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Mechanics of Gear Hobbing

The exacting demands for producing accurate gears make it necessary to determine the cutting forces coming on the hob-shaft which is the weakest element subjected to severe bending and torsion. The authors, with the help of a specially designed hobbing dynamometer, have investigated the magnitude and nature of the tangential and radial component of cutting force during the conventional hobbing process.

Introduction

THE EXACTING demands for producing accurate gears make it necessary to determine the cutting forces coming on the hob-shaft. Further, recent work of Slavicek [1][2] has indicated that the determination of the cutting force behavior is essential for analyzing the stability of hobbing machine, particularly the nature of variation of forces with respect to principal variables.

Literature research [2] indicates that the process of hobbing has been invented over a hundred years ago, and since then many investigations have been carried out for developing gear cutting machines on hobbing principle. However, not much work has been found until recently in the field of mechanics of hobbing during gear cutting process.

Sidorenko [3] reported the variation of contact area during the rotation of a blank and axial feed of the hob. Thammer [4] calculated the mean chip thickness and plotted it's variation with respect to feed, rate, and size of gears.

It has been reported that the number of gashes, also, influence the chip thickness. Sunajev, Gavrilov, and Sumulevic [5] have determined the shapes of the side-edge of chips by machining a gear blank made of wax with a simple one-tooth hob. The collected chips have been unfolded and their shapes drawn. Certain conclusions regarding the mean width of chip can be drawn from Sunajev's result:

(*a*) The mean width of chip on both the side edges are equal.

(*b*) The mean width of chip on one side edge is approximately equal to that on the peripheral edge.

(*c*) The peripheral edge removes 60 percent of total chip volume, while side edge removes only 20 percent.

Sidorenko and Adam [6] and Adam [7] have investigated the torque in the hob-shaft. Rozenberg and Nekrasov and Saifullin [8] also attempted to experimentally determine an empirical equation for the torque in hobbing process. However, there have been some contradictions in their reports which the authors wanted to investigate and compare.

Cutting Process During Hobbing

The generating action of a hob while cutting a gear is exactly the same as the conjugate action between two helical gears whose axes are not parallel.

A single-threaded hob is a single-tooth helical gear. The helical gear which forms the basis of the cutter has a lead angle such that the gear resembles a worm. Then the threads are gashed and relieved at regular intervals along their length to form cutting edges.

It is important to distinguish how the chips are removed in the process of hobbing. Like milling, there are two methods, the first one being the conventional manner of hobbing, the other being the climb-hobbing. In the conventional hobbing, the so-called "comma-chip" is obtained from the thin end, while in climb-hobbing, the chip starts at its thicker end.

In metal cutting practice, it is assumed that the cutting forces primarily vary in milling and similar processes because of the variation of chip thickness. However, the variation of cutting forces also depends upon many other parameters involved with the type of process itself, like changing rake, changing velocity, and so forth. In turning and similar operations, the width and depth of cut are clearly defined and measured. However, in hobbing, both the quantities of width and depth of cut vary during the cutter revolution. In hobbing, chips are being removed from the periphery as well as from both the flanks of the tooth profile. Hence, the chips are nonuniform and irregular in shape. The variation of chip thickness is due to many factors involved in the gear hobbing process, such as the blank diameter, module, number of gashes in hob, axial feed-rate, and so forth.

During hobbing, both peripheral and side edges of the hob remove the chips. However, the main part of chip volume is cut off by peripheral edges. The side edges merely shape the teeth. During rotation of the blank and axial feed of the hob, the cutting edges have differential interference resulting in a variation of contact area. The edges do not cut simultaneously, and the width of cut is different at the different positions of the cutting edge. The shape and size of the contact area at the cutting edge continually change and depend upon: (*a*) number of teeth of the

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[2] Numbers in brackets designate References at end of paper.

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gear, X, (*b*) module of the gear, m, (*c*) the axial feed rate, S_0, and (*d*) the diameter of the hob (D_h). Sidorenko and Adam [6] have investigated the variation in contact area by exploring the successive position of the basic racks of the hob.

