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Air-driven reverse buoyancy

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Abstract

Air-induced fluidization in vibrated granular beds can be responsible of the phenomenon of reverse buoyancy. We show that the air flow induced across the bed, as the gap in the bottom of the cell evolves, can fluidize the bed in a similar way as in gas-fluidized *static* beds and consequently a buoyancy force can arise. To quantify this mechanism we use Kroll's model. © 2005 Elsevier B.V. All rights reserved.

Keywords: Granular matter; Vibration; Reverse buoyancy; Solid–fluid transitions

1. Introduction

A forced collection of grains can exhibit very rich and striking behavior that sometimes resembles the properties of fluids and solids [1,2]. Fluid-like behavior can be induced, among other means, by shaking [2,3] or by injecting gas through the grains [4–6]. A basic understanding of phase changes as a response of these dissipative systems to a change in forcing conditions, grains properties and of any relevant parameter is of practical and fundamental interest.

The influence of interstitial air in phenomena related to vibrated granular media have been studied in different contexts [7–14]. Under certain conditions, interstitial air has proved to be a crucial ingredient in segregation phenomena [10–14], and in this line some efforts to include air in molecular dynamics simulations have appeared

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recently [13]. However, a coherent description that would allow us to understand the role played by the air in vibrated granular systems is still lacking.

Shinbrot and Muzzio [15] reported the surprising observation that, under certain experimental conditions, large heavy objects immersed in a vibrated granular medium rise and similar light ones sink to the bottom. This phenomenon was called reverse buoyancy. In Ref. [16], it was proposed that this phenomenon could be explained by assuming that the granular bed behaves as a fluid in only a fraction of the cycle of vibration. In this article, we present a simple model that describes the role of air in the above fluidization. The key ingredient is not just the presence of air but the way it flows through the vibrated bed. We discuss the consequences of this kind of fluidization for the phenomenon reverse buoyancy.

We assume that the vibrated grain pack acts like a porous piston so the acceleration of the container induces a pressure difference between its ends as the pack moves relative to the bottom of the container, which in turn produces an air flow across the bed. We use Kroll's model [7] to describe the evolution of the gap under the grain pack, within one period of vibration. This model takes into account a force that arises as a consequence of the pressure difference across the bed that develops as the gap in the bottom evolves. We propose that the flow of air induced by this pressure difference can fluidize the medium in a similar way as it occurs in gas-fluidized *static* beds, where gas is injected against gravity. The model of Kroll will allow us to present explicit relations between the flow of air through the bed, the pressure difference between its ends and the gap formed at the bottom of the bed. From these relations we develop a quantitative description of the proposed mechanism of fluidization.

This article is organized as follows: in Section 2 we review the model of Kroll; in Section 3 we present a mechanism of fluidization; in Section 4 we discuss the consequences of this kind of fluidization in the context of the phenomenon of reverse buoyancy and our conclusion is given in Section 5.

2. Kroll's model

In the well-known Kroll's model [7], the granular bed is treated as a porous piston. When the effective gravity starts to point upward the piston takes off. According to Kroll's model, in the noninertial reference frame placed on the container, only two forces are taken into account during the flight: the effective weight of the granular bed and the force due to the pressure difference across the piston. The following evolution equation for the gap at the bottom of the bed is obtained (see Fig. 1):

$$\frac{d^2s}{dt^2} + (p_a - p_i) \frac{A}{m} = - \frac{d^2w}{dt^2} - g, \quad (1)$$

where s is the size of the gap, p_a is the atmospheric pressure, p_i is the time dependent pressure in the gap, A is the transversal area of the container, m is the mass of the piston, g is the acceleration of gravity and w is the position as a function of time of the container.

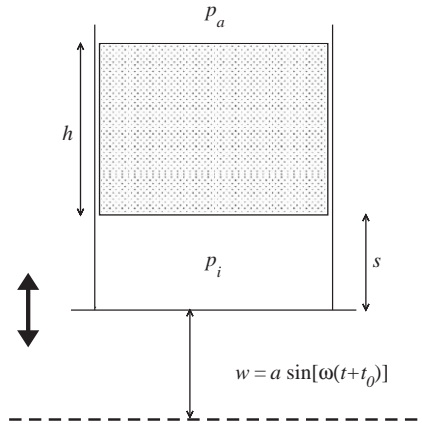


Fig. 1. The Kroll’s picture: the granular bed is treated as a porous piston. The container is subjected to sinusoidal vibrations of frequency ω and amplitude a . At the top of the bed we have atmospheric pressure p_a and at the bottom p_i is the time dependent pressure in the gap.

