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Redesign of Structural Vibration Modes by Finite-Element Inverse Perturbation

A previously developed technique for redesigning the vibrational properties of structures, by inverting the first-order perturbation analysis of the equations of motion, has been applied to a NASTRAN finite element analysis for plates and shells. The program finds the minimal changes to the thicknesses of the plate elements necessary to effect a given set of changes in the modal frequencies and shapes. Results have been obtained for a flat cantilever plate, a cantilever segment of a cylinder, and for a compressor blade for a jet engine.

Introduction

In a previous paper [1], we described a computer program for redesigning structures to effect desired changes in the shapes and frequencies of their vibration modes, by making the smallest possible change in the structure. This was the outgrowth of a general perturbation analysis of the equations of motion of a structure [2] and the development of an inversion of that analysis [3]. The program described in reference [1] accepted holographically obtained data on the vibration modes of a physical model. As a consequence, it was limited to plate-like structures with no in-plane vibrations. It was further limited as a design tool by the need to build a new physical structure after each application of the program. This was required to determine the degree to which the redesign was successful and to obtain a new starting point should an iterative continuation of the procedure be desired [1].

Many of these limitations have been eliminated by reformulating our inverse perturbation design program to accept vibration mode data provided by a NASTRAN finite element analysis. This paper presents a discussion of this new program and the results that have been obtained for three test structures, the last of which is an actual compressor blade designed for a jet engine. In its present form, the program accepts mesh point displacements and rotations in global coordinates from the NASTRAN output, accepts physical constants and a set of desired changes from the designer, and processes these data to obtain an ensemble of new element thicknesses so as to effect the desired changes with the minimum change to the structure. Because the solution is based on first-order perturbation theory, it is approximate, and a new NASTRAN analysis of the redesign can be used as a starting point for a second application of the procedure, etc. This program is limited only to structures that can be analyzed with triangular plate elements and for which changes in element thicknesses alone are desired.

We will begin with a discussion of the equations involved in this new program; this will be followed by a discussion of its component programs; and, finally, the numerical results will be presented and discussed.

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Mathematical Formulation

Let us present here the specific equations that relate to this program. First, let the design variable be the percentage change in thickness of the plate or shell, $\Delta h(x, y)/h(x, y)$, where $h(x, y)$ is the thickness of the structure, and x and y are spatial coordinates in its surface. This design variable is expanded as a series of functions, $\theta(x, y)$, called perturbation functions, which are functions of the coordinates, x and y. The coefficients of the series are determined, in part, by the set of changes that are desired, (Δ), which can be written as a row matrix. The Δ's are related to percentage changes in frequency, and to coefficients of admixture between mode shapes. (Admixture between mode shapes implies that the mode shapes of an altered structure may be expressed as linear combinations of the mode shapes of an unaltered structure). The final set of factors in the equation that determines the percentage changes in thickness is the inverse of what we may call a perturbation matrix, $[B]$. Thus, the primary equation of inverse perturbation is

$$\Delta h(x,y)/h(x,y) = (\Delta)[B]^{-1}(\theta(x,y))^T. \quad (1)$$

In equation (1), the necessary set of perturbation functions have been arranged as a row matrix, $(\theta(x, y))$ and the superscript T denotes the transposed matrix; i.e., a column matrix. In order to define the matrix $[B]$, it is necessary to specify the set of changes (Δ) and the set of perturbation functions (θ). (Note: the functional dependence of θ on x and y is implied throughout.) The perturbation functions are formed from products of parameters obtained from pairs of vibration modes, or from squares and products of parameters obtained from a single mode. Thus, we may identify perturbation functions by a double subscript θ_{nk}, or θ_{nn}, depending on whether the perturbation function is formed from two vibration mode shapes $\Phi_n(x, y)$ and $\Phi_k(x, y)$, or from $\Phi_n(x, y)$ alone. Similarly, the change parameters (Δ) may be characterized by double subscripts, Δ_{nk} being related to the amount of mode shape Φ_k that is found added to Φ_n as a result of the structural change, and Δ_{nn} being related to the change in frequency of the nth mode. The specification of any change parameter requires the inclusion of the corresponding perturbation function in equation (1). In this way, the number of degrees of freedom in changing the structure (i.e., the perturbation functions) matches the number of constraints (i.e., the change parameters).