The width of chip at the cutting edges have been determined by Sunajev, et al. [5] and is given by,

$$L = 0.273 m^{1.18} S_0^{0.17} Z^{0.64}$$

where:

S_0 = feed of the hob, mm/rev
m = module of the hob
Z = number of teeth of gear blank

During the conventional hobbing process, the chip thickness changes like the peripheral milling process. The contact angle, similar to the angle of engagement in up-milling, can be determined from,

$$\cos \psi = \left(1 - \frac{2h}{D_h}\right) \quad (1)$$

where:

ψ = angle of engagement of one tooth
h = depth of gear tooth being machined, mm
D_h = diameter of the hob, mm

The module (m) of hobs is conventionally related to the diameter (D_h) of hobs by:

$$D_h = C_1 m^k \quad (2)$$

where K is an exponent depending on the type of gear being cut, and C_1 is a constant of proportionality.

From the manufacturing practice of hobs, the values of the constants are evaluated as

$$C_1 = 43, \qquad K = 0.48$$

The mean angle of engagement can be obtained by assuming that

$$\psi_{\text{avg}} = \frac{\psi}{2} \simeq \sin\frac{\psi}{2} \simeq \sqrt{\frac{1 - \cos\psi}{2}} = \sqrt{\frac{h}{D_h}} \quad (3)$$

Substituting for D_h from equation (2) and noting that h = 2.157 m, the mean angle of engagement is given by:

$$\psi_{\text{avg}} = \sqrt{\frac{2.157m}{43m^{0.48}}} = 0.217 m^{0.26} \quad (4)$$

Hence, the mean chip thickness per hob-tooth will be

$$S_0 = \frac{S_h}{Z_h}(0.217 m^{0.26}) \quad (5)$$

where:

S_h = feed per revolution of the hob, mm
Z_h = number of gashings in the hob

From equation (5), it is seen that the mean thickness of cut at the periphery increases when the axial hob-feed is increased. The mean chip thickness decreases if the number of gashes is increased. Calculations of Thammer [4] confirmed these observations.

The true feed at the sides is given by,

$$S_s = S_0 \cos\lambda \sin\phi \quad (6)$$

where:

λ = lead angle of hob
ϕ = pressure angle of the basic rack
S_s = true feed at the sides
S_0 = true feed at the periphery

Forces During Hobbing Process

The forces on the plane of the basic rack can be obtained by simulation with respect to fundamental metal cutting process. It may be assumed that each tooth of the hob removes material from the workpiece: (1) at the periphery of the teeth, and (2) at the sides of the teeth.

From the schematic force diagram shown in Fig. 1,

$$P_{yhi} = R_i \sin\eta_1 \sin\phi \quad (7)$$

where:

η_i = friction angle at the ith side
R_i = resulting cutting load on the ith side
ϕ = pressure angle

The numerical suffixes (i) denote:

1 = leading side of the tooth
2 = trailing side of the tooth
3 = periphery of the tooth

The axial components are

$$P_{xhi} = R_i \sin\eta_i \cos\phi \quad (8)$$

The tangential components are

$$P_{Ti} = P_{Xi} = R_i \cos\eta_i \quad (9)$$

Nomenclature

C_1, C_2, C_3 = various constants of proportionality
D_h = diameter of hob
h = depth of gear tooth being machined
i = number of starts in hob
K = exponent, depending on the type of gear being cut
M = Torque on the hob shaft
m = module of the hob
n_w = blank rpm
n_H = hob rpm
p_{xh} = component of hobbing force along x coordinate of hob
P_{yh} = component of hobbing force along y coordinate of hob
P_{zh} = component of hobbing force along Z coordinate of hob
P_{xw} = component of the hobbing force along the direction of workpiece
P_{yw} = component of the hobbing force along y direction of the workpiece
P_T = tangential cutting force on the hob arbor
P_R = radial cutting force on the hob arbor
R = resultant cutting force
S_0 = axial or downward feed rate
S_h = feed per revolution of the hob
U = specific energy during hobbing
V = volume of material being removed per revolution of the blank
W = work done in hobbing per revolution of the blank
X = exponent of module, m
y = exponent of feed, S_0
Z = number of teeth of the gear being cut
Z_H = number of gashings in the hob
ψ = angle of engagement of one tooth
ψ_{avg} = mean angle of engagement or mean contact angle
ϕ = pressure angle
λ = lead angle of the hob
η = friction angle

