On the Spatial Structure of the Antenna Electromagnetic Near Field

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Abstract

This paper presents a general theory of the antenna near field in the spatial domain. The approach is based on using the Wilcox expansion of the radiated field to define a set of asymptotic spherical regions covering the entire exterior space of the antenna problem. Careful examination of the energy expression within this picture revealed the rich and complex interaction mechanisms between the various spherical regions indicated above. The multipole expansion is then utilized to construct nonrecursively the full near field in the exterior region starting from the far field only. The analysis led to interesting theorems regrading energy exchange processes in the near zone and also to a completely analytical evaluation of the antenna reactive energy in terms of the TE and TM modes of the antenna system.

1 The Structure of the Antenna Near Field in the Spatial Domain

We assume that an arbitrary electric current $\mathbf{J}(\mathbf{r})$ exists inside a volume V_0 enclosed by the surface S_0. Let the antenna be surrounded by an infinite, isotropic, and homogenous space with permittivity ε and permeability μ. The antenna current will radiate electromagnetic fields everywhere and we are concerned with the region outside the source volume V_0. We consider two characteristic regions. The first is the region V enclosed by a spherical surface S and this will be the setting for the near fields. The second region V_∞ is the one enclosed by the spherical surface S_∞ taken at infinity and it corresponds to the far fields. The complex Poynting theorem states that $\nabla \cdot \mathbf{S} = -(1/2)\mathbf{J}^* \cdot \mathbf{E} + 2i\omega\,(w_h - w_e)$, where the complex Poynting vector is defined as $\mathbf{S} = (1/2)\,\mathbf{E} \times \mathbf{H}^*$ and the magnetic and electric energy densities are given, respectively, by $w_e = (1/4)\varepsilon\mathbf{E} \cdot \mathbf{E}^*$ and $w_h = (1/4)\mu\mathbf{H} \cdot \mathbf{H}^*$, and ω is the radian frequency [2]. By eliminating the source-field interaction (work) term in order to focus entirely on the fields, it is possible to obtain after some manipulations the following well-known relation

$$\int_S d\mathbf{s}\frac{1}{2}\left(\mathbf{E} \times \mathbf{H}^*\right) = P_{\mathrm{rad}} - 2i\omega \int_{V_\infty - V} dv\,(w_h - w_e)\,, \tag{1}$$

where the radiated power is defined as $P_{\mathrm{rad}} = \mathrm{Re}\int_S d\mathbf{s}\frac{1}{2}\left(\mathbf{E} \times \mathbf{H}^*\right)$. The result above will be taken up again in Section 3. In the remaining part of this section we try to gain a more detailed insight into the nature of the near field that goes beyond the simple picture presented by (1).

We consider the fields generated by the antenna lying in the intermediate zone, i.e., the interesting case between the far zone $kr \to \infty$ and the static zone $kr \to 0$. We aim to attain a conceptual insight into the nature of the near field by mapping out its inner structure in details. Since the fields in the exterior region satisfy the homogenous Helmholtz equation, we can expand the electric and magnetic fields as [1]

$$\mathbf{E}(\mathbf{r}) = \frac{e^{ikr}}{r}\sum_{n=0}^{\infty}\frac{\mathbf{A}_n(\theta,\varphi)}{r^n}, \quad \mathbf{H}(\mathbf{r}) = \frac{e^{ikr}}{r}\sum_{n=0}^{\infty}\frac{\mathbf{B}_n(\theta,\varphi)}{r^n}, \tag{2}$$

where $\mathbf{A}_n$ and $\mathbf{B}_n$ are vector angular functions dependent on the far-field radiation pattern of the antenna and $k = \omega\sqrt{\varepsilon\mu}$ is the wavenumber.[1] Let us then divide the entire exterior region surrounding the antenna into an infinite number of spherical layers. The outermost layer R_0 is identified with the far zone while the innermost layer R_∞ is defined as the minimum sphere totally enclosing the antenna current distribution. In between these two regions, an infinite number of layers exists, each corresponding to a term in the Wilcox

[1] The far fields are the asymptotic limits $\mathbf{E}(\mathbf{r}) \underset{r\to\infty}{\sim} (e^{ikr}/r)\mathbf{A}_0(\theta,\varphi)$, and $\mathbf{H}(\mathbf{r}) \underset{r\to\infty}{\sim} (e^{ikr}/r)\mathbf{B}_0(\theta,\varphi)$.