With this in mind, let us define the elements of the perturbation matrix, b_{pq}. The double subscripts of the change variables, and the

perturbation functions, may be ordered in some arbitrary, but mutually consistent way. Let the double subscript nk correspond to the single index p, and some other subscript pair lm correspond to 'q'. Then, the element b_{pq} is defined by

$$b_{pq} = \int\int \theta_p(x, y)\theta_q(x, y)dxdy, \tag{2}$$

where the integration is carried out over the surface of the structure. In this particular program, however, the structure has been divided into discrete elements so that the integrations will extend only over the elements, and the contributions of the elements summed to obtain the coefficient b_{pq}.

In order to define the perturbation functions, we must first define the mode functions according to the form they take in the NASTRAN program. Each element of the shell-like structures considered in this program is a triangular platelet of uniform thickness which may bend out of its plane to any shape describable by a cubic polynomial (minus the x^2y term), and may deform and rotate homogeneously within its plane. In terms of a coordinate system local to each element, we may define the mode function, vectorially, as

$$\underline{\Phi}_n = \hat{i}\Phi_{nx} + \hat{j}\Phi_{ny} + \hat{k}\Phi_{nz}, \tag{3}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors in the x, y and z direction, with z being out of the plane of the element, and the origin located at one of the corners of the element. The components of $\underline{\Phi}_n$ are, in their polynomial forms:

$$\Phi_{nx} = a_{n1} + a_{n2}x + a_{n3}y, \tag{4}$$

$$\Phi_{ny} = b_{n1} + b_{n2}x + b_{n3}y, \text{ and} \tag{5}$$

$$\Phi_{nz} = c_{n1} + c_{n2}x + c_{n3}y + c_{n4}x^2 + c_{n5}xy + c_{n6}y^2 + c_{n7}x^3 + c_{n8}xy^2 + c_{n9}y^3. \tag{6}$$

The perturbation functions, which may now be defined, are most easily expressed as the sum of four terms: (1) an inertial term, θ_m, (2) a term related to bending strain, θ_{sb}; (3) a term related to membrane strain, θ_{sm}, and (4) a term relating to transverse sheer strain, θ_{ss}. Thus,

$$\theta_p = \theta_{mp} + \theta_{sbp} + \theta_{smp} + \theta_{ssp} \tag{7}$$

where the subscript p denotes the mode pair nk. The perturbation function, it should be noted, is a scalar. The first term of equation (7) is

$$\theta_{mp} = -(\omega_n \rho h/\omega_k M_k)\underline{\Phi}_n \cdot \underline{\Phi}_k, \tag{8}$$

where ω_n and ω_k are the natural frequencies, in radians, of modes n and k, ρ is volume density, and M_k is the model mass of the kth mode, defined by

$$M_k = \int\int \rho h \underline{\Phi}_k \cdot \underline{\Phi}_k \, dx \, dy. \tag{9}$$

To define the second and third terms, let us first define the matrix $\mathbf{G}_s$ as

$$\mathbf{G}_s = [Eh/\omega_n\omega_k M_k(1-\nu^2)]\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}, \tag{10}$$

where E is Young's modulus and ν is Poisson's ratio. We may now define $\mathbf{G}_b$ as

$$\mathbf{G}_b = \mathbf{G}_s h^2/4. \tag{11}$$

Now,

$$\theta_{sbp} = (\Phi''_{nz})\mathbf{G}_b(\Phi''_{kz})^T, \tag{12}$$

where

$$(\Phi''_{kz}) = (\Phi^{xx}_{kz}, \Phi^{yy}_{kz}, 2\Phi^{xy}_{kz}), \text{ and}$$
$$(\Phi''_{nz}) = (\Phi^{xx}_{nz}, \Phi^{yy}_{nz}, 2\Phi^{xy}_{nz}), \tag{13}$$