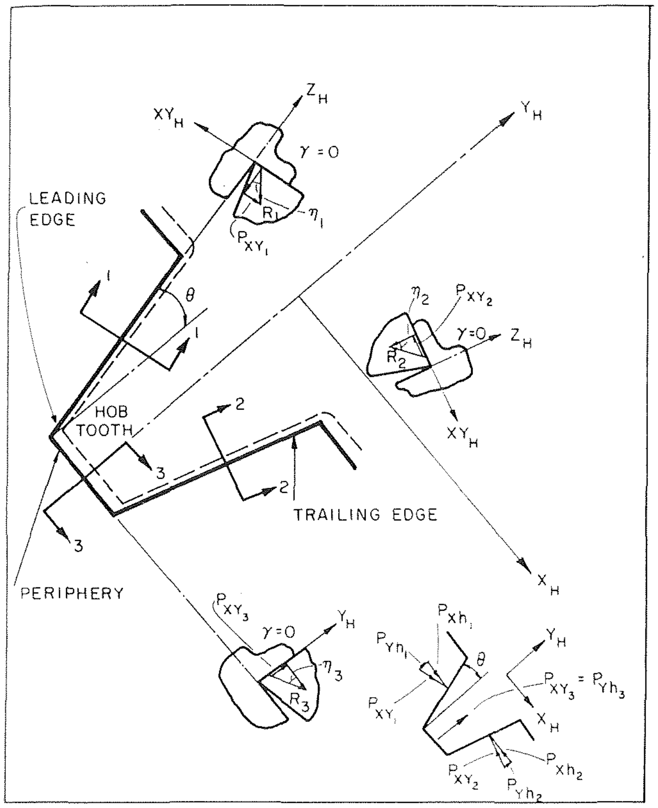

Fig. 1 Geometry of chip formation and forces acting at the different locations of the cutting edges of a hob

Fig. 1 shows the overall force system in the plane of basic rack. It is observed that under the assumption of equal cutting at the sides resultant P_{xh} is zero, while the total P_{yh} is obtained from

$$P_{yh}|_{\text{total}} = \sum_i P_{yhi}$$

$$= (R_1 \sin \eta_1 + R_2 \sin \eta_2) \sin \phi + R_3 \sin \eta_3 \quad (10)$$

Similarly,

$$P_T = P_{Zh}|_{\text{total}} = \Sigma R_i \cos \eta_i \quad (11)$$

Assuming that

$$R_i = \bar{K} A_i$$

where $\bar{K}$ is a constant depending on the tool-work pair, cutting conditions, environment, and so forth, and A_i is the area of cut at the ith side.

If n_i and K are assumed to be approximately constant for a given set, then for Z_n number of teeth in actual contact, the tangential force is given by:

$$P_T = P_Z = \bar{K} \cos \eta \sum_{z_n} \sum_i A_i \quad (12)$$

However, it is very difficult to estimate the instantaneous variation of the area of cut of equation (12). Hence, a convenient approach for estimating the tangential cutting force P_z is to find it from the specific energy of cutting.

