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A Flow Model for the Effect of a Slanted Base on Drag

Experiments by Morel have shown that slanting the base of a bluff body causes large variations in the drag, as the slant angle changes. For a particular, critical slant angle the drag changes discontinuously. This phenomenon was correlated with a drastic change in the type of base flow. A mechanism to explain this change, and therefore, the discontinuity in drag, is proposed. The basis of the mechanism is breakdown of the side edge vortices. An estimate of the swirl angle in these vortices is obtained using a swept-back mixing zone solution. This swirl angle and the other elements of the theory provide an estimate for the critical slant angle that is entirely consistent with the observed value.

Introduction

In the search to reduce drag of road vehicles, it has become clear that a more basic understanding of the complex flows about such bodies is required. Many investigations of drag and flow phenomena exist; see [1] for surveys of recent work.

A particularly interesting phenomenon is the effect that slanting the base has on the drag of a bluff body. Following up on work by Janssen and Hucho, partially reported in reference [1], Morel did a comprehensive study of that effect, see [1] and [2]. Janssen and Huccho observed an overshoot in drag and a change in separation pattern in tests on a model of a hatch-back car when the angle of the slanted portion of the roof was varied over a small range. In order to examine this effect more closely and gain some understanding of it, Morel, [1] and [2], made extensive tests on two models (see Fig. 1): (*a*) an ogive cylinder with a slanted base, mounted in the center of a wind tunnel to minimize wall effects; and (*b*) a vehicle-like model simulating a hatch-back car, mounted in the center of the tunnel and close to the tunnel wall. Because the wind tunnel models are simpler to discuss and there are more experimental details, the results of Morel will be used in this paper.

The most striking result from the tests of Morel was the extremely rapid change in drag coefficient, C_D, as the slant angle, β, is varied. In fact the data show, essentially, a discontinuity in C_D for a certain $\beta = \beta_c$. The results of Janssen and Hucho are qualitatively the same, but the variation of c_D with β is smooth, i.e., there exists no discontinuity. (The term discontinuity is used here for convenience and its descriptive accuracy, even though a mathematical discontinuity, does not exist.)

Visualizing the flow with smoke, Morel showed that there are two distinct types of base flow. For $\beta > \beta_c$ a closed base flow, typical of blunt-based axisymmetric bodies, was found. For $\beta < \beta_c$ streamwise vortices were formed at the side edges with a resultant 3-D separation pattern. It was concluded that switching from one separation pattern to the other caused the discontinuity in C_D.

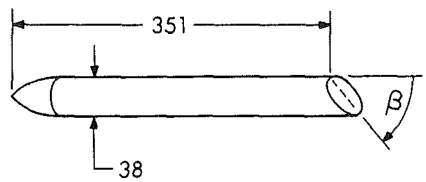

Fig. 1(*a*) Ogive-cylinder model

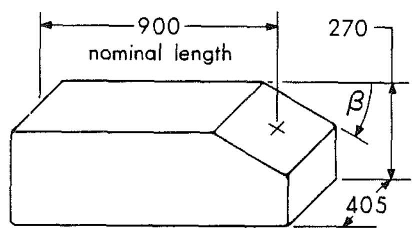

Fig. 1(*b*) Vehicle-like model

Fig. 1 Models tested by Morel [2]. Dimensions in mm.

Additional evidence for the existence of the streamwise vortices springing from the side edges is given by Carr, [3]. He used the surface indicator (oil flow) method to visualize the flow and found clear and distinct edge vortices for $\beta = 25$ deg but not for $\beta = 35$ deg. Carr also discusses the downwash produced by, and the effect on the rear lift force of the side edge vortices.

The primary purpose of this paper is to propose a flow mechanism to explain the change in separation pattern and the discontinuity in C_D. The mechanism involves breakdown of the side-edge vortices. Vortex breakdown has been relatively well studied and is partially understood; the work on this subject through 1972 has been reviewed by Hall, [4], and later work by Leibovich, [5]. To make use of the empirical knowledge on vortex breakdown, a model for the flow over a side edge is required. For an idealized side edge configuration, a flow model is constructed which allows a simple estimate of the swirl in the vortex. Empirical evidence shows that breakdown occurs for a value of swirl corresponding to $\beta = \beta_c$, approximately. The consistency of this theoretical result and the experimental data is sufficient to warrant further examination of this mechanism and to test it in additional experiments.