We can use an equation of state to relate the gap, and hence the volume of the air in the gap, with the pressure p_i . The air is assumed to behave as an incompressible fluid [7,8]. This results in the additional equation

$$s = \frac{G}{\rho_a A}, \tag{2}$$

where G is the mass of air in the gap and ρ_a is the density of air. As is usually done, it is assumed that the flux of air through the bed is given by Darcy’s law [7,17] which gives the following equation

$$\frac{dG}{dt} = \rho_a A \left[\frac{k(p_a - p_i)}{\mu h} \right], \tag{3}$$

where the permeability k of the piston is assumed to be constant, h is the height of the piston and μ is the viscosity of air. Eqs. (1)–(3) can be integrated for the unknowns $s(t)$, $p_i(t)$ and $G(t)$. We assume that the container is subjected to sinusoidal oscillations of amplitude a and angular frequency ω , so we have that $w = a \sin[\omega(t + t_0)]$, where $\sin(\omega t_0) = 1/\Gamma$ and $\Gamma = a\omega^2/g$. At $t = 0$ the medium takes off.

It is convenient to introduce the dimensionless quantities $s' = s/a$ and $t' = \omega t$, to find the evolution equation for the dimensionless gap s' :

$$\ddot{s}' + \frac{1}{\tau_K} \dot{s}' = \frac{1}{\Gamma} [\Gamma \sin(t' + t'_0) - 1], \tag{4}$$

where $\dot{s}' = ds'/dt'$, $t'_0 = \omega t_0$ and the dimensionless number τ_K is given by

$$\tau_K = \frac{\omega k \rho_m}{\mu}. \tag{5}$$

We see that the effect of air in the piston movement is a usual frictional factor, with a friction coefficient given by $1/\tau_K$. We expect then that if τ_K is small then the gap will be small. The dimensionless number τ_K depends on the grains diameter through the permeability k of the medium [17]. An interesting prediction that arises is that the gap's dynamics depends only on the dimensionless numbers Γ and τ_K . Integrating Eq. (4) with initial conditions $s'(0) = s(0) = 0$ we obtain the evolution of the dimensionless gap $s'(t')$:

$$s' = \frac{\tau_K}{\Gamma} [C - t' - \tau_K(1 + \lambda\Gamma \sin \delta)e^{-t'/\tau_K} - \lambda\Gamma \cos(t' - \delta)], \quad (6)$$

where $C = \tau_K + \lambda\Gamma(\tau_K \sin \delta + \cos \delta)$, $\lambda = (1 + \tau_K^2)^{-1/2}$, and $\delta = \arctan \tau_K - \arcsin 1/\Gamma$. Eq. (6) is valid while $s' > 0$.

The gap is small [18] when a large pressure difference between the ends of the bed is induced. From Eqs. (2) and (3) we see that in this model the pressure difference across the bed is proportional to the first derivative of s and is given by

$$\frac{p_a - p_i}{gh\rho_m} = \frac{\Gamma}{\tau_K} s'. \quad (7)$$

From Darcy's law, the flow of air through the grains is proportional to this pressure difference.

3. Gas-fluidized beds and cyclic fluidization in vibrated granular beds

In the well-known technique of fluidizing beds by injecting gas against gravity a fluidized state is achieved when the force due to the pressure difference across the bed is high enough to cancel the weight of the bed [4–6]. An important control parameter in this technique is the velocity U at which gas is injected upward at the bottom of the *static* bed. As the gas flows up through the granular medium it induces a pressure difference that results in an effective force on the bed that points upward. This force rises as U is increased from zero and when it equals the weight of the bed fluid-like behavior starts to be observed [4,5]. This fluidization condition can be written by

$$\Delta P A \geq mg, \quad (8)$$

where ΔP is the pressure difference between the ends of the bed.

In Kroll's model the pressure difference and the effective gravitational field are time dependent. This can be seen if we rewrite the evolution Eq. (1) for the gap as

$$m \frac{d^2 s}{dt^2} = mg_{ef}(t) - [p_a - p_i(t)]A, \quad (9)$$

where $g_{ef}(t) = g\{\Gamma \sin[\omega(t + t_0)] - 1\}$ is the effective gravitational field. Before taking off, when g_{ef} points downward and the bed is in contact with the bottom of the container, there is no significant air flow, the bed is being pushed downward against the bottom of the container and there is no relative movement between the grains; under these conditions we consider the bed as being in a solid state. When g_{ef} starts to point upward the bed takes off, the size of the gap and the air flow through the

grains starts to increase. Introducing condition (8), for this case we can write the fluidization condition as follows ($g_{ef} \neq 0$):

$$\frac{[p_a - p_i(t)]A}{mg_{ef}(t)} \geq 1. \tag{10}$$

Combining Eqs. (9) and (10), the fluidization condition (10) is initially satisfied at the inflection point of the curve s' vs. t' shown in Fig. 2. This result is independent of the validity of Eqs. (2) and (3) and, consequently, of the assumptions made in deriving them. If we assume that Eqs. (2) and (3) hold, the beginning of the fluidized state occurs at the dimensionless time t'_f (measured from the time when g_{ef} starts to point upwards) which is given by

$$\cos(t'_f - \delta)e^{t'_f/\tau_K} = \frac{1 + \lambda\Gamma \sin \delta}{\lambda\Gamma\tau_K}. \tag{11}$$