The work done per revolution of the cutter is given by

$$W = 2\pi M \frac{z}{i} \quad (13)$$

where:

W = the work done per revolution of the blank
M = the torque in Kgm
Z = number of teeth in the gear
i = the number of starts in the hob
Z/i = revolution of hob cutter per revolution of blank

During this period, when the blank has rotated once, the volume of material removal is obtained from

$$V = 0.5\pi hmZS_0 \quad (14)$$

where:

h = depth of one tooth, mm
= 2.157 m
S_0 = feed, mm/rev
Z = number of teeth
m = module

The specific energy is obtained from

$$U = \frac{W}{V} = \frac{2\pi MZ}{0.5h\pi mZS_0 i} = \frac{4M}{hmS_0 i} \quad (15)$$

Substituting for h,

$$U = \frac{1.8M}{m^2 S_0 i} \quad (16)$$

Rewriting equation (16),

$$M = \left(\frac{Ui}{1.8}\right) m^2 S_0 \quad (17)$$

This is the expression for torque acting on the hobshaft. Noting,

$$P_T = \frac{2M}{D_h} \quad (18)$$

where D_h is obtainable from the relation shown in equation (2).

Now, substituting for M from equation (17),

$$P_T = \frac{2Ui}{1.8} m^2 S_0 \left(\frac{1}{43m^{0.48}}\right) = C_2 m^{1.52} S_0 \quad (19)$$

This expression shows that the tangential component of the cutting force, P_T, is directly dependent on the feed, while with module the effect is more pronounced. This is due to the variation of the width of cut when module changes. The exponents of m and S_0 are checked from experiments, as well as compared with previous results of other research workers.

Experimental Set-Up and Procedure

General Set-Up

A Hv-14 type "Sykes" universal hobbing machine has been employed in the present project. Single start hobs conforming to BS 2062/1963 (Grade B) specification have been used. The particulars of the three hobs used in the present project are shown in Table 1.

From the manufacturing practice of hobs, a relationship between the outside diameter of the hobs and their module is often observed. By plotting the Sykes' data per size of hobs, a relationship of the form

$$D = 43m^{0.48} \quad (20)$$

has been obtained. This relationship has been used in the theoretical calculations and analysis for cutting forces and energy consumption during the hobbing process.

Table 1 Hob Specifications

	1	2	3
Normal D.P.	8	10	12
Rake	0	0	0
Helix angle	2 deg 57 min	2 deg 36 min	2 deg 21 min
Pressure angle	$14^1/_2$ deg	$14^1/_2$ deg	$14^1/_2$ deg
Addendum, in.	0.125	0.100	0.083
Dedendum, in.	0.1446	0.1157	0.0964
Bore, in.	$1^1/_4$	$1^1/_4$	$1^1/_4$

