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The Synthesis of Planar Parallel Manipulators with a Genetic Algorithm

This paper presents a genetic algorithm approach for the synthesis of planar three-degree-of-freedom parallel manipulators. A genetic algorithm is an optimization method inspired by natural evolution. As in nature, the fittest members of a population are given better chances of reproducing and transmitting part of their genetic heritage to the next generation. This leads to stronger and stronger generations which evolve towards the solution of the problem. For the applications studied here, the individuals in the population consist of the architectural parameters of the manipulators. The algorithm optimizes these parameters to obtain a workspace as close as possible to a prescribed working area. For each individual of the population, the geometric description of the workspace can be obtained. The algorithm then determines the intersection between the prescribed workspace and the actual workspace, and minimizes the area of the regions that do not intersect. The method is applied to two planar three-degree-of-freedom parallel manipulators, one with prismatic joints and one with revolute joints.

1 Introduction

Many papers have been published on the optimization of the workspace of manipulators (e.g. Lin and Freudenstein, 1986; Paden and Sastry, 1988; Tsai and Soni, 1984). Few researchers have studied the synthesis of manipulators to try to fit a specified workspace as closely as possible. This approach would produce more compact manipulators, or a more economical solution. Gosselin and Guillot (1991) presented an algorithm and synthesized a serial and a parallel manipulator with two degrees of freedom (dof) for prescribed workspaces. Merlet (1997) introduced a numerical procedure to determine all the possible geometries of Gough-type 6 dof parallel manipulators whose workspace must include a desired one. Murray et al. (1997) proposed a technique using planar quaternions for designing planar parallel manipulators with platforms capable of reaching any number of desired poses.

This paper presents a genetic algorithm approach for the synthesis of planar three-degree-of-freedom parallel manipulators. A genetic algorithm (GA) is used because of its robustness and good convergence properties. It is shown that the architectural parameters can be optimized to produce a manipulator that has a workspace as close as possible to a prescribed one.

2 Workspace Intersection

The determination of the workspace of parallel mechanisms is an important issue which has been addressed by several researchers (see for instance: Bajpai and Roth, 1986; Gosselin, 1990; Kumar, 1992). Several types of workspace regions can be defined such as the reachable workspace, the constant orientation workspace, the dextrous workspace and the total orientation workspace (Kumar, 1992; Merlet et al., 1998). The constant orientation workspace will be used here. Moreover, the workspace prescribed for a given orientation will be specified using a set of circular arcs in the plane.

The optimization objective is to determine the architectural parameters such that the actual workspace is as close as possible to the prescribed one. The procedure consists in minimizing the area of the region that is not part of the intersection between the prescribed and actual workspaces. Let $\Re_p$ denote the region corresponding to the prescribed workspace and $\Re_a$ the region corresponding to the actual workspace.

The following quantities are defined.

$$\Re_i = \Re_p \cap \Re_a \quad \Re'_p = \Re_p \backslash \Re_i \quad \Re'_a = \Re_a \backslash \Re_i \tag{1}$$

where $\cap$ denotes an intersection and $\backslash$ a subtraction of regions.

3 Optimization

To obtain an actual workspace as close as possible to the prescribed one, the regions $\Re'_p$ and $\Re'_a$ have to be minimized. The performance index η will use the surface area of these regions to optimize the architectural parameters and will be given by

$$\eta = A'_p + A'_a \tag{2}$$

where A'_p and A'_a represent the surface area of regions $\Re'_p$ and $\Re'_a$, respectively. The optimization problem can therefore be formulated as

$$\min_{k} \eta \tag{3}$$

where **k** represents the vector containing the architectural parameters of the manipulator. Since the boundaries of the workspaces are described by a series of arcs, the area can be computed by integration on the boundary (Gosselin, 1990).

