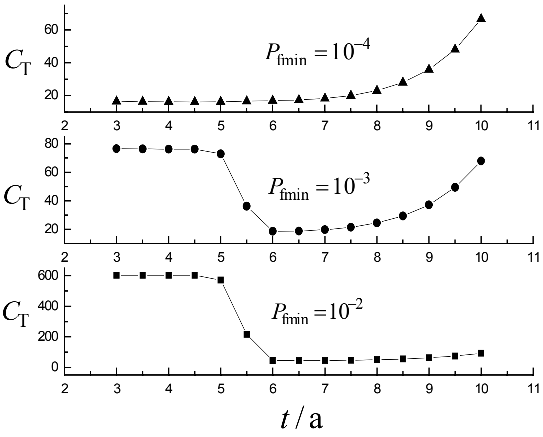

Fig.22. Optimal inspection instants for different failure criteria

5.2.7 Sensitivity of the failure criteria

Three values of $P_{\text{fmin}} = 10^{-2}$, $P_{\text{fmin}} = 10^{-3}$ and $P_{\text{fmin}} = 10^{-4}$ are adopted for investigating the effect of the variation of the failure criteria on the total expected costs. The results are given in Fig. 22. It is seen that different failure criteria have a significant influence on the optimal inspection instant. While different failure criteria have no significant influence on the minimum total expected costs. It is also seen that the variation law of the total expected costs for

$P_{\mathrm{fmin}} = 10^{-4}$ is different from those for $P_{\mathrm{fmin}} = 10^{-2}$ and $P_{\mathrm{fmin}} = 10^{-3}$.

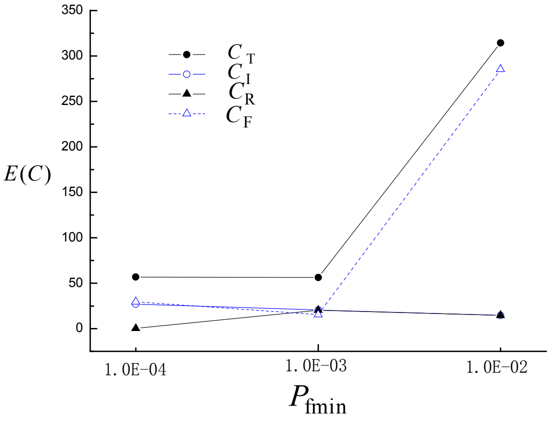

Fig. 23. The expected total cost for different failure

Landet et al., [4] suggested that an optimal failure criterion is the target failure probability resulting in the lowest total expected costs. This approach is used to the numerical example studies herein. Fig.23 gives the total expected costs for different failure criteria. It is found that the failure probability of 10^{-3} is the optimal failure criterion, which results in the lowest total expected costs. This is consistent with the result given by Ayyub et al.,[15]

Due to many assumptions made herein, the numerical results obtained and conclusions drawn are applicable to the example problem. Should any of these assumptions change, the results will change accordingly. Therefore, these results cannot be applied to any other ship structures subjected to corrosion deterioration. However, the method for obtaining the results can be applied to any corroded ship structures subjected to corrosion deterioration, using relevant data for that ship structure.

6 Conclusions

This paper develops the risk based inspection and repair planning for ship structures. The optimal inspection and repair planning is obtained by minimizing the total expected costs in the expected design lifetime. The event margins are developed as well. The Hohenbichler method is adopted for the calculation of the probabilities of intersection events. A component subjected to corrosion deterioration is investigated to illustrate the application of the proposed method. The results show that the optimal inspection instant is not the instant when the reliability index reaches the minimum acceptable reliability index. An inspection should be performed before the instant when the reliability index reaches the minimum acceptable reliability index. Furthermore, the proposed method can result in a better resource allocation to develop an effective inspection and repair planning. At the same time, an optimal failure criterion is determined by comparing the total expected costs associated with each inspection and repair strategy. For the numerical example studied in this paper, from the analysis of the sensitivities for various parameters, it is found that the decreases of the inspection and repair costs do not have much influence on the optimal inspection and repair planning if we take the values of the inspection, repair and failure costs into consideration. However, the increases of the inspection and repair costs have very significant influence on the optimal inspection and repair planning. At the same time, the decrease of the failure costs has a very significant influence on the optimal inspection and repair planning. However, the increase of the failure costs does not have much influence on the optimal inspection and repair planning.

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