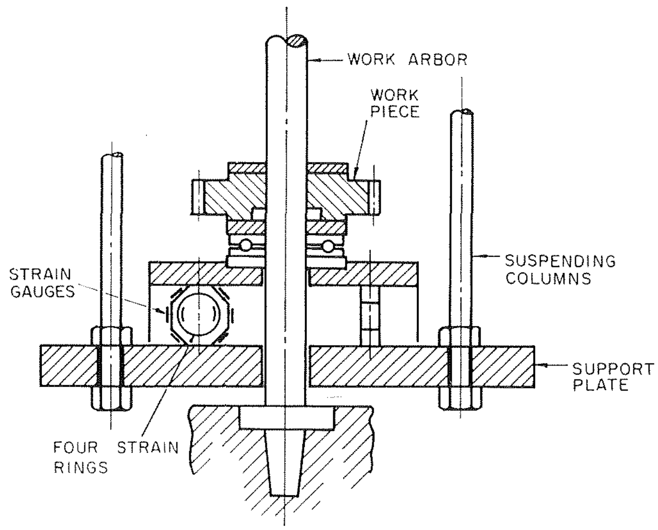

Fig. 2 Scheme of the hobbing dynamometer using four strain rings

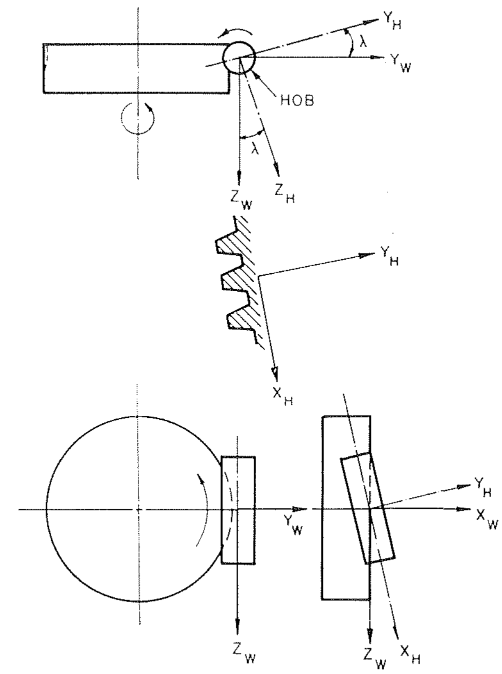

Fig. 3 Scheme of coordinate system in gear hobbing

Table 2 Gear blank specifications

Workpiece number	1	2	3
Number of teeth	71	71	71
D.P.	8	10	12
Pitch diameter, in.	8.8750	7.1000	5.9166
Outside diameter, in.	9.1250	7.3000	6.0830
Width of blank, in.	0.75	0.75	0.75

Three different sizes of workpieces of "constant" number of teeth have been chosen. The material for the workpieces has been hot-rolled AISI-1025 steel. The blank sizes are given in Table 2.

Measurement of Hobbing Forces

Unlike others [6] where the tangential component of hobbing force has been determined from the twisting moment of the hob-shaft, a three-component ring-type dynamometer has been specially designed for the present investigation. The general scheme of the hobbing dynamometer employing strain-rings is shown in Fig. 2. The principle of strain-rings is well known and can be seen from references [10-11].

For defining the force system, it is necessary to assign a suitable coordinate system either with respect to machine or with respect to tool. In the machine reference system, the job can be taken as a suitable reference datum being laid in the same plane of the hobbing machine as shown in Fig. 3. The $X_m - y_m - Z_m$ coordinates of machine reference system are assumed to be the same as $X_w - y_w - Z_w$ of the workpiece.

The force, p_{Xw}, acting along X_w coordinate is measured by means of the strain gages placed in vertical arms of the strain rings and form the measuring bridge. The force, P_{yw}, is picked up by the strain gages placed on the slanting arms of the octagonal strain-rings.

However, P_{xw}, P_{yw}, P_{zw} are the forces measured by the dynamometer along the chosen coordinates of the workpiece. A transformation is necessary to obtain the average tangential and radial components P_T and P_R of hobbing force from the average observed values of the dynamometer readings.

For generating straight or helical tooth, the hob has got to be inclined either at its lead angle or at some suitable angle, depending on the nature of helix to be cut. Assume that the X_h and y_h coordinates of the hobs describe the section of the hob in its inclined position so that the basic rack can be obtained in the $X_h - y_h$ plane. Thereby, the axes of Z_h and y_h of the hob will be inclined to machine reference system by the inclination of the hob. The tool-reference coordinates with respect to hob $X_h - y_h + Z_h$ are indicated in Fig. 3.

The forces in the tool reference system are related to the forces in machine reference coordinates by a transfer matrix derived with respect to Fig. 4(*a*):

$$\begin{bmatrix} P_{xw} \\ P_{yw} \end{bmatrix} = \begin{bmatrix} \cos\lambda, & -\sin\lambda \\ \sin\lambda, & \cos\lambda \end{bmatrix} \begin{bmatrix} P_{zh} \\ P_{yh} \end{bmatrix} \quad (21)$$

The axis of Z_h is, however, transitory in nature because of the motion of the cutter tooth during the hobbing process. If this variation is neglected and a mean quasi-static tangential direction, T, is assumed to act at an angle of ψ_{avg} with respect to Z_h, then the force along a mean direction T is deviated from Z_h by an angle ψ_{avg}. Hence, if the forces at the periphery of the hob are indicated by P_T and P_R, as shown in Fig. 4(*b*), the transfer matrix is given by:

$$\begin{bmatrix} P_T \\ P_R \end{bmatrix} = \begin{bmatrix} \cos\psi, & \sin\psi \\ \sin\psi, & -\cos\psi \end{bmatrix} \begin{bmatrix} P_{zh} \\ P_{yh} \end{bmatrix} \quad (22)$$