4 Genetic Algorithms

Genetic algorithms were developed by Holland (1975). These consist of optimization procedures based on principles inspired by natural evolution. Each member of the population constitutes a possible solution to the problem to be solved. The chromosome of an individual, which contains its characteristics, will be recombined with the chromosome of another individual to create offspring. For many generations, reproduction, crossover and mutation operations are performed, as the method gradually converges towards the solution. According to Davis (1991), Real-Coded GA using real numbers usually outperform GA using bit string encoding. Details of the GA operations for Real-Coded GA, which are used in this work, can be found in Boudreau and Turkkan (1996).

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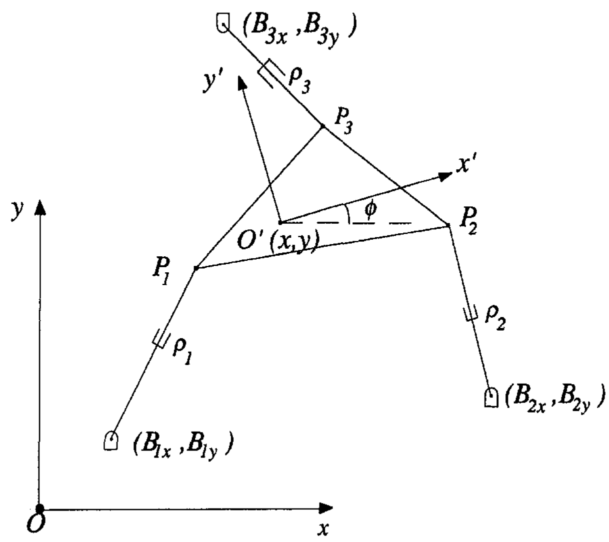

Fig. 1 Planar 3 dof parallel manipulator with prismatic joints

5 Example Applications

5.1 Planar Three-Degree-of-Freedom Manipulator with Prismatic Joints

5.1.1 Manipulator Workspace. A planar three-dof parallel manipulator with prismatic joints is shown in Fig. 1. It consists of a movable platform, or end effector, $P_1P_2P_3$ and a fixed base $B_1B_2B_3$. The unactuated revolute joints located at points B_i and P_i, $i = 1, 2, 3$, are connected by three actuated prismatic joints, whose lengths are noted ρ_i, $i = 1, 2, 3$, and which allow the positioning (x, y) and the orientation (ϕ) of the end effector in a plane.

It can be shown that for a given orientation of the platform, i.e. for a specified value of angle ϕ, if all the actuators are identical and their range of motions varies from a minimum length $\rho_{\min}$ to a maximum length $\rho_{\max}$, then the locus of points attainable by point O' considering the constraints on each actuator individually will have the two following concentric circles as boundaries.

$$(x - a_i)^2 + (y - b_i)^2 = \rho^2_{\min}, \quad i = 1, 2, 3 \tag{4}$$

$$(x - a_i)^2 + (y - b_i)^2 = \rho^2_{\max}, \quad i = 1, 2, 3 \tag{5}$$

The intersection of the three regions described by Eqs. (4) and (5) represents the workspace of the manipulator. The centres of the circles (a_i, b_i) depend on the architectural parameters and on the orientation of the platform.

5.1.2 Procedure and Results. The prescribed workspace is described by a list of circular arcs. To obtain the manipulator workspace, the twelve architectural parameters defining the dimensions, (B_{ix}, B_{iy}) and (P'_{ix}, P'_{iy}), $i = 1, 2, 3$, are necessary, as well as the minimum actuator lengths $(\rho_{\min})$ and a factor (α) for specifying the maximum actuator lengths as a function of the minimum actuator lengths $(\rho_{\max} = \alpha\rho_{\min})$. The vector of optimization variables is

$$\mathbf{k} = [B_{1x}, B_{1y}, B_{2x}, B_{2y}, B_{3x}, B_{3y}, P'_{1x}, P'_{1y}, P'_{2x}, P'_{2y}, P'_{3x}, P'_{3y}, \rho_{\min}, \alpha] \tag{6}$$

In the GA, each individual consists of these fourteen architectural parameters. A search domain must be specified for each variable to create an initial population. The limits of the search domain are set by inspection of the prescribed area. Values for the multiplication factor α were set between 1.5 and 2.0. The actual workspace is computed for each individual as described in Section 5.1.1 and the area that is not part of the intersection between the actual and prescribed workspaces is determined. The GA minimizes this area to obtain the best architectural parameters.