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expansion as we now explain. The boundaries between the various regions are not sharply defined, but taken only as indicators in the asymptotic sense to be described momentarily.[2] The outermost region R_0 corresponds to the far zone. As we start to descend toward the antenna, we enter into the next region R_1, where the mathematical expression of the far field is no longer valid and has to be augmented by the next term in the Wilcox expansion. Indeed, we find that for $\mathbf{r} \in \mathrm{R}_1$, the electric field takes (approximately, asymptotically) the form $\mathbf{A}_0 \exp{(ikr)}/r + \mathbf{A}_1 \exp{(ikr)}/r^2$. Subtracting the two fields from each other, we obtain the difference $\mathbf{A}_1 \exp{(ikr)}/r^2$. Therefore, it appears to us very natural to interpret the region R_1 as the "seat" of a field in the form $\mathbf{A}_1 \exp{(ikr)}/r^2$. Similarly, the nth region R_n is associated (in the asymptotic sense just sketched) with the field form $\mathbf{A}_n \exp{(ikr)}/r^{n+1}$. By dividing the exterior region in this way, we become able to mentally visualize progressively the various contributions to the total near field expression *as they are mapped out spatially.* To be sure, this spatial picture will remain a mere definition unless it is corroborated by some interesting consequences. This actually turns out to be the case: It is possible to show that certain theorems about the physical behavior of each layer can be proved. Better still, it is possible to investigate the issue of the mutual *electromagnetic interaction* between different regions defined above. It turns out that a general theorem (proved in [3]) can be established, which shows that exactly "half" of these layers don't electromagnetically interact with each other. In order to understand the meaning of this remark, we need first to define precisely what is expressed in the term 'interaction.' Let us use the Wilcox expansion (2) to evaluate the electric and magnetic energies. The result is[3]

$$w_e = \frac{\varepsilon}{4}\mathbf{E}\cdot\mathbf{E}^* = \frac{\varepsilon}{4}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\frac{\mathbf{A}_n\cdot\mathbf{A}_{n'}^*}{r^{n+n'+2}},\ w_h = \frac{\mu}{4}\mathbf{H}\cdot\mathbf{H}^* = \frac{\mu}{4}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\frac{\mathbf{B}_n\cdot\mathbf{B}_{n'}^*}{r^{n+n'+2}}. \tag{3}$$

We rearrange the terms of these two series to produce the following illuminating form[4]

$$w_e(\mathbf{r}) = \frac{\varepsilon}{4}\sum_{n=0}^{\infty}\frac{\mathbf{A}_n\cdot\mathbf{A}_n^*}{r^{2n+2}} + \frac{\varepsilon}{2}\sum_{\substack{n,n'=0\\ n>n'}}^{\infty}\frac{\mathrm{Re}\left\{\mathbf{A}_n\cdot\mathbf{A}_{n'}^*\right\}}{r^{n+n'+2}}, w_h(\mathbf{r}) = \frac{\mu}{4}\sum_{n=0}^{\infty}\frac{\mathbf{B}_n\cdot\mathbf{B}_n^*}{r^{2n+2}} + \frac{\mu}{2}\sum_{\substack{n,n'=0\\ n>n'}}^{\infty}\frac{\mathrm{Re}\left\{\mathbf{B}_n\cdot\mathbf{B}_{n'}^*\right\}}{r^{n+n'+2}}. \tag{4}$$