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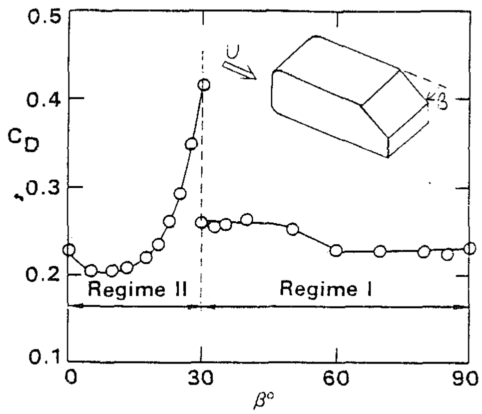

Fig. 2 Drag coefficient of the vehicle-like model in the free stream location, from [2]

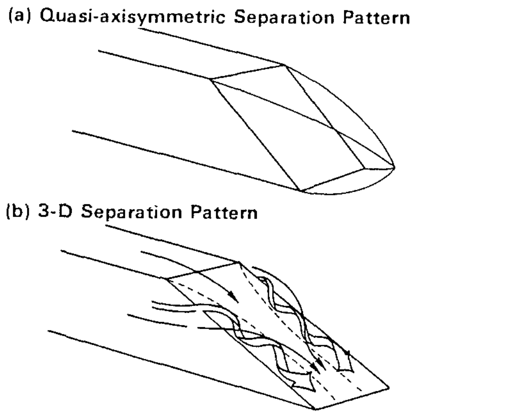

Fig. 3 The two types of base flows for a body with a slanted base: (a) The closed type exists in regime I; (b) the open type with side edge vortices exists in regime II.

The Experimental Evidence

For the two models tested by Morel, (see Fig. 1) the β_c were different. It is simpler to discuss the vehicle-like model because it has a straight side edge. Some discussion of the ogive-cylinder model is given later.

The drag coefficient, based on projected frontal area, as a function of β is shown in Fig. 2. The base flows corresponding to Regimes I and II are shown in Fig. 3. In Fig. 3(*a*), for Regime I, the base flow is essentially like that of an axisymmetric body; in the mean the base flow is a closed region. In Fig. 3(*b*), for Regime II, a vortex springs from each side edge; between them there is attached longitudinal flow. In [3], Carr quotes the work of Potthoff, 1969, who pointed out the existence of the side edge vortices and the fact that they can prevent the slanted base flow from separating for β as large as 30 deg or even greater with suitable shaping of the body sides.

For Carr's model, which had a short "boot length" at the end of the slant base, $\beta = 25$ deg. The surface flow pattern clearly showed the trace of the side edge vortices. A part of this pattern is sketched in Fig. 4, which shows the rear view of the slanted part of the roof, i.e., slanted base. The flow along

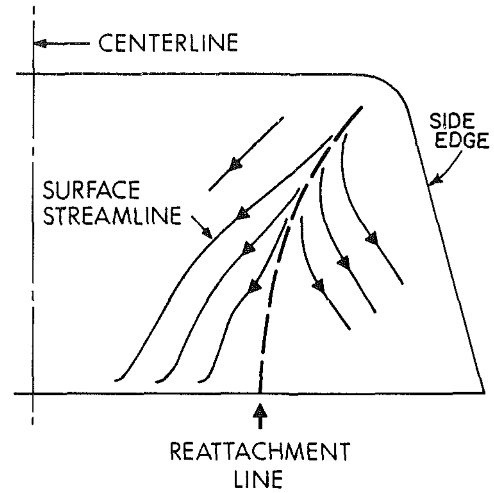

Fig. 4 Rear view of a slanted base showing part of the surface flow pattern. The flow along the side (out of the plane of the paper) separates at the side edge and reattaches at the dashed line; sketched from a photograph in [3].