with the superscripts denoting partial differentiation with respect to the superscript variables. Next,

$$\theta_{smp} = (\Phi'_{nxy})\,\mathbf{G}_s(\Phi'_{kxy}), \tag{14}$$

where

$$(\Phi'_{nxy}) = (\Phi^x_{nx}, \Phi^y_{ny}, \Phi^x_{ny} + \Phi^y_{nx}), \text{ and}$$
$$(\Phi'_{kxy}) = (\Phi^x_{kx}, \Phi^y_{ky}, \Phi^x_{ky} + \Phi^y_{kx}). \tag{15}$$

Finally,

$$\theta_{ssp} = (\Phi'''_{nz})\,\mathbf{G}_{ss}(\Phi'''_{kz})^T \tag{16}$$

where

$$\mathbf{G}_{ss} = [Eh^5/\omega_n\omega_k M_k\, 12(1-\nu^2)(1-\nu)]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and} \tag{17}$$

$$(\Phi'''_{nz}) = [(\Phi^{xxx}_{nz} + \Phi^{yyx}_{nz}), (\Phi^{yyy}_{nz} + \Phi^{xxy}_{nz})], \text{ and}$$
$$(\Phi'''_{kz}) = [(\Phi^{xxx}_{kz} + \Phi^{yyx}_{kz}), (\Phi^{yyy}_{kz} + \Phi^{xxy}_{kz})].$$

Define the change parameters, (Δ), in terms of the percentage changes in frequency and coefficients of admixture:

$$\Delta_{nn} = [(1 + \Delta\omega_n/\omega_n)^2 - 1], \text{ and} \tag{19}$$

$$\Delta_{nk} = C_{nk}(\omega_n^2 - \omega_k^2)/\omega_n\omega_k, \tag{20}$$

where C_{nk} is the admixture coefficient. The new mode shape, $\underline{\Phi}'_n$, is expressable in terms of the old mode shapes by the series

$$\underline{\Phi}'_n = \underline{\Phi}_n + \sum C_{nk}\underline{\Phi}_k, \tag{21}$$

where the term for $n = k$ is omitted from the summation.

The format of the NASTRAN analysis requires some degree of approximation in implementing equation (1). First of all, whatever new design is generated, it should have plate elements of constant thickness, or else the analysis of the new structure becomes very difficult. This means that the values of the perturbation functions in equation (1) should be made constant for each element by taking the average value over the element. Another difficulty arises in the computation of the matrix coefficients, b_{pq}, via equation (2). If the integrations over each element are carried out as indicated, with products of the polynomials of equations (4–6), then the resulting integrands may be as high as twelfth order polynomials. To avoid the awkwardness of having to deal with such large polynomials, we decided to approximate the perturbation functions within elements by their average values therein. This made evaluation of the matrix coefficients, via equation (2), simpler and, simultaneously, provided constant changes in thickness for the plate elements.

Computer Program Descriptions

A number of component programs were required in order to implement the design procedure. As described in the following paragraphs, they included the primary routines for: (1) accepting the NASTRAN data, processing them and forming coefficient data files (POSTPR); (2) accepting the data files and computing the associated perturbation functions (NASTY); and (3) accepting the perturbation functions and performing the redesign (CHANGNAST), as well as various ancillary routines for performing checks (TESTNASTY, C2 and DISP2V), and subroutines for simplifying some of the programming.

POSTPR Program. This program accepts, as input, the displacements and rotations of the grid points of the finite elements in a NASTRAN vibration-mode analysis, in what is referred to as the global coordinate system. These displacements and rotations are converted to sets of coordinates that are local to each element. The displacements and rotations are then used, together with equations (4–6), to solve for the coefficients that determine the internal deformations of the elements; i.e., the a's, b's, and c's of equations (4–6). This is done for each NASTRAN vibration mode, and the resulting arrays of coefficients are stored in data files for use in subsequent programs.