The following parameters were used in the GA:

Population:	100
Probability of crossover, p_c:	0.85
Probability of mutation, p_m:	0.05
Relative weight factor, a:	1.05
Degree of dependancy factor, b:	5
Maximum number of generations:	200

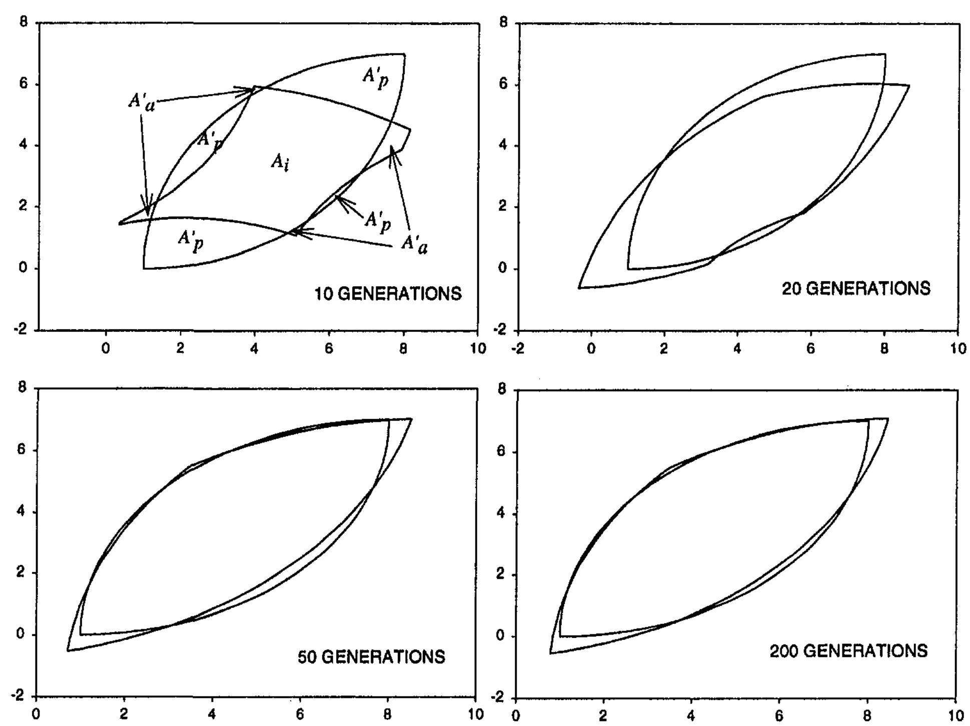

Fig. 2 Evolution of actual workspaces, $\phi = 20°$

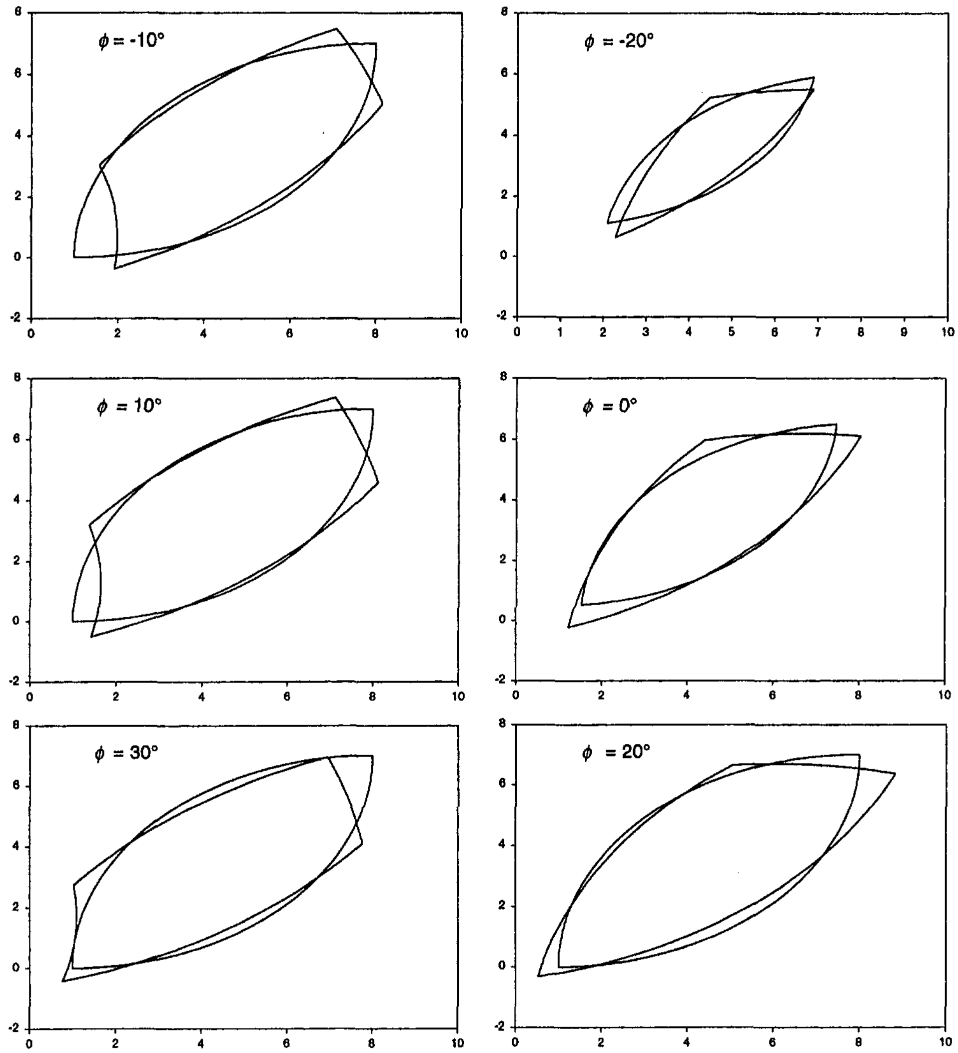

Fig. 3 **(*a*) Actual and prescribed workspaces for different orientations (left column); (*b*) Actual and prescribed workspaces, prescribed workspaces of different sizes (right column)**

Results are given for three cases. First, architectural parameters that lead to an actual workspace as close as possible to the prescribed one, when the orientation of the platform is given by $\phi = 20$ deg, are sought. Next, the parameters are optimized on the same prescribed workspace when the orientation of the platform varies from -10 deg to $+30$ deg, using increments of 10 deg. For each individual of the population, the area that is not part of the intersection between the actual and prescribed workspaces is computed for each of the orientation angles. The objective function is then the sum of the areas at the five specified orientations. Finally, the parameters are optimized for three different prescribed areas at orientations of -20 deg, 0 deg and 20 deg. Similarly, here the objective function is the sum of the areas at the three specified orientations.

Figure 2 shows the prescribed workspace and the evolution of the workspace of the best individual after 10, 20, 50 and 200 generations, when an orientation angle of 20 deg is used. The architectural parameters found by the GA after 200 generations are

$$\mathbf{k} = [4.86, -7.03, 11.64, -3.52, -1.75, 12.98, -5.23, -2.70, 0.25, -2.48, 0.78, 3.90, 5.60, 1.75] \quad (7)$$

From Fig. 2, we can see that the parameters found produce an actual workspace that is very similar to the prescribed one when the orientation is 20 deg. However, if the orientation of the platform is changed, the actual workspace is significantly different. For the parameters found, the manipulator has no workspace when $\phi = -10$ deg.