Within Kroll's assumptions, at t'_f the pressure difference across the bed is maximum. From Eq. (4) or (7) and the discussion above, it is expected that the mechanism proposed will be effective if τ_K is sufficiently small. Eq. (11) gives, to our knowledge, a new result. With this equation, we can estimate when the new

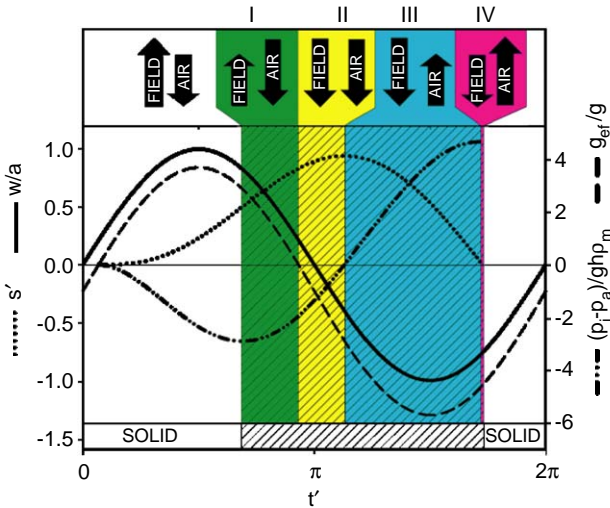


Fig. 2. Within a period of vibration, we show in this figure the evolution in time of the following dimensionless variables: the gap s' (from Eq. (6)), the pressure difference across the bed $(p_i - p_a)/gh\rho_m$ (from Eq. (7)), the effective gravitational field acting on the bed g_{ef}/g and the position of the container w/a , for the experimental conditions of reference [16] ($a = 8.5$ mm, $\omega = 2\pi 11.7$ rad/s, $\rho_m = 1.4$ g/cm³ and a mean grain's diameter of 230 μ m). The beginning of the fluidized state occurs at the inflection point of the curve s' vs. t' . It is shown in, which fraction of the cycle, the bed is considered to be in a solid state. Within the fraction of the cycle that runs from t'_f until the bed hits the bottom of the cell at the end of its flight, we identify four zones depending on the direction and intensity of the effective gravitational field and of the air flow. The direction of these quantities is shown at the top of the figure. Relative movement between the intruder and the grains is expected to be possible in the four zones, but only in zones I and IV the fluidization condition (10) is satisfied.

mechanism of fluidization is initiated. This can be experimentally verified by measuring what happens to the granular medium at the inflection point of the curve that represents the evolution of the gap.

In Fig. 2, we show the evolution in time of the gap (from Eq. (6)), the pressure difference across the bed (from Eq. (7)), the effective gravitational field acting on the bed and the position of the container for the experimental conditions of Ref. [16]. Within the fraction of the cycle that runs from t'_f until the bed hits the bottom of the cell at the end of its flight, we can identify four zones depending on the direction and intensity of the effective gravity g_{ef} and of the air flow. The fluidization condition (10) is satisfied in zones I and IV (see figure). If the flight time of the bed is small compared with the period of vibration, then we have a granular bed that alternates cyclically between a solid and a fluidized state.

4. Cyclic fluidization and reverse buoyancy

We are now going to explore the consequences of the above mechanism of fluidization in the context of the phenomenon of reverse buoyancy. Under certain conditions large heavy objects immersed in a vibrated granular medium rise and similar light ones sink to the bottom. This was called reverse buoyancy [15]. In Ref. [16], it was proposed that this phenomenon could be explained by assuming that the granular bed behaves as a fluid in only a fraction of the cycle of vibration. The necessary feature is that during most of the time in which the bed is in a fluidized state the effective gravity points upward.