NASTY Program. This program forms perturbation functions from equations (7–15), and takes their average value over each triangle. Two subroutines (POLVOL and TRIANG) were written to facilitate this computation. POLVOL finds the value of the integral

of the product of two polynomials over a triangular region in the x, y plane, while TRIANG, a subroutine to POLVOL, integrates any term of the two-dimensional polynomial over the triangular region. NASTY itself is a subroutine to the main inverse perturbation program (CHANGNAST).

CHANGNAST Program. This is the primary inverse perturbation design program, and it has been written so that it may use perturbation functions generated either by NASTY (and, therefore, NASTRAN) or by PERTNAST; an updated version of the original program that generates perturbation functions from experimental holographic data. CHANGNAST accepts the various material parameters (Young's modulus, Poisson's ratio, density, and the vibration mode frequencies), as well as the desired change parameters, i.e., the Δ_{nk} values. It thereupon computes the necessary perturbation functions, forms the perturbation matrix, and generates the new design via equation (1), which it can make available on punched cards. It also prints the modal masses, the perturbation matrix, the determinant of the perturbation functions that comprise the new design, and the root-mean-square change in the element thicknesses. Finally, via a subroutine called NASTDL, it computes the various perturbations that may be expected, even for the parameters that are unconstrained, by a formula that is more accurate than the perturbation formula used for the inversion process [1].

TESTNASTY Program. This program will print the values of the perturbation functions formed by NASTY, and compute the Rayleigh quotients (the ratio of maximum potential energy to maximum kinetic energy in the vibration cycle) and the check for orthogonality of the stiffness functions [1]. It will also compare NASTRAN computations of the variation modes with holographic data when the vibration mode is a scalar function. It does this by computing the mode-function value of the NASTRAN solution at the center of each section of the structure for which the holographic data have been digitized.

C2 Program. This program takes NASTRAN solutions for the vibration modes of an original and modified structure, and computes the admixture coefficients that best describe the new modes as a series of the old modes.

DISP2V Program. This program computes vectorial displacements, in one plane, of a plate or shell from data provided by two holographic interferograms of a vibration mode. (The third component is assumed to be zero). By selecting the centers of the sections of the interferograms to be digitized so that they correspond to the NASTRAN grid points, direct comparison can be made to the NASTRAN analysis.

Analytical and Experimental Investigations

NASTRAN Modeling of the Vibration Modes. The first computations performed were the NASTRAN analyses of both a uniform plate and a uniform shell segment. The plate was 15.24 cm long by 12.7 cm wide by 0.317 cm thick, and the shell segment was a 45 degrees arc of a cylindrical shell, 7.62 cm long, with an outer radius of 7.62 cm and a thickness of 0.317 cm. Schematic illustrations of the structures, as divided into triangular elements for the NASTRAN analyses, are presented in Fig. 1. Material parameters were taken to be: Young's modulus, E, 6.1×10^{11} dynes/cm^2; density, ρ, 2.7 grams/cm^3; and Poisson's ratio, ν, 0.31. (These were experimentally determined for an aluminum alloy available in stock). The boundary conditions for the clamped edges were imposed by requiring that the displacements and rotations of the grid points along the clamped edges by zero.

Real structures were built to check these models. The comparison of the mode frequencies of the structures, as obtained via NASTRAN and experimentally, are presented in Table 1 for the first three vibration modes.