The first sums in the RHS of (4) represent the *self* interaction of the nth layer with itself. Those are the self interaction of the far field, the so-called radiation density, and the self interactions of all the remanning (inner) regions R_n with $n \geq 1$. The second sum in both equations represents the interaction between *different* layers. Now because we are interested in the spatial structure of near field, it is natural to average over all the angular information contained in the energy expressions (4). The *radial energy density* is given by

$$w_e(r) = \frac{\varepsilon}{4}\sum_{n=0}^{\infty}\frac{\langle\mathbf{A}_n,\mathbf{A}_n\rangle}{r^{2n+2}} + \frac{\varepsilon}{2}\sum_{\substack{n,n'=0\\ n>n'}}^{\infty}\frac{\langle\mathbf{A}_n,\mathbf{A}_{n'}\rangle}{r^{n+n'+2}},\ w_h(r) = \frac{\mu}{4}\sum_{n=0}^{\infty}\frac{\langle\mathbf{B}_n,\mathbf{B}_n\rangle}{r^{2n+2}} + \frac{\mu}{2}\sum_{\substack{n,n'=0\\ n>n'}}^{\infty}\frac{\langle\mathbf{B}_n,\mathbf{B}_{n'}\rangle}{r^{n+n'+2}}, \tag{5}$$

where the mutual interaction between two angular vector fields $\mathbf{F}$ and $\mathbf{G}$ is defined as[5]

$$\langle\mathbf{F}(\theta,\varphi),\mathbf{G}(\theta,\varphi)\rangle \equiv \int_{4\pi} d\Omega\,\mathrm{Re}\left\{\mathbf{F}(\theta,\varphi)\cdot\mathbf{G}^*(\theta,\varphi)\right\}. \tag{6}$$

The total energy is obtained by integrating over the remaining radial variable, which is possible in closed form as we will see later in Section 3. A particulary interesting observation, however, is that almost "half" of the mutual interaction terms appearing in in (5) are exactly zero. Indeed, as we proved in [3], if the integer $n+n'$ is odd, then the interactions are identically zero, i.e., $\langle\mathbf{A}_n,\mathbf{A}_{n'}\rangle = \langle\mathbf{B}_n,\mathbf{B}_{n'}\rangle = 0$ for $n+n' = 2k+1$ and k is integer. This represents, in our opinion, a significant insight on the nature of antenna near fields in general.

[2]To be precise, by definition only region R_∞ possesses a clear-cut boundary.

[3]Since the series expansion under consideration is absolutely convergent, and the conjugate of an absolutely convergent series is still absolutely convergent, the two expansions of $\mathbf{E}$ and $\mathbf{E}^*$ can be freely multiplied and the resulting terms can be arranged as we please.

[4]In writing equations (4), we made use of the reciprocity in which the energy transfer from layer n to layer n' is equal to the corresponding one from layer n' to layer n.

[5]In deriving (5), we made use of the fact that the energy series is uniformly convergent in θ and φ in order to interchange the order of integration and summation.

2 Direct Construction of the Antenna Near-Field Starting from a Given Far-field Radiation Pattern

The localization of the electromagnetic field within each of the regions R_n suggests that the outermost region R_0, the far zone, corresponds to the simplest field structure possible, while the fields associated with the regions close to the antenna exclusion sphere, R_∞, are considerably more complex. In this section we show that the entire region field can be determined from the far field directly, i.e., nonrecursively, by a simple construction based on the analysis of the far field into its spherical wavefunctions. Our point of departure is the far-field expressions, where we observe that because $\mathbf{A}_0(\theta,\varphi)$ and $\mathbf{B}_0(\theta,\varphi)$ are well-behaved angular vector fields tangential to the sphere, it is possible to expand their functional variations in terms of infinite sum of vector spherical harmonics [2]. That is, we write

$$\mathbf{E}\left(\mathbf{r}\right) \underset{r\to\infty}{\sim} \eta\frac{e^{ikr}}{kr}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left(-1\right)^{l+1}\left[a_E\left(l,m\right)\mathbf{X}_{lm}\ -a_M\left(l,m\right)\hat{r}\times\mathbf{X}_{lm}\right], \tag{7}$$