To obtain a manipulator that has a workspace similar to the prescribed one for orientations from -10 deg to $+30$ deg, the optimization was performed using an objective function based on the workspace at different orientations, as mentioned previously. The architectural parameters found by the GA after 200 generations are

$$\mathbf{k} = [-6.92, -0.08, 13.46, -7.00, -4.13, 13.63, -1.77, -1.03, 1.02, -1.65, -0.70, 1.19, 7.00, 1.98] \quad (8)$$

Figure 3*a* shows the results with the parameters given in Eq. (8) when the orientation is given by $\phi = -10$ deg, 10 deg, and 30 deg. The results at 0 deg and 20 deg are not shown but are similar. The actual workspace is similar to the prescribed workspace for all of the angles. Figure 3*b* indicates the workspaces obtained when three different prescribed areas are specified at orientations of -20 deg, 0 deg and 20 deg. The prescribed areas are all similar and comprised of two arcs but increase in size from -20 deg to 20 deg. The architectural parameters found by the GA after 200 generations are

$$\mathbf{k} = [4.99, -6.90, 13.91, -4.79, -3.19, 13.26, -1.71, -1.42, 1.20, -2.56, -0.55, 2.50, 6.27, 1.86] \quad (9)$$

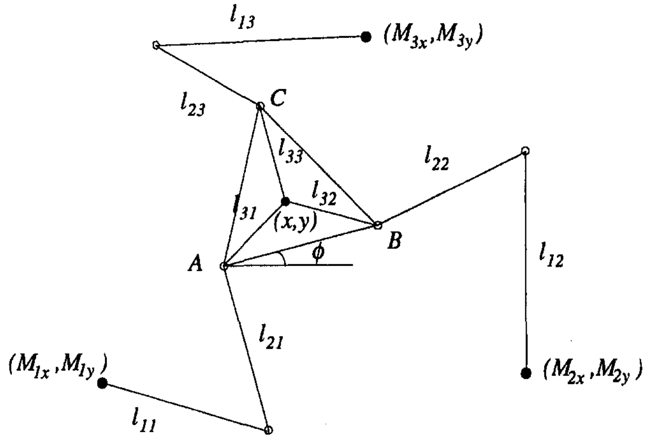

Fig. 4 Planar 3 dof parallel manipulator with revolute joints

5.2 Planar Three-Degree-of-Freedom Manipulator with Revolute Joints

5.2.1 Manipulator Workspace. A planar three-dof parallel manipulator with revolute joints is shown in Fig. 4. Motors M_1, M_2 and M_3 are fixed while triangle ABC constitutes the end effector of the manipulator. The position and orientation of the end effector are given by its centroid (x, y) and by the angle ϕ. A symetric end effector is assumed for the manipulator in this study ($l_{31} = l_{32} = l_{33} = l_3$). Similarly to the development shown in Section 5.1.1, it can be shown that the workspace for a given orientation can be found from the intersection of three pairs of concentric circles.

5.2.2 Procedure and Results. For this manipulator, the optimization variables are the motor coordinates (M_{ix}, M_{iy}), the link lengths (l_{1i}, l_{2i}), $i = 1, 2, 3$, and the end effector length l_3. The vector of optimization variables is therefore

$$\mathbf{k} = [M_{1x}, M_{1y}, M_{2x}, M_{2y}, M_{3x}, M_{3y}, l_{11}, l_{21}, l_{12}, l_{22}, l_{13}, l_{23}, l_3] \quad (10)$$

The procedure is similar to the one used for the preceding manipulator. The parameters used in the GA were the same.

In this case case, two optimizations were performed: a first one with the orientation constant at 0 deg and another with orientations varying from −30 deg to +30 deg in increments of 10 deg. To compare each procedure, three trials were computed for each type of optimization. For each optimum result obtained, the area of the regions that do not intersect between the prescribed and actual regions was computed for each orientation of the platform comprised between $\phi = -30$ deg and $\phi = 30$ deg with increments of 10 deg. For the manipulators obtained using the optimization based on different orientations, the average area of the regions that did not intersect was 131 units squared. However, for the manipulator obtained when the orientation is constant at $\phi = 0$ deg during the optimization, the average value was 207 units squared. This result demonstrates the benefit of including different orientations in the performance index if it is desired to obtain a workspace that is similar throughout a range of orientations.