The angle ψ is determined by equation (4). Combining equation (21) and (22),

$$\begin{bmatrix} P_T \\ P_R \end{bmatrix} = \begin{bmatrix} \cos\psi, & \sin\psi \\ \sin\psi, & -\cos\psi \end{bmatrix} \begin{bmatrix} \cos\lambda, & \sin\lambda \\ -\sin\lambda, & \cos\lambda \end{bmatrix} \begin{bmatrix} P_{xw} \\ P_{yw} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\psi+\lambda), & \sin(\psi+\lambda) \\ \sin(\psi+\lambda), & -\cos(\psi+\lambda) \end{bmatrix} \begin{bmatrix} P_{xw} \\ P_{yw} \end{bmatrix} \quad (23)$$

Thus, by employing equation (23), the force system measured by the dynamometer is transformed into the force system in T-R coordinates describing the average values of forces and torques acting on the hob shaft.

Test Results and Discussion

The observed components of cutting force during the process of hobbing for different feeds and modules have been transformed by employing the transformation matrix of equation (23) for finding the tangential and radial components of the hobbing force. The forces, P_T and P_R, are plotted in double-log coordinates in Figs. 5

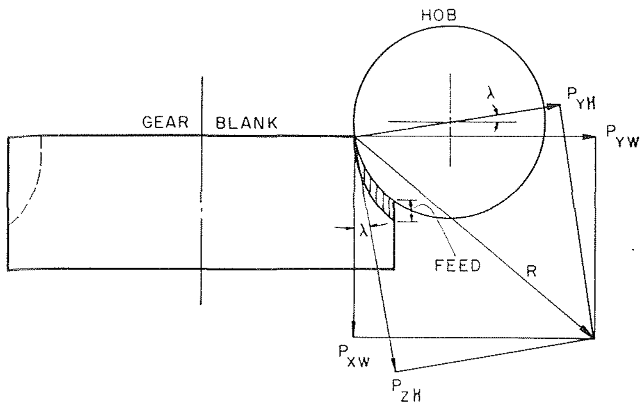

Fig. 4(a) Transfer of force system from machine reference system coordinates to tool reference coordinates

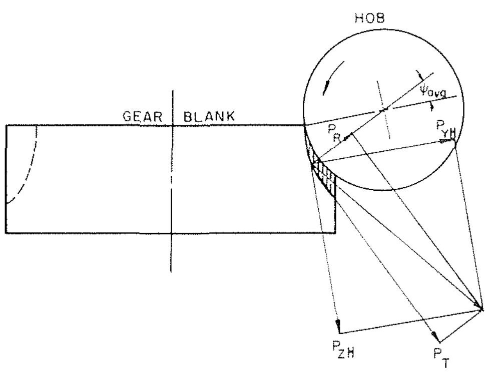

Fig. 4(b) Tangential and radial component of forces acting at the mean contact angle