In the fluidization condition explained in the previous section, the air flow points downward just after the bed takes off, while the effective gravity points upward [19]. If a buoyancy force arises in the above fluidized state, large heavy objects immersed in it may *sink* upward while similar light ones may *float* downward. This case is portrayed in zone I shown in Fig. 2. In zone IV the fluidization condition (10) is also satisfied, but now g_{ef} is pointing downwards and we expect to observe regular buoyancy in this zone (heavy objects sink to the bottom of the container, while similar light ones float to the top). In zone II the fluidization condition (10) is not satisfied because the air flow and the effective gravity point in the same direction, while in zone III (10) is not satisfied because the air flow is not strong enough to produce a pressure difference across the bed that cancels the effective weight of the piston. Further work is needed to anticipate how the bed would behave in zones II and III, but it is expected that relative movement between the grains and the intruder is possible in these zones. In particular, we can ask ourselves: is there a buoyancy force in zones II and III? In this line, valuable information can be obtained by resolving the motion of an intruder immersed in the bed within a period of vibration and comparing it with the evolution of the measured pressure difference across the bed. Our model predicts, for the case of reverse buoyancy, a change in the motion of an intruder at time t'_f . Other mechanisms in which interstitial air is considered can play a role in these zones [14]. When interstitial air is not relevant [20] (because of a large bed particle's diameter or density) other models may apply [21,22].

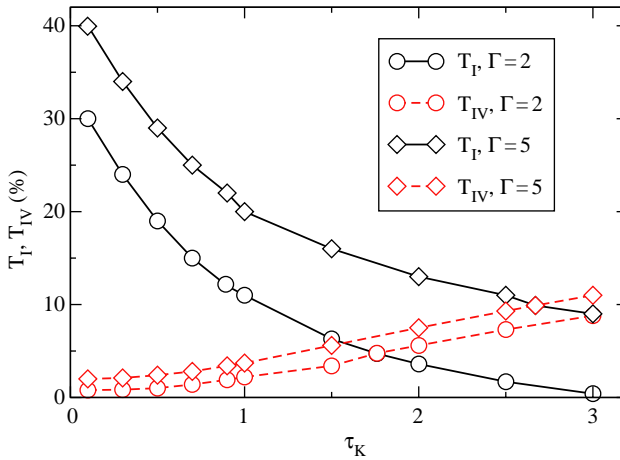


Fig. 3. Time that the bed will spend in zone I (T_I) and in zone IV (T_{IV}) of Fig. 2 as a function of τ_K numerically calculated from the model of Kroll, for $\Gamma = 2$ and $\Gamma = 5$. T_I and T_{IV} are reported as its percentual fraction with respect to $T_I + T_{II} + T_{III} + T_{IV}$, that is the time in which an intruder immersed in the bed can move with respect to the small grains. For $\tau_K > 3$ the flight time of the bed is close to the period of vibration.

The time that the bed would spend in each of the zones shown in Fig. 2 will depend on experimental conditions. Within the model of Kroll, it would depend on Γ and τ_K . If reverse buoyancy is observed, then it is a sign that zone I prevails. In Fig. 3, we show the time that the bed would spend in zones I (T_I) and IV (T_{IV}) as predicted by the model of Kroll. It can be seen that if τ_K is small and Γ is large then we have an scenario that favors reverse buoyancy. The model predicts that there is a particular value $\tau_{K,c}(\Gamma)$ for which $T_I = T_{IV}$. For $\tau_K = \tau_{K,c}$ the average motion of an intruder immersed in the bed would depend on how the bed behaves in zones II and III.

In Ref. [12], reverse buoyancy was suppressed by evacuating the air of the system. If we quench the flow of air through the bed for the experimental conditions of Ref. [16] by keeping the top of the container open and introducing a filter permeable to air in its bottom, as was made in Ref. [11], we do not observe reverse buoyancy: light and heavy intruders rise. In both procedures the proposed fluidization mechanism cannot take place.

5. Conclusion

In this article, we have presented a mechanism by which a vibrated granular bed can be fluidized in the presence of interstitial air in a similar way as it occurs in gas-fluidized *static* beds. This mechanism results from an interplay between the flow of air through the bed and the effective gravitational field acting on the bed. Since the mechanism proposed is an extension of an static one, and since the bed cannot switch between a fluidized state and a solid state instantly, we expect that this results are

applicable when the period of vibration is large. Under this condition, we can predict when a vibrated granular system initially becomes gas fluidized during each cycle of oscillation. We conclude, from the graph shown in Fig. 3, that a favorable scenario for reverse buoyancy is small frequency of vibration, large amplitude of vibration, small grain density and small grain diameter. In the above description reverse buoyancy should not be very sensitive to the height of the bed. Despite its simplicity, the proposed model constitutes a promising quantitative framework that gives some relevant information about the process of cyclic fluidization that occurs in a vibrated granular bed. It would be interesting to monitor experimentally the evolution of the granular medium, within one period of oscillation of the system.

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