the series being absolutely-uniformly convergent. Here, $\eta=\sqrt{\mu/\varepsilon}$ is the wave impedance. $a_E(l,m)$ and $a_M(l,m)$ stand for the coefficients of the expansion TE_{lm} and TM_{lm} modes, respectively. Since the asymptotic expansion of the spherical vector wavefunctions is exact, the electromagnetic fields throughout the *entire* exterior region of the antenna problem can be expanded as a series of complete set of of vector multipoles [2]. The upshot of our argument, which is very technical, is the unique determinability of the antenna near field in the various spherical regions R_n by a specified far field taken as the starting point of the engineering analysis of general radiating structures. Therefore, *the Wilcox series can be derived from the multipole expansion and the exact variation of the angular vector fields* $\boldsymbol{A}_n$ *and* $\boldsymbol{B}_n$ *are directly determined in terms of the spherical far-field modes of the antenna* [3].[6] Antenna designers usually specify the goals of their devices in terms of radiation pattern characteristics. It appears from our analysis that an exact analytical relation between the near field and these design goals do exist in the form derived above. Since the designer can still choose any type of antenna that fits within the enclosing region R_∞, the results of this paper should be viewed as a kind of canonical machinery for generating fundamental relations between the far-field performance and the lower bound formed by the field behavior in the entire exterior region compatible with any antenna current distribution that can be enclosed inside R_∞.

3 Construction of the Reactive Energy Densities

We call any energy density calculated with the point of view of the imaginary part of (1) *reactive* densities. When trying to calculate the total electromagnetic energies in the region $V_\infty - V$, the result is divergent integrals. However, condition (1) clearly suggests that there is a *common* term between w_e and w_h which is the source of the trouble. We postulate then that $w_e \equiv w_e^1 + w_{\mathrm{rad}}$, $w_h \equiv w_h^1 + w_{\mathrm{rad}}$. Here w_e^1 and w_h^1 are taken as *reactive energy densities*, which we prove to be finite. The common term w_{rad} is divergent in the sense $\int_{V_\infty - V} dv w_{\mathrm{rad}} = \infty$. Therefore, it is obvious that the integral of $w_h - w_e = w_h^1 - w_e^1$ is finite. Next, we observe that $w_h\left(r\right) \underset{r\to\infty}{\sim} w_{\mathrm{rad}}\left(r\right),\quad w_e\left(r\right) \underset{r\to\infty}{\sim} w_{\mathrm{rad}}\left(r\right)$. In other words, the common term w_{rad} is easily identified as the radiation density *at the far-field zone.* It is well-known that the integral of this density is not convergent and hence our original assumption is confirmed. The final step consists in showing that the total energy is finite. To prove this, we make use of the Wilcox expansion in the energy densities expression. It is found that

$$w_e\left(r\right) = w_{\mathrm{rad}}\left(r\right) + \frac{\varepsilon}{2}\frac{\left\langle \mathbf{A}_0,\mathbf{A}_1\right\rangle}{r^3} + \frac{\varepsilon}{4}\sum_{n=1}^{\infty}\frac{\left\langle \mathbf{A}_n,\mathbf{A}_n\right\rangle}{r^{2n+2}} + \frac{\varepsilon}{2}\sum_{\substack{n,n'=1\\ n>n'}}^{\infty}\frac{\left\langle \mathbf{A}_n,\mathbf{A}_{n'}\right\rangle}{r^{n+n'+2}}, \tag{8}$$

[6] We bring to the reader's attention the fact that this derivation does *not* imply that the radiation pattern determines the antenna itself, if by the antenna we understand the current distribution *in*side the innermost region R_∞. There is an infinite number of current distributions that can produce the same far-field pattern. Our results indicate, however, that the entire field in the *exterior* region, i.e., outside the region R_∞, is determined exactly and nonrecursively by the far field.