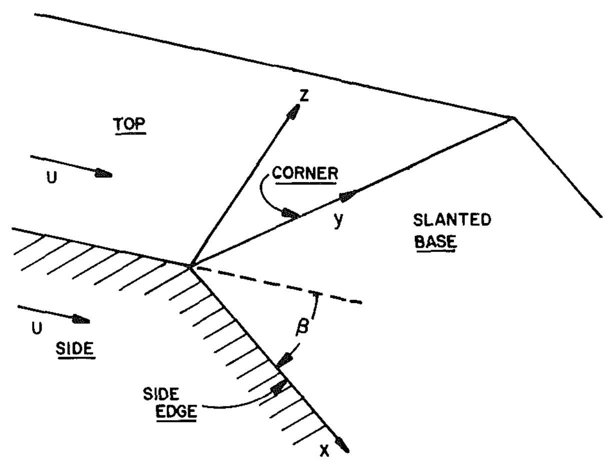

Fig. 5 A wedge, semi-infinite in the *y*-direction, as an idealized configuration for side edge flow. The freestream velocity, *U*, is parallel to the top and the side. The slanted base is $z = 0, x > 0, y > 0$.

the side separates at the side edge and reattaches along the dashed line. The surface streamlines, which are easily visible in the oil film used for the visualization and which emanate from the dashed line, are typical of reattachment, forming what is often called a herringbone pattern close to the reattachment line. Using the reattachment line to indicate the inboard boundary of the vortex, this result shows that the vortices extend over about one-half of the width of the slant base, for this case.

There is enough evidence to conclude that side edge vortices exist for $\beta < \beta_c$. For smaller β, say $\beta < 10$ deg, they probably exist but would be difficult to detect because they are weak. As pointed out by Morel, [1] and [2], the initial decrease in C_D, for $0 \leq \beta \leq 10$ deg, may be a boattail type of effect, familiar in the design of projectiles, rather than a side edge vortex effect. The latter probably begin to dominate for $\beta >$ 10 deg. For $\beta = \beta_c$ the discontinuity in C_D occurs and the side

Nomenclature

C_D = drag coefficient based on frontal area
C_P = pressure coefficient
O = order of magnitude symbol
S = see (2)
u = axial velocity in vortex and velocity in x-direction
U = free stream velocity
V_θ = azimuthal velocity in vortex
w = velocity in z-direction
x,y,z = Cartesian coordinates, see Fig. 5
β = angle of slanted base
β_c = angle of slanted base dividing Regimes I and II
ϕ = swirl angle in vortex
σ = mixing coefficient

Subscripts

r,s = reattachment and separation

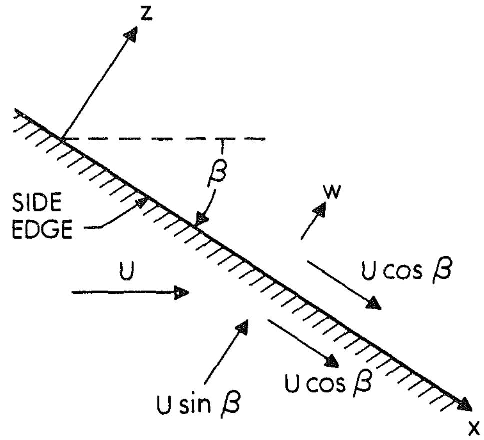

Fig. 6(*a*) The view in the plane $y = 0$

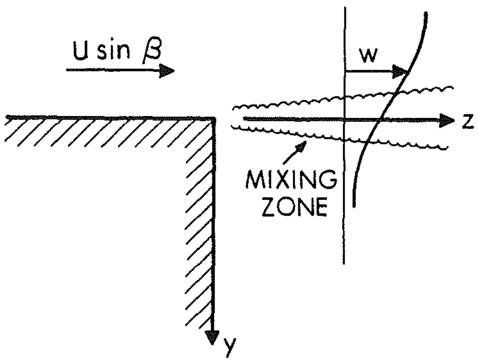

Fig. 6(*b*) The view in the plane x = constant. The flow separates from the side edge, forming a free shear layer or mixing zone.

Fig. 6 Flow over an infinite, yawed side edge at $y = 0, z = 0$

edge vortices disappear. This massive change in the slanted base flow field occurs suddenly, for a small change in β.

Before discussing the flow mechanism for this sudden change, some estimates of the side edge flow are necessary.

The Side Edge Flow

A description of the complete flow field over either of the models shown in Fig. 1 by analytical methods would be very difficult. To obtain a tractable idealization the local flow at the side edge should be examined. One possibility is shown in Fig. 5 where the side edge is along the x-axis and the leading edge of the slanted base, or corner, is along the y-axis. The z-axis is normal to the base and forms a right-handed, rectangular, Cartesian coordinate system. The external flow, U, is parallel to the top and side surfaces. The boundary layer would be neglected until the flow reaches the side edge and corner. This is an idealization, however, but will not yield the simple estimates needed here.

