

Optimal Trajectories of Mobile Remote Sensors for Parameter Estimation in Distributed Cyber-Physical Systems

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Abstract—In this paper, we present a method to obtain the optimal trajectories of a team of robots monitoring a distributed parameter system located in a different domain. The mobile robots are equipped with remote sensors capable of measuring the considered system's state from a separate space. The purpose of trajectory planning of the team of robots is to obtain measurements so as to estimate the parameters of the considered system. From a given set of partial differential equation the dynamics of the distributed system's behavior, the optimal path and steering of the team of mobile nodes is obtained by minimizing the D-optimality criteria associated with the expected accuracy of the obtained parameter values. From this original problem, an optimal control problem is derived with the advantage of being solvable by readily available commercial softwares. The method is illustrated on a diffusive distributed systems.

I. INTRODUCTION

A. Literature Review

The juxtaposition of “real-life” physical systems and communication networks has brought to light a new generation of engineered systems: Cyber-Physical Systems (CPS) [1]. A definition of CPS was given in [2] in the following way: “Computational thinking and integration of computation around the physical dynamic systems form CPS where sensing, decision, actuation, computation, networking and physical processes are mixed”. Given its recent emergence and wide array of applications, the topic and study of CPS is believed to become a highly researched area in the years to come including its conferences [3][4] and journals [5]. “Applications of CPS arguably have the potential to dwarf the 20-th century IT revolution” [6]. The application of CPS are numerous and include medical devices and systems, patient monitoring devices, automotive and air traffic control, advanced automotive systems, process control, environmental monitoring, avionics, instrumentation, oil refineries, water usage control, cooperative robotics, manufacturing control, buildings, etc.

Within these potential applications, the one we are interested in belongs to the environmental monitoring category. It is believed that applied remote sensing can help determine the evapotranspiration of a given agricultural field and hence give improved information on crop condition and yield

to better irrigation control. In the same vein of research, remote sensing can give information correlated to the water stress level of the crops [7]. Remote sensing could provide important information to the farmers or even be use as feedback for a global irrigation control algorithm. Our on-going project consist of developing unmanned air vehicles (UAVs) equipped with aerial imagers to develop such control algorithm [8].

In the considered framework, the system is a distributed parameter system (DPS), that is to say the states are evolving along both time and space. Consequently, the traditional finite-dimensional input-output relationships have to be put aside and partial differential equations (PDEs) have to be used to model the system. This increased complexity of he system leads to challenging problems. Whereas the location of sensors is rather straightforward when considering a finite dimensional system, determining where measurement should be done is not a straightforward task in a DPS. One needs to consider the location of the sensors so that the gathered information best helps the parameter estimation. Therefore, it is a necessity to develop systematic approaches in order to increase the efficiency of PDE parameter estimation techniques.

The problem of sensor location in DPS has been studied before as one can find in review paper [9]. However, most publications are limited to stationary sensors. Newer approaches take advantage of spatially-movable sensors because when sensors are not assigned to fixed positions, the optimality is not achieved in the average sense but instead allow the sensors to follow the optimal location a given time moment. This leads to the tracking of the best information about the parameters to be identified. This leads to improved performance in parameter estimation as illustrated in [9].

The literature of mobile sensor trajectory planning in CPS is growing. Even though few approaches have been introduced, a large collection of problems have been considered. In [10], Rafajóvicz solves optimal path planning problem using the determinant of the Fisher Information Matrix (FIM) associated with the parameters he wants to estimate. His formulation leads to results consisting in an optimal time-dependent measure rather than a pure sensor trajectory. In [9] and [11], Uciński reformulates the problem of time-optimal path planning into a state-constrained optimal-control one which allows the addition of different constraints on the dynamics of the sensor. Increased observability of the system is considered in [12]. In [13], Uciński tries to properly formulate and solve the time-optimal problem for moving sensors which observe the state of a DPS in order to

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estimate its parameters' value. In [14], [15], the detection of a moving source within of distributed system is considered for a sensor network. In [16], the state estimation of a distributed system is considered; mobile sensors steering policy is decided so as to improve the state estimate. In [17], both source detection and process estimation are combined into a single framework. In [18], the Turing's Measure of Conditioning is used to obtain optimal sensor trajectories. The problem is solved for heterogeneous sensors (i.e. with different measurement accuracies) in [19]. Limited power resource is considered in [20]. In [21], Demetriou considers the optimal trajectories of mobile sensors in unison so as to improve state estimation. In [22], a so-called "closed-loop" scheme was considered for the case where initial estimates of the system's parameters were inaccurate. This paper opened a window towards networking considerations as computations were not prior to the experiment anymore but "on-line". For the first time, actuators trajectories were considered and optimized in [23] for known sensor trajectories. In [24], both actuators and sensors trajectories were optimized for improved performance.