The NASTRAN computations of the frequencies for the vibration modes of the plate are all lower than the experimental values by approximately 14 percent. Four percent of the difference is due to the thickness of the experimental plate (0.33 cm not 0.317), while the remaining 10 percent is quite probably due to a premature cutoff of

Table 1

Vibration Mode	Plate	
	NASTRAN	Experimental
f_1	105.6 Hz	120.6 Hz
f_2	293.3 Hz	333.8 Hz
f_3	642.2 Hz	730.6 Hz
Vibration Mode	Shell	
	NASTRAN	Experimental
f_1	898.7 Hz	806.0 Hz
f_2	1280.0 Hz	1380.0 Hz
f_3	4118.0 Hz	4080.0 Hz

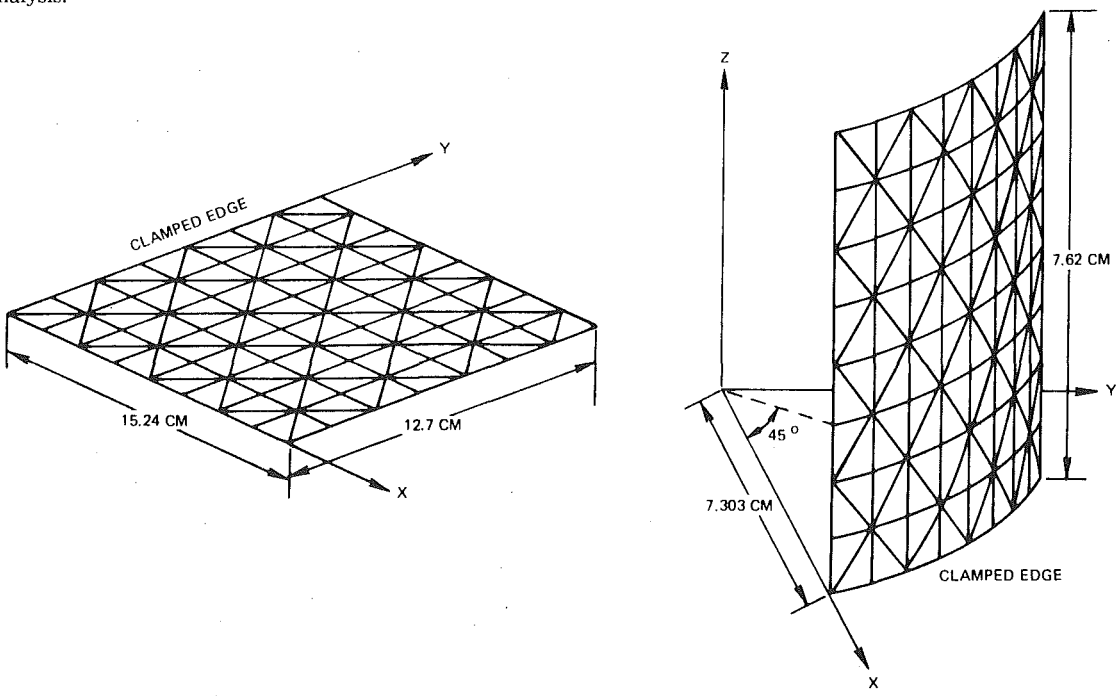

Fig. 1 Element break-up of a cantilevered plate and a cantilevered segment of a cylinder

the iterative solution for the eigenvector in the NASTRAN computations. The consistency suggests a systematic error, and this is supported by the Rayleigh quotient checks presented later.

The NASTRAN computations for the modal frequencies of the shell give results that lie both above and below the experimental values. It is suspected that the first mode frequency, which is in error by over 10 percent, may be the victim of a difficulty in adequately modeling the boundary condition where the shell is attached to the base.

System Checks. The mode shapes for both the plate and the shell were compared to holographic results. The errors found were quite small and within the accuracy of holographic measurement. In the case of the shell, the holograms were recorded with two illuminations so as to provide vectorial mode shapes.

The next computations involved the orthogonality checks and Rayleigh quotient checks. The former measure the degree to which the stiffness functions between modes are orthogonal, and the latter measure the ratio of maximum potential to maximum kinetic energy during the vibration cycle of a mode [1]. The orthogonality parameters should be zero and the Rayleigh quotients should be unity. These results are presented in Table 2.