As a further simplification, appropriate to the side edge flow away from the corner, consider an infinite, slanted, or yawed, side edge. That is, there is no corner. The side view, $y = 0$ and the back view, x = constant, are shown in Figs. 6(*a*) and 6(*b*) respectively. The solution to this problem is independent of x. This kind of idealization is basic in the study of swept wings. As an additional approximation the incoming flow, U, is taken to be the free stream velocity over the model in Fig. 1(*b*). The thickness of the boundary layer, before it separates from the edge, is neglected. This is the same assumption made in the classic Görtler solution for the 2-D, free, turbulent, shear layer, often called the mixing zone problem. We obtain this problem, discussed in [6], if $\beta = 90$ deg. This idealization of the side edge flow can be called a combination of the 2-D mixing zone solution with the sweep-back principle.

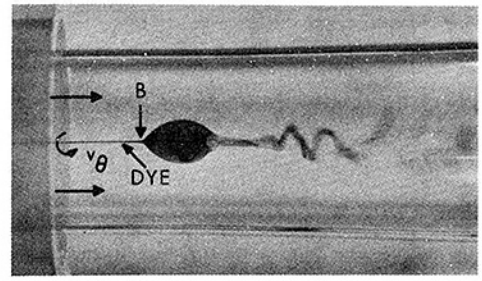

Fig. 7(*a*) Bubble type

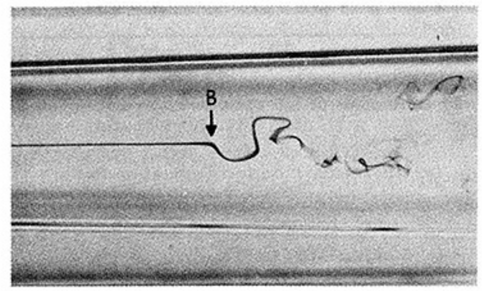

Fig. 7(*b*) Spiral type

Fig. 7 Vortex breakdown in a duct flow, made visible with dye introduced into the flow, from [6]. *B* is the breakdown point.

However, an additional caution must be discussed. In most applications of the sweep-back theory the independence principle holds, i.e., the flow in the (y,z) plane can be computed, and then the flow in x-direction determined. For laminar flow this is always possible but, because of the Reynolds stress terms in the momentum equations, the independence principle is not strictly valid for turbulent flow. It will be assumed here that the independence principle holds; this assumption should cause a small error in the swept-back mixing zone problem.

Therefore, from Fig. 6, the mixing zone flow is determined for the external velocity $U \sin \beta$. The velocity, w, in the z-direction, in the mixing zone is given by the well known solution, (see [6])

$$w = (1/2)U\sin\beta[1 + \text{erf}(-\sigma y/z)] \quad (1)$$

where $\sigma \simeq 12$ is the mixing coefficient. The vorticity, $\partial w/\partial y$, is solely in the x-direction. The velocity parallel to the edge, $U \cos \beta$, is constant for all z. This is as far as the idealization can be carried. Specifically, in the absence of pressure gradients, the free shear layer cannot reattach to the wall $z = 0$.

On the actual model, the separated flow must reattach on the slanted base in order to form the vortex. Only empirical criteria for reattachment are available and these require knowledge of the pressure. The criterion given by Crabtree [7] for leading edge bubbles on airfoils is $S \lesssim 0.35$ where

$$S = (C_{pr} - C_{ps})/(1 - C_{ps}) \quad (2)$$

and C_{pr} and C_{ps} are the pressure coefficients at reattachment and separation, respectively. Using Morel's pressure data [1] reattachment is indicated by Crabtree's criterion. At this stage this is only a consistency statement. It would be useful deductively if estimates of the pressure could be obtained.

The most important quantity that can be estimated from the solution to the swept-back mixing zone problem is the swirl in the side edge vortex. Assume that the flow over the actual model reattaches on the slanted base. The swirl angle is defined as

$$\phi = \tan^{-1}(v_\theta/u),$$

where v_θ is the azimuthal velocity in the vortex and u is the velocity in the x-direction, i.e., axial velocity in the vortex. ϕ varies with position in the vortex because v_θ and u vary. The estimate for v_θ is obtained from (1) and that for u from $U \cos \beta$. Thus,

$$v_\theta = O\ (U \sin \beta)$$
$$u = O\ (U \cos \beta)\quad ,$$

so that

$$\phi = O\ (\beta)\quad .$$

This result for ϕ will be used as approximate relation

$$\phi \simeq \beta\quad . \qquad (3)$$

The velocities v_θ and u could be measured using a laser Doppler velocimeter to obtain a check on this result.