So far, the literature has limited the movements of the sensors within the domain of the distributed parameter system. However, with the emergence of remote sensing, we should extend the framework to mirror this new way of taking measurements. Our main motivation comes from our own projects [25]. With the help of small UAVs, we are capable of taking pictures and obtain information on the amount of soil-moisture on a specific plot of land. Such UAVs could also be used to gather information on soil dynamics and help for better prediction of soil-moisture. However, the computation required for solving this complex problem (three dimensional PDE, imprecise knowledge of initial conditions, complex nonlinear UAV dynamics) does not allow us to yet to implement to proposed methodology to an actual application. This approach is instead reflected in an illustrative example used later in this paper.

B. Problem Formulation for PDE Parameter Estimation

Consider a distributed parameter system (DPS) described by the partial differential equation

$$\frac{\partial y}{\partial t} = \mathcal{F}(\mathbf{x}, t, y, \boldsymbol{\theta}) \quad \text{in } \Omega_{sys} \times T, \quad (1)$$

with initial and boundary conditions

$$\mathcal{B}(\mathbf{x}, t, y, \boldsymbol{\theta}) = 0 \quad \text{on } \Gamma_{sys} \times T, \quad (2)$$

$$y = y_0 \quad \text{in } \Omega_{sys} \times \{t = 0\}, \quad (3)$$

where $y(\mathbf{x}, t)$ stands for the scalar state at a spatial point $\mathbf{x} \in \bar{\Omega}_{sys} \subset \mathbb{R}^n$ and time instant $t \in \bar{T}$. $\Omega_{sys} \subset \mathbb{R}^n$ is a bounded spatial domain with sufficiently smooth boundary Γ , and $T = (0, t_f]$ is a bounded time interval. \mathcal{F} is assumed to be a known well-posed, possibly nonlinear, differential operator which includes first- and second-order spatial derivatives and include terms for forcing inputs. \mathcal{B} is a known operator acting on the boundary Γ and $y_0 = y_0(\mathbf{x})$ is a given function.

We assume that the state y depends on the parameter vector $\boldsymbol{\theta} \in \mathbb{R}^m$ of unknown parameters to be determined from measurements made by N moving sensors. Those mobile sensors are assumed to ambulate in a spatial domain $\Omega_{sens} \neq \Omega_{sys}$. The sensors are able to remotely take measurements in $\Omega_{meas} \subset \Omega_{sys}$ over the observation horizon T . We call $\mathbf{x}_s^j : T \rightarrow \Omega_{sens}$ the position/trajectory of the j -th sensor, where Ω_{sens} is a compact set representing the domain where the sensors can move. We call $\mathbf{z}_s^j : T \rightarrow \Omega$ the collection of measurements in Ω_{meas} where the j -th sensor is observing. We assume that a function $\mathbf{f} : \Omega_{sens} \rightarrow \Omega_{meas}$ linking the position of the sensor and measurements exists. The observations for the j -th sensor are assumed to be of the form

$$\mathbf{z}_s^j(t) = \mathbf{y}(\mathbf{f}(\mathbf{x}_s^j(t)), t) + \boldsymbol{\varepsilon}(\mathbf{f}(\mathbf{x}_s^j(t)), t), \quad t \in T, \quad j = 1, \dots, N, \quad (4)$$

where \mathbf{y} is the vector containing the values of the state y at the different locations $\mathbf{f}(\mathbf{x}_s^j(t))$ and $\boldsymbol{\varepsilon}$ represents the measurement noise assumed to be white, zero-mean, Gaussian and spatial uncorrelated with the following statistics

$$\mathbb{E}\{\boldsymbol{\varepsilon}(\mathbf{f}(\mathbf{x}_s^j(t)), t)\boldsymbol{\varepsilon}(\mathbf{f}(\mathbf{x}_s^i(t')), t')\} = \sigma^2 \delta_{ji} \delta(t - t'), \quad (5)$$

where σ^2 stands for the standard deviation of the measurement noise, δ_{ij} and $\delta(\cdot)$ are the Kronecker and Dirac delta functions, respectively.