Figure 5 shows the prescribed and actual workspaces at −30 deg, 0 deg and +30 deg when the optimization is performed by specifying different orientations. The corresponding vector of optimization variables, after 200 generations is

$$\mathbf{k} = [-8.43, -10.45, 15.17, -1.61, -4.85, 12.39, 9.56, 6.44, 11.51, 6.26, 9.91, 5.96, 3.02] \quad (11)$$

Figure 5 with an orientation of 30 deg shows the manipulator corresponding to the parameters of Eq. (11) when the origin of the moving coordinate frame is at (2.2, 5.3).

The computer code for the GA was written in C/C++, compiled with a Watcom C/C++ compiler and executed on a Pentium Pro 200 MHz computer. Execution time is approximately 2 minutes for 200 generations when a constant orientation is specified. When the performance index is computed using a set of different orientations, the execution time mentioned is approximately multiplied by the number of specified orientations.

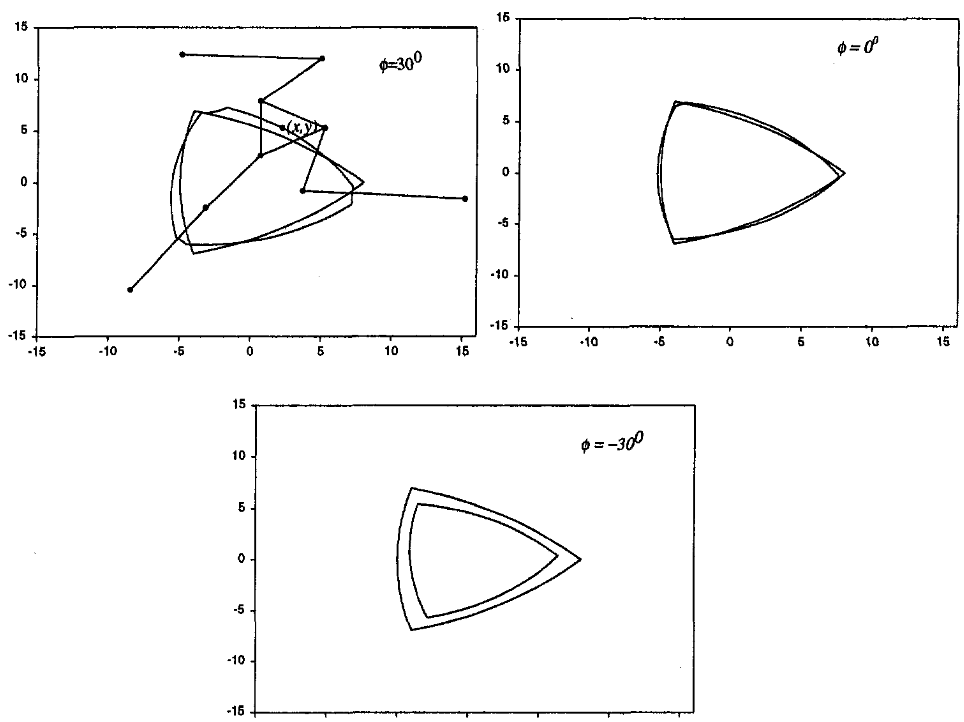

Fig. 5 Prescribed and actual workspaces, manipulator with revolute joints

6 Conclusion

A method for the optimization of the architectural parameters of planar three-dof parallel manipulators has been presented. The method was applied to two manipulators, one with prismatic joints and one with revolute joints. The results show that GA can determine the architectural parameters of manipulators that provide a workspace very similar to a prescribed one. The optimization procedure may include the specification of the prescribed workspace at a specified orientation or over a range of orientations. The GA showed very good convergence for all of the trial runs. Since the workspace of a parallel manipulator is far from being intuitive, the method developed should be very useful as a design tool.

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