An estimate of ϕ is needed to relate breakdown of the side edge vortex to the discontinuity in C_D. The error in this estimate may be considerable, but it should decrease as x increases. Since only the ratio, v_θ/u, is used to estimate ϕ and it may have smaller error than v_θ and u.

The Vortex Breakdown Hypothesis

Vortex breakdown has been observed in a number of flows, e.g., flow over highly swept wings and flow in ducts. Breakdown is one of the more remarkable aspects of vortex cores. Although it has been studied vigorously, there is no completely satisfactory theory for it. The description of it given by Hall [4] will be quoted here; he refers to flow in a duct with swirl imparted by vanes at the duct entrance. "If we follow the fluid as it spirals along the duct we find, typically, that the structure of the vortex, as indicated for example by the velocity distribution over a cross section of the duct, varies only slowly in the axial direction and then, suddenly and, at first sight, unexpectedly, there is an abrupt change in the structure with a very pronounced retardation of the flow along the axis and a corresponding divergence of the stream surfaces near the axis." This abrupt change is called vortex breakdown. Since Hall's review many papers on the subject have appeared, too numerous to mention here. Leibovich [5] reviews the recent work. The important facts for present purposes are the abruptness of the change in the vortex core flow and its sensitivity to small changes in flow conditions. Vortex breakdown occurs in two forms, mainly seen in reference [5]. One is called the bubble type and is nearly axisymmetric, at least close to the breakdown region; the second is called the spiral type and is highly asymmetric. These are illustrated in Fig. 7(*a*) and 7(*b*), respectively. In the duct flow a filament of dye is introduced along the axis which then gives a visual record of the breakdown.

Most of the quantitative information on vortex breakdown is obtained from duct flows because it is more difficult to run a controlled experiment in other flows. It seems that the phenomenon was first discovered in flows over highly swept wings; see [4] for references and a smoke flow photograph. This photograph shows a bubble type breakdown on one side of a delta wing and a spiral type on the other. Clearly the flow over a wing, with vortices on each side of the plane of symmetry, is more representative of the slanted base flows discussed here than duct flows. For typical wings, the spanwise separation of the vortices is much greater than in slanted base flows; the interaction beween the vortices is then quite different.

According to Hall [4] the necessary conditions for breakdown are: 1) the maximum $\phi \geq$ 40 deg; 2) an adverse pressure gradient; 3) stream tube divergence in the vortex core. Conditions (2) and (3) are satisfied for the side edge vortices. The estimate obtained from the swept-back mixing zone problem gave $\phi \simeq \beta$ so that condition (1) would give $\beta \simeq$ 40 deg for breakdown.

Condition (1) is clearly approximate. It was deduced by Hall from the limited experimental data available at the time. Since then laser-Doppler anemometry has come into wide use

Table 1*

Re	Type of breakdown	ϕ_m (deg)	Core expansion ratio
1920	spiral	31.7	1.64
	bubble	30.4	2.54
2812	spiral	30.6	1.78
	bubble	30.8	1.97
3348	spiral	33.5	2.10
	bubble	29.9	2.76

*Data from [5].

and is ideal for velocity measurements in vortex breakdown flow fields because no probe is introduced into the flow; a probe in the breakdown region can introduce large perturbations in the flow field. Results from many experiments on duct flows, using laser-Doppler anemometry, are presented by Leibovich [5]. In particular, his Table 1 summarizes data for the velocity in the approach flow, i.e., the flow to within about 1.5 vortex core diameters upstream of the breakdown point. Both bubble and spiral types of breakdown were observed and measured at three Reynolds numbers, which is based on the axial velocity far from the axis and the vortex core diameter. From those data, Table 1 was constructed.

The swirl angle, ϕ_m, presented in Table 1 is defined as

$$\phi_m \equiv \tan^{-1}(v_{\theta_{\max}}/u_{\max}).$$

It is calculated using certain functional forms for v_θ and u; the parameters in those forms are determined from a best fit to the data. The value of ϕ_m is a measure of the swirl angle for vortex breakdown; it is more reliable than the criterion given in condition (1).