With the above settings, similar to [9], the optimal parameter estimation problem is formulated as follows: Given the model (1)–(3) and the measurements \mathbf{z}_s^j from the sensors \mathbf{x}_s^j , $j = 1, \dots, N$, determine an estimate $\hat{\boldsymbol{\theta}} \in \Theta_{ad}$ (Θ_{ad} being the set of admissible parameters) of the parameter vector which minimizes the generalized output least-squares fit-to-data functional given by

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta} \in \Theta_{ad}} \sum_{j=1}^N \int_T [\mathbf{z}_s^j(t) - \hat{\mathbf{y}}(\mathbf{f}(\mathbf{x}_s^j(t)), t; \boldsymbol{\theta})]^2 dt \quad (6)$$

where $\hat{\mathbf{y}}$ is the solution of (1)–(3) with $\boldsymbol{\theta}$ replaced by $\boldsymbol{\vartheta}$.

By observing (6), it is possible to foresee that the parameter estimate $\hat{\boldsymbol{\theta}}$ depends on the number of sensors N and the mobile sensor trajectories \mathbf{x}_s^j . This fact triggered the research on the topic and explains why the literature so far focused on optimizing both the number of sensors and their trajectories. The intent was to select these design variables so as to produce best estimates of the system parameters after performing the actual experiment.

Since our approach is based on the methodology developed for optimal sensor location, we display it here as an introduction to the theory from [9] and [26]. In order to achieve optimal sensor location, some quality measure of sensor configurations based on the accuracy of the parameter estimates obtained from the observations is required. Such a measure is usually related to the concept of the Fisher Information Matrix (FIM), which is frequently referred to in the theory of optimal experimental design for lumped parameter systems [27]. Its inverse constitutes an approximation of the covariance matrix for the estimate of $\boldsymbol{\theta}$. Given the

assumed statistics of the measurement noise, the FIM has the following representation [9]:

$$\mathbf{M} = \sum_{j=1}^N \int_T \mathbf{g}(\mathbf{f}(\mathbf{x}_s^j(t)), t) \mathbf{g}^\top(\mathbf{f}(\mathbf{x}_s^j(t)), t) dt, \quad (7)$$

where

$$\mathbf{g}(\mathbf{x}, t) = \nabla_{\boldsymbol{\theta}} y(\mathbf{x}, t; \boldsymbol{\theta}) \Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \quad (8)$$

denotes the vector of the so-called *sensitivity coefficients*, $\boldsymbol{\theta}^0$ being a prior estimate to the unknown parameter vector $\boldsymbol{\theta}$ [11], [9].

However, the FIM can hardly be used in an optimization as is. Therefore, it is necessary to maximize some scalar function Ψ of the information matrix to obtain the optimal experiment setup. The introduction of the scalar criterion allows us to pose the sensor location problem as an optimization problem. Several choices for such a function can be found in the literature [28], [27] and the most popular one is the D-optimality criterion

$$\Psi[\mathbf{M}] = -\log \det(\mathbf{M}). \quad (9)$$

Its use yields the minimal volume of the uncertainty ellipsoid for the estimates of the parameters. In this paper, only the D-optimality criterion is considered.

II. OPTIMAL MEASUREMENT PROBLEM

A. Mobile Sensor Model

1) *Sensor Dynamics*: We assume that the sensing devices are equipped on vehicles whose dynamics can be described by the following differential equation

$$\dot{\mathbf{x}}_s^j(t) = \mathbf{f}_s(\mathbf{x}_s^j(t), \mathbf{u}_s^j(t)) \quad \text{a.e. on } T, \quad \mathbf{x}_s^j(0) = \mathbf{x}_{s0}^j. \quad (10)$$

With this nomenclature, the function $\mathbf{f}_s : \mathbb{R}^N \times \mathbb{R}^{r_s} \rightarrow \mathbb{R}^N$ has to be continuously differentiable, the vector $\mathbf{x}_{s0}^j \in \mathbb{R}^N$ represents the initial disposition of the j -th sensor, and $\mathbf{u}_s : T \rightarrow \mathbb{R}^{r_s}$ is a measurable control function satisfying the following inequality

$$\mathbf{u}_{sl} \leq \mathbf{u}_s(t) \leq \mathbf{u}_{su} \quad \text{a.e. on } T, \quad (11)$$

for some known constant vectors \mathbf{u}_{sl} and \mathbf{u}_{su} . Let us introduce,

$$\mathbf{s}(t) = (\mathbf{x}_s^1(t), \mathbf{x}_s^2(t), \dots, \mathbf{x}_s^N(t))^T, \quad (12)$$

where $\mathbf{x}_s^j : T \rightarrow \Omega_{sens}$ is the trajectory of the j -th sensor. Additionally, we define $\mathbf{s}(0) = \mathbf{s}_0$.