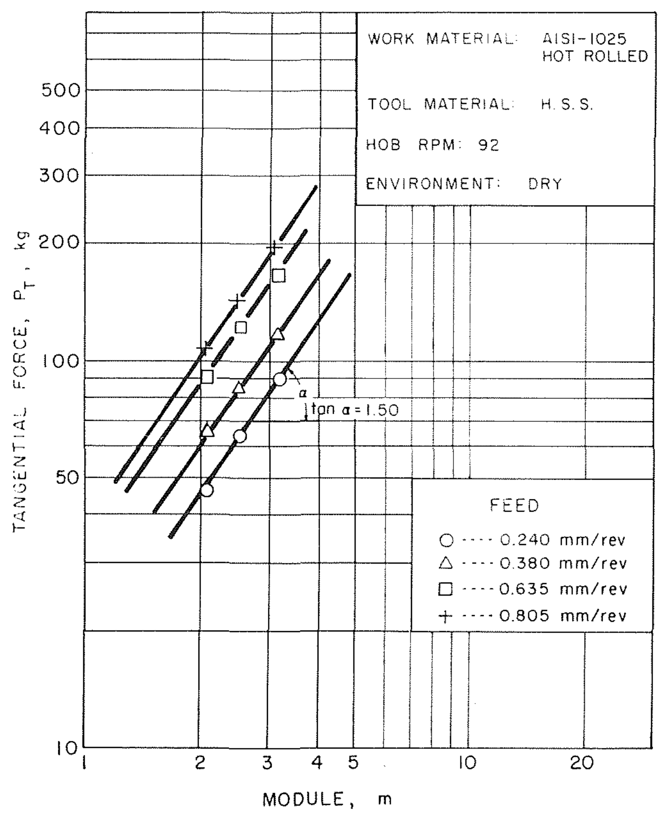

Fig. 5 Effect of module and feed on the tangential component, p_T

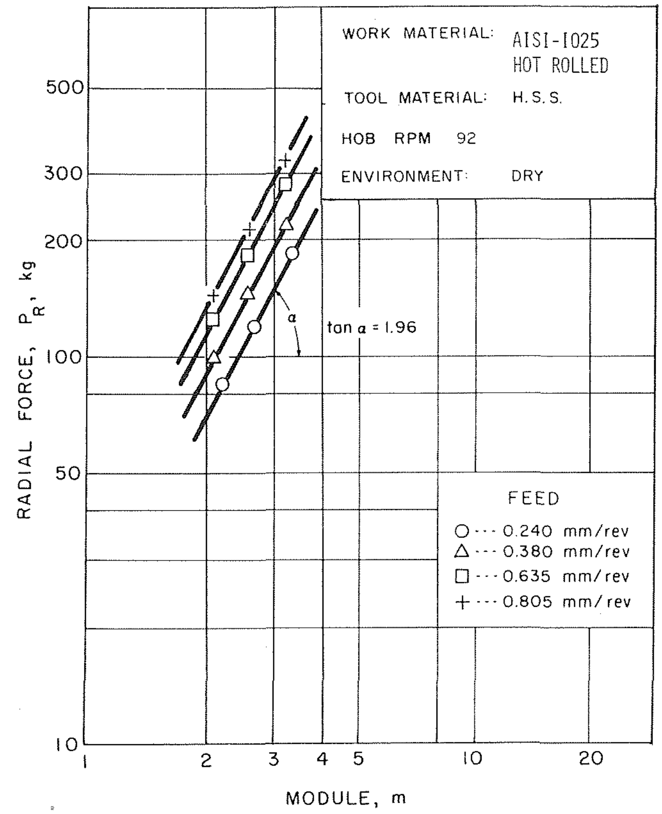

Fig. 6 Effect of module and feed on the radial component, p_R

Table 3 Exponents x and y in the equation $P_T = C_2 m^x S^y$

	Results of			
			Author	
Exponents	Rozenberg	Nekrasov	Analytical	Experimental
x	1.30	1.41	1.52	1.50
y	0.90	0.92	1.00	0.75

and 6. Analyses of Figs. 5 and 6 show that the effect of module and feed on the tangential and radial components of hobbing force are given by:

$$P_T = 43m^{1.50}S_0^{0.75},\ \text{Kg} \tag{24}$$

$$P_R = 40m^{1.96}S_0^{0.55},\ \text{Kg} \tag{25}$$

A comparative chart shown in Table 3 indicates the comparison between the various values of exponents of tangential force, P_T, obtained by various techniques by several research workers.

From Table 3, it is seen that the exponent of module agrees well with the analytically estimated exponent. Results of previous investigators indicate values of x lying between 1.12 to 1.41. The exponent of feed is 0.75, while some of the previous investigators have reported the value of 0.9.

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