and a similar equation for the magnetic energy density. By carefully examining the radial behavior of the total energy densities, we notice that the divergence of their volume integral arises from two types of terms. The first type is that associated with the radiation density $w_{\rm rad}$, which takes a functional form like $\langle \mathbf{A}_0, \mathbf{A}_0 \rangle / r^2$ and $\langle \mathbf{B}_0, \mathbf{B}_0 \rangle / r^2$. The volume integral of such terms will give rise to *linearly* divergent energy. The second type is that associated with functional forms like $\langle \mathbf{A}_0, \mathbf{A}_1 \rangle / r^3$ and $\langle \mathbf{B}_0, \mathbf{B}_1 \rangle / r^3$. The volume integral of these terms will result in energy contribution that is *logarithmically* divergent. However, we make use of the fact proved in [3] stating that the interactions $\langle \mathbf{A}_0, \mathbf{A}_1 \rangle$ and $\langle \mathbf{B}_0, \mathbf{B}_1 \rangle$ are identically zero. Therefore, only singularities of the first type will contribute to the total energy. Making use of the equality of the electric and magnetic radiation densities in the far zone, the remaining singularities can be eliminated and we are then justified in reaching the following series expansions for the *reactive* radial energy densities

$$w_e^1(r) = \frac{\varepsilon}{4} \sum_{n=1}^{\infty} \frac{\langle \mathbf{A}_n, \mathbf{A}_n \rangle}{r^{2n+2}} + \frac{\varepsilon}{2} \sum_{\substack{n,n'=1 \\ n>n'}}^{\infty} \frac{\langle \mathbf{A}_n, \mathbf{A}_{n'} \rangle}{r^{n+n'+2}}, w_h^1(r) = \frac{\mu}{4} \sum_{n=1}^{\infty} \frac{\langle \mathbf{B}_n, \mathbf{B}_n \rangle}{r^{2n+2}} + \frac{\mu}{2} \sum_{\substack{n,n'=1 \\ n>n'}}^{\infty} \frac{\langle \mathbf{B}_n, \mathbf{B}_{n'} \rangle}{r^{n+n'+2}}. \quad (9)$$

Denote by a the radius in $\mathrm{R}_\infty = \{(r, \theta, \varphi) : r \leq a\}$. After integrating term by term, we finally arrive to the following result for the total reactive energy[7]

$$W_e^1 = \sum_{n=1}^{\infty} \frac{(\varepsilon/4) \langle \mathbf{A}_n, \mathbf{A}_n \rangle}{(2n-1) a^{2n-1}} + \sum_{\substack{n,n'=1 \\ n>n'}}^{\infty} \frac{(\varepsilon/2) \langle \mathbf{A}_n, \mathbf{A}_{n'} \rangle}{(n+n'-1) a^{n+n'-1}}, \quad (10)$$

and a similar equation for the total magnetic energy. Therefore, the total reactive energy is finite. It follows then that the definitions postulated above for the reactive energy densities w_h^1 and w_e^1 are consistent. Moreover, from the results of [3], we now see that total reactive energies (10) are evaluated completely in analytical form and that *in principle* no computation of infinite numerical integrals is needed here.[8]

4 Conclusion

We stress here that the contribution of the expressions (10) is not merely having at hand a means to calculate the reactive energy of the antenna. The main insight here is the fact that the same formulas contain information about the mutual dependence of 1) the quality factor Q (through the reactive energy), 2) the size of the antenna (through the dependence on a), and 3) the far-field radiation pattern (through the interaction terms and the results of [3].) The derivation above points to the *relational* structure of the antenna from the engineering point of view in the sense that the quantitative and qualitative interrelations of performance measures like directivity and polarization (far field), matching bandwidth (the quality factor), and the size become all united within one outlook. The analysis of the antenna is not reduced to merely computing few numbers, but rather understood by the elaborate interconnection of all performance measures within an integral whole.

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[7] The expansions (9) are uniformly convergent in r and therefore we can interchange the order of summation and integration.

[8] Special cases of (10) appeared throughout literature. For example, see [4].