2) *Mobility Constraints*: We assume that all the mobile nodes equipped with sensors are confined within an admissible region Ω_{sensAD} (a given compact set) where the sensors are allowed to travel. Ω_{sensAD} can be conveniently defined as

$$\Omega_{sensAD} = \{\mathbf{x} \in \Omega_{sens} : b_{si}(\mathbf{x}) = 0, i = 1, \dots, I\}, \quad (13)$$

where the b_{si} functions are known continuously differentiable functions. That is to say that the following constraints have to be satisfied:

$$h_{ij}(\mathbf{s}(t)) = b_{si}(\mathbf{x}_s^j(t)) \leq 0, \forall t \in T, \quad (14)$$

where $1 \leq i \leq I$ and $1 \leq j \leq N$. For simpler notation, we reformulate the conditions described in (14) in the following way

$$\gamma_{sl}(\mathbf{s}(t)) \leq 0, \forall t \in T, \quad (15)$$

where $\gamma_{sl}, l = 1, \dots, \nu$ tally with (14), $\nu = I \times N$. It would be possible to consider additional constraints on the path of the vehicles such as specific dynamics, collision avoidance, communication range maintenance and any other conceivable constrains.

3) *Remote Sensing Constraints*: As mentioned earlier, we assume that the sensors are capable of taking measurements in Ω_{sys} , while being physically in Ω_{sens} . For that purpose, we introduce a remote sensing function \mathbf{f} giving the location(s) of the measurement based on the location of the sensor. Similarly to path constraints, we assume that the remote sensing is only allowed within an admissible region Ω_{measAD} where the measurements are possible. The constraints on remote sensing can be defined as constraints on measurement location and then transformed into mobility ones. We can define Ω_{measAD} as

$$\Omega_{measAD} = \{\mathbf{x} \in \Omega_{sens} : b_{mi}(\mathbf{f}(\mathbf{x})) = 0, i = 1, \dots, I\}, \quad (16)$$

where the b_{mi} functions have the same properties as b_{si} . Similarly, we can regroup the remote sensing constraints into an inequality

$$\gamma_{mi}(\mathbf{s}(t)) \leq 0, \forall t \in T. \quad (17)$$

Remark: For our project [8], UAVs equipped with multispectral imagers are used for collecting aerial images of agricultural fields. The purpose of remote sensing is to gather data about the ground's surface while avoiding to come in contact with it. Multispectral imagers can generate an image for each different wavelength bands ranging from visible spectra to infra-red or thermal based for various applications. Having such a diverse and wide range of wavelengths allow for a better analysis of the ground's surface properties. Under such circumstances, the domain where the sensors ambulate (space), is different from the domain where measurements are taken (ground). The constraints on mobility (such as collision avoidance between UAVs and/or environment) are different from the constraints on remote sensing (such as maintaining the images within the domain of interest that is the crop field).

B. Problem Definition

The purpose of the optimal measurement problem is to determine the forces (controls) applied to each vehicle, which minimize the design criterion $\Psi(\cdot)$ defined on the FIMs of the form (7), which are determined unequivocally by the corresponding trajectories, subject to constraints on the magnitude of the controls and induced state constraints. To increase the degree of optimality, our approach considers \mathbf{s}_0 as a control parameter vector to be optimized in addition to the control function \mathbf{u}_s .

Given the above formulation we can cast the optimal measurement policy problem as the following optimization

problem: Find the pair (s_0, \mathbf{u}_s) which minimizes

$$J(s_0, \mathbf{u}_s) = \Psi[M(\mathbf{s})] \quad (18)$$

over the set of feasible pairs

$$\mathcal{P} = \{(s_0, \mathbf{u}_s) \mid \mathbf{u} : T \rightarrow \mathbb{R}^r \text{ is measurable,} \quad (19)$$

$$\mathbf{u}_{sl} \leq \mathbf{u}_s(t) \leq \mathbf{u}_{su} \text{ a.e. on } T, s_0 \in \Omega_{sens}\},$$

subject to the constraint (15) and (17).

