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## Extension of Mappings and Pseudometrics

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*Abstract:* The aim of this paper is to show a development of various methods of extensions of mappings and their interrelations. We shall point out some methods and relations entailing more general results than those originally stated.

*Key words:* Extension of function, extension of pseudometric, metric space.

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