In Table 1, ϕ_m varies between 29.9 and 33.5 deg. No trend in the variation of ϕ_m with Reynolds number can be detected; the average value of ϕ_m is 31.9 deg for the spiral type, 30.4 deg for the bubble type, and 31.2 deg if both types are considered. For purposes of estimation, $\phi_m \simeq$ 30 deg can be used. Assuming that ϕ_m approximates the estimate of ϕ in (3), breakdown of the side edge vortex is estimated to occur at $\beta =$ 30 deg.

Another feature of vortex breakdown that enters the proposed mechanism to explain the discontinuity in C_D is the increase in size of the vortex core after breakdown. Downstream of the breakdown region a new vortex is formed, in the wake region. The ratio of the wake-core radius to that in the flow upstream of breakdown is called the core expansion ratio, given in Table 1. The core radius is defined as the radial coordinate of the maximum in the azimuthal velocity. The smallest entry in the table gives a 64 percent increase in the size of the core.

One of the major results of the aforementioned theory is that breakdown of the side edge vortices is possible for the range of parameters covered in Morel's experiments. Assuming it does occur, its relations to the sudden change in C_D can be described as follows. Before breakdown the two vortices occupy a substantial part of the slanted base; see the discussion of Fig. 4, above. The sudden billowing of these vortices, as measured by the core expansion ratio, will cause the mutual interaction between them and their interaction with the outer stream to increase. The adverse pressure gradient acting on the vortices will increase and cause the breakdown region to move further upstream, etc. Rapid movement of the breakdown region is observed in duct flow experiments. The 3-D separation pattern, or open base flow, then collapses into the quasi-axisymmetric base flow. The final stage of this collapse cannot be described by the model.

The estimates of swirl, the relationship between swirl and slant angle, $\beta \simeq \phi$, and the model proposed here can now be combined. They give the critical slant angle for the discontinuity in C_D, $\beta_c \simeq 30$ deg. Morel's experiments give (Fig. 2) $\beta_c = 30$ deg. Considering the idealizations made in arriving at the theoretical result, its agreement with the experimental value must be considered fortuitous. The proper conclusion is that the theory is consistent with the available experimental findings.

For the ogive-cylinder wind tunnel model tested by Morel ([2], Fig. 1(*a*)) the C_D versus β variation was generally the same as that in Fig. 2. However, the discontinuous decrease in C_D, from 0.62 to 0.3 in this case, occurs at $\beta_c = 43$ deg rather than $\beta_c = 30$ deg as for the straight side-edge. Development of a flow model for the ogive-cylinder slanted base, where the edge is an ellipse, will not be attempted. Some understanding of the flow for $\beta < \beta_c$ can be obtained by isolating three modules of the base flow field, assuming steady flow. 1) At the top of the base the flow separates. The scale of this separated region, measured by the location of reattachment line, is a small fraction of the major axis. 2) In the neighborhood of the side of the base the flow separates and side-edge vortices are formed. The flow pattern would be topologically the same as that in Fig. 4. The scale of the separation region is a significant fraction of the minor axis. 3) At the bottom of the base the flow has, at most, a small scale separated region.

It appears reasonable to neglect first order interaction between these three modules; then some portion of the base surface streamlines can be sketched. For small enough β the side-edge vortices must depart from the base either by reaching the edge or by lifting off. It is conjectured that for $\beta \simeq 30$ deg either the swirl is decreased below the critical value before they depart the surface or they break down off the surface. In the former, larger β is required for breakdown on the surface. In the latter, their mutual interaction and that with the outer stream, which is necessary to explain the sudden change in base flow and C_D, is relatively weak. Stronger vortices, i.e., larger β, would be required to have breakdown on the surface and the required interaction. In either case the conclusion is consistent with the observed larger β_c.

Discussion

More refined estimates for the swirl in the side edge vortices are possible, e.g., using the configuration of Fig. 5, but would require much more analysis. It would be preferable to test the central hypothesis of this theory experimentally. Introduction of a blunt body along the center of the vortex core, which is known to promote breakdown, could be useful as a diagnostic tool.

An explanation of the discontinuity in C_D at $\beta = \beta_c$ requires a process with the abruptness (almost explosive nature) of vortex breakdown. Theories based on some other hypothesis are possible, however. One of these could be based on the inability of the free shear layer to reattach for $\beta > \beta_c$.

In addition to an understanding of the phenomena that lead to the discontinuity in C_D, it would be desirable to be able to predict drag, especially for $\beta < \beta_c$. Linearized theory might be useful for the case of small β. Otherwise, only numerical methods could be successful.

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