The solution to this problem can hardly have an analytical solution. It is therefore necessary to rely on numerical techniques to solve the problem. A wide variety of techniques are available [29]. However, the problem can be reformulated as a classical Mayer problem where the performance index is defined only via terminal values of state variables.

III. OPTIMAL CONTROL FORMULATION

In this section, the problem is converted into a canonical optimal control one making possible the use of existing optimal control problems solvers.

To simplify our presentation, we define the function $\text{svec} : \mathbb{S}^m \rightarrow \mathbb{R}^{m(m+1)/2}$, where \mathbb{S}^m denotes the subspace of all symmetric matrices in $\mathbb{R}^{m \times m}$ that takes the lower triangular part (the elements only on the main diagonal and below) of a symmetric matrix A and stacks them into a vector \mathbf{a} :

$$\mathbf{a} = \text{svec}(A) \quad (20)$$

$$= \text{col}[A_{11}, A_{21}, \dots, A_{m1}, A_{22}, \dots, A_{32}, \dots, A_{m2}, \dots, A_{mm}]. \quad (21)$$

Reciprocally, let $A = \text{Smat}(\mathbf{a})$ be the symmetric matrix such that $\text{svec}(\text{Smat}(\mathbf{a})) = \mathbf{a}$ for any $\mathbf{a} \in \mathbb{R}^{m(m+1)/2}$.

Consider the matrix-valued function

$$\Pi(\mathbf{s}(t), t) = \sum_{j=1}^N \mathbf{g}(f(\mathbf{x}_s^j(t)), t) \mathbf{g}^T(f(\mathbf{x}_s^j(t)), t). \quad (22)$$

Setting $r : T \rightarrow \mathbb{R}^{m(m+1)/2}$ as the solution of the differential equations

$$\dot{\mathbf{r}}(t) = \text{svec}(\Pi(\mathbf{s}(t), t)), \quad r(0) = 0, \quad (23)$$

we obtain

$$M(\mathbf{s}) = \text{Smat}(\mathbf{r}(t_f)), \quad (24)$$

i.e., minimization of $\Phi[M(\mathbf{s})]$ thus reduces to minimization of a function of the terminal value of the solution to (23). Introducing an augmented state vector

$$\mathbf{q}(t) = \begin{bmatrix} \mathbf{s}(t) \\ \mathbf{r}(t) \end{bmatrix}, \quad (25)$$

we obtain

$$\mathbf{q}_0 = \mathbf{q}(0) = \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{0} \end{bmatrix}. \quad (26)$$

Then the equivalent canonical optimal control problem consists in finding a pair $(\mathbf{q}_0, \mathbf{u}_s) \in \bar{\mathcal{P}}$ which minimizes the performance index

$$\bar{J}(\mathbf{q}_0, \mathbf{u}_s) = \phi(\mathbf{q}(t_f)) = \Phi[\text{Smat}(\mathbf{r}(t_f))] \quad (27)$$

subject to

$$\begin{cases} \dot{\mathbf{q}}(t) = \varphi(\mathbf{q}(t), \mathbf{u}_s(t), t) \\ \mathbf{q}(0) = \mathbf{q}_0 \\ \bar{\gamma}_{sl}(\mathbf{q}(t)) \leq 0 \\ \bar{\gamma}_{ml}(\mathbf{q}(t)) \leq 0 \end{cases} \quad (28)$$

where

$$\bar{\mathcal{P}} = \{(\mathbf{q}_0, \mathbf{u}) \mid \mathbf{u} : T \rightarrow \mathbb{R}^r \text{ is measurable,} \quad (29)$$

$$\mathbf{u}_l \leq \mathbf{u}(t) \leq \mathbf{u}_u \text{ a.e. on } T, s_0 \in \Omega_{sens}^M\},$$

and

$$\varphi(\mathbf{q}, \mathbf{u}, t) = \begin{bmatrix} f(\mathbf{s}(t), \mathbf{u}(t)) \\ \text{svec}(\Pi(\mathbf{s}(t), t)) \end{bmatrix}, \quad (30)$$

$$\bar{\gamma}_{sl}(\mathbf{q}(t)) = \gamma_{sl}(\mathbf{s}(t)) \quad (31)$$

$$\bar{\gamma}_{ml}(\mathbf{q}(t)) = \gamma_{ml}(\mathbf{s}(t)). \quad (32)$$

The problem formulated above in normal form can be solved with readily available software packages for solving dynamic optimization problems in a numerical way. Like in most of our work, we use RIOTS_95, which is designed as a MATLAB toolbox written mostly in C and runs under Windows 98/2000/XP/vista and Linux. The theory behind RIOTS_95 and its numerical methods can be found in [30].

IV. AN ILLUSTRATIVE EXAMPLE

In this section, we use a demonstrative example to illustrate the method developed earlier. The system we consider here consists of the two-dimensional diffusion equation

$$\frac{\partial y}{\partial t} = \nabla \cdot (\kappa \nabla y) + 20 \exp(-50(x_1 - t)^2) \quad (33)$$

for $\mathbf{x} = [x_1 \ x_2]^T \in \Omega_{sys} = (0, 1)^2$ and $t \in [0, 1]$, subject to homogeneous zero initial and Dirichlet boundary conditions. The spatial distribution of the diffusion coefficient is assumed to have the form

$$\kappa(x_1, x_2) = \theta_1 + \theta_2 x_1 + \theta_3 x_2. \quad (34)$$

In this example, the values of the diffusion coefficient parameters (which we want to estimate) are $\theta_1 = 0.1$, $\theta_2 = -0.05$ and $\theta_3^0 = 0.2$. They are assumed to be known prior to the experiment. The dynamics of the mobile sensors follow the given dynamical model

$$\dot{\mathbf{x}}_s^j(t) = \mathbf{u}_s^j(t), \quad \mathbf{x}_s^j(0) = \mathbf{x}_{s0}^j, \quad (35)$$

for $\mathbf{x} = [x_1 \ x_2 \ x_3]^T \in \Omega_{sens} = (0, 1)^3$ and additional constraints

$$|u_i^j(t)| \leq 0.7, \quad \forall t \in T, \quad j = 1, \dots, N, \quad i = 1, 2, \quad (36)$$

$$|u_i^j(t)| \leq 0.2, \quad \forall t \in T, \quad j = 1, \dots, N, \quad i = 3. \quad (37)$$

We can notice that Ω_{sens} is of dimension 3 and Ω_{sys} is of dimension 2, and that Ω_{sys} lies in the boundary of Ω_{sens} . The remote sensing function f is defined in a way that is very similar to a camera embedded on an unmanned air vehicle. We assume the mobile node's attitude is determined by an orthogonal basis directed by the control input \mathbf{u}_s^j . \mathbf{u}_s^j gives us the direction the robot is facing, the second axis is taken

parallel to the $x_3 = 0$ plane and the third axis is obtained by completing the orthogonal basis in a direct way. The obtained basis is $\{e_{j1}, e_{j2}, e_{j3}\}$, with $e_{j1} = u_s^j$. The view vector of the j -th sensor is taken as $-e_{j3}$ which can be seen as a camera facing downward. The vertical field of view is chosen as $\frac{\pi}{3}$ and the horizontal field of view is taken as $\frac{\pi}{2}$. Since we decided to model our remote sensor as a camera, we choose a resolution of 3×3 . Measurements are taken at the intersection of the field of view and Ω_{sys} . To give the reader a better insight of the remote sensing function, we provide a visual description in Fig.1. The orthogonal basis is in black, the view vector is represented by a red line and the visual footprint is represented by a blue trapezoid.

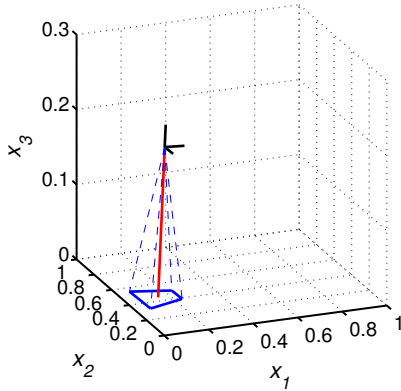


Fig. 1. Description of the remote sensing function.

The purpose of our optimization is to obtain the trajectories of a team of three sensors so as to determine the best possible estimates of the parameters θ_1 , θ_2 and θ_3 .

The determination of the Fisher information matrix for a given experiment requires the knowledge of the vector of the sensitivity coefficients $\mathbf{g} = \text{col}[g_1, g_2, g_3]$ along sensor trajectories. The FIM can be obtained using the direct differentiation method [9] by solving the following set of PDEs:

$$\begin{aligned} \frac{\partial y}{\partial t} &= \nabla \cdot (\kappa \nabla y) + 20 \exp(-50(x_1 - t)^2), \quad (38) \\ \frac{\partial g_1}{\partial t} &= \nabla \cdot \nabla y + \nabla \cdot (\kappa \nabla g_1), \\ \frac{\partial g_2}{\partial t} &= \nabla \cdot (x_1 \nabla y) + \nabla \cdot (\kappa \nabla g_2), \\ \frac{\partial g_3}{\partial t} &= \nabla \cdot (x_2 \nabla y) + \nabla \cdot (\kappa \nabla g_3), \end{aligned}$$

in which the first equation represents the original state equation and the next three equations are obtained from the differentiation of the first equation with respect to the parameters θ_1 , θ_2 and θ_3 , respectively. The initial and Dirichlet boundary conditions for all the four equations are homogeneous.

Since the sensing function is not pointwise, we reformulate

Eqn.8 for our illustrative example.

$$\mathbf{g}(\mathbf{x}, t) = \sum_{i=1}^{res} \sum_{j=1}^{res} \nabla_{\boldsymbol{\theta}} y(\mathbf{x}_{ij}, t; \boldsymbol{\theta}) \Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} / res^2, \quad (39)$$

where res stands for the resolution of the sensor (3 in our case). In addition, to prevent the mobile nodes from intersecting with the systems's domain Ω_{sys} , which would be equivalent to a crash, the optimality criteria is reformulated as,

$$J(\mathbf{s}_0, \mathbf{u}) = \Phi[M(\mathbf{s})] + \frac{1}{|x_3|}. \quad (40)$$

The resulting optimal trajectory of one mobile sensor can be observed in Fig.2. The results for a team of two sensors is displayed in Fig.3, and the case for three sensors is given in Fig.4.

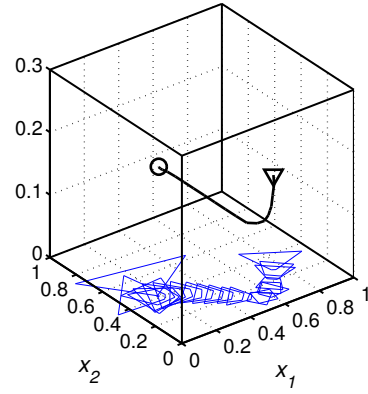


Fig. 2. D-Optimal trajectory of one sensor. The initial positions are marked with open circles and the final positions are designated by triangles. The measured area is delineated by a blue trapezoid.

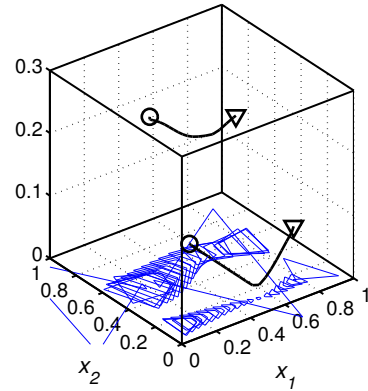


Fig. 3. D-Optimal trajectories of two sensors. The initial positions are marked with open circles and the final positions are designated by triangles. The measured area is delineated by a blue trapezoid.

V. CONCLUSION

We have extended the existing framework of the design of mobile sensor trajectories which minimizes the volume of the confidence ellipsoid for the estimates to the emerging

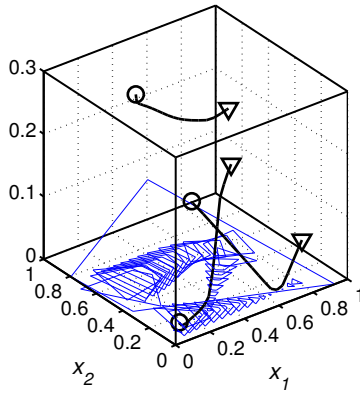


Fig. 4. D-Optimal trajectories of three sensors. The initial positions are marked with open circles and the final positions are designated by triangles. The measured area is delineated by a blue trapezoid.

field of remote sensing. For that purpose, we introduced a remote sensing function linking the mobility domain and the sensing domain. It is important to notice that the introduced formulation can still be transformed into a canonical optimal control problem. This reformulation allows the problem to be solved by the MATLAB toolbox RIOTS_95, a collection of routines capable of solving a large class of finite-time optimal control problems, with the help of the MATLAB Partial Differential Equation Toolbox. The method was then applied to an illustrative example to demonstrate its applicability.

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