Table 2

Mode Index (nk)	Orthogonality Parameter	
	Plate	Shell
1, 2	-.01141	-.000004
1, 3	-.00019	+.085832
2, 3	+.00309	.000000
Mode Index (nk)	Rayleigh Quotient	
	Plate	Shell
1, 1	1.0735	1.0282
2, 2	1.1047	1.0245
3, 3	1.1082	1.0852

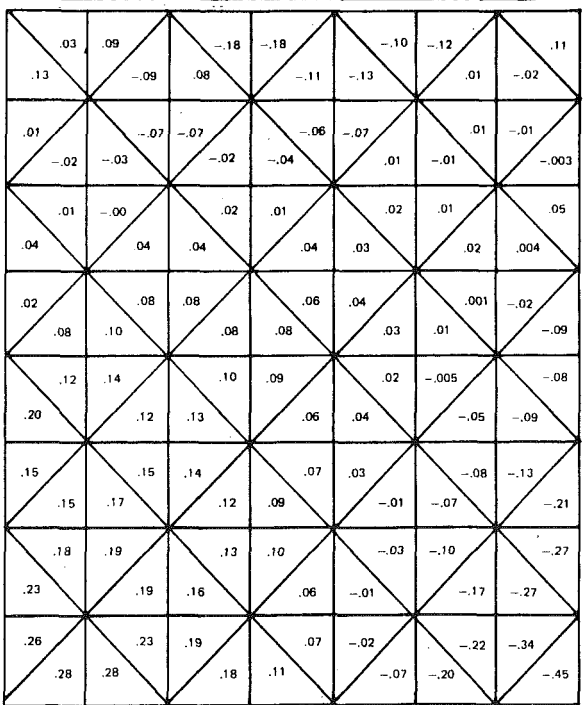

Fig. 2(*a*) New design for the flat plate. The numbers represent the fractional change in thickness of each element ($\Delta h/h$) and the clamped edge is at the top

Structural Redesign and Correlation. The next two tasks were to redesign the structures and evaluate the results via a second NASTRAN analysis. Both the cantilever plate and the cylindrical shell segment were redesigned subject to four constraints: (1) lowering of the first mode frequency by 5 percent ($\Delta\omega_1/\omega_1 = -.049$) (2) raising of the second mode frequency by 5 percent ($\Delta\omega_2/\omega_2 = +.051$), (3) an admixture coefficient of the first to the second mode of magnitude 0.125 ($C_{21} = \pm.125$), and (4) an admixture coefficient of the third to the second mode of magnitude 0.025 ($C_{23} = -.025$). (For the plate, the C_{21} parameter was positive, and for the shell it was negative. These sign conventions were necessary to generate the same shift in the node line of the second mode on both the plate and the shell, and they resulted only from a change in the sign conventions between the NASTRAN analyses of the plate and the shell.) The results provided structures which could be fabricated, and which had root-mean-square thickness changes of 12.7 percent for the plate and 14.5 percent for the shell.

The new designs, as arrays of the fractional changes in thickness of the triangular elements of the plate and shell, respectively, are presented in Figs. 2(*a*) and 2(*b*).

The designs of Figs. 2(*a*) and 2(*b*) were analyzed by NASTRAN to determine the new frequencies and mode shapes of these structures. The modes of the original structures were then used to form a series to approximate (to least-square-error) the shape of the new second mode, and, therefore, to give the resulting admixture coefficients C_{21} and C_{23}.

The final results are presented in Table 3 together with the results predicted by the more accurate perturbation calculation mentioned earlier in the description of the CHANGNAST program, and considered further in reference [1].

Pratt and Whitney Aircraft provided us with an opportunity to evaluate the inverse perturbation technique on a compressor blade, which they had analyzed using NASTRAN. A diagram of the mesh of blade elements used in the analysis is shown in Fig. 3, and a resonance diagram of the compressor, made up of a disk and a set of these blades, is presented in Fig. 4. The mode frequencies are plotted as a function of speed, along with the engine-order excitation curves. It was felt that the vibratory performance of the rig could be improved by increasing the frequency of the seventh mode, which, at the maximum rig speed, had the same value as the eighteenth engine order excitation. Thus, the design requirements were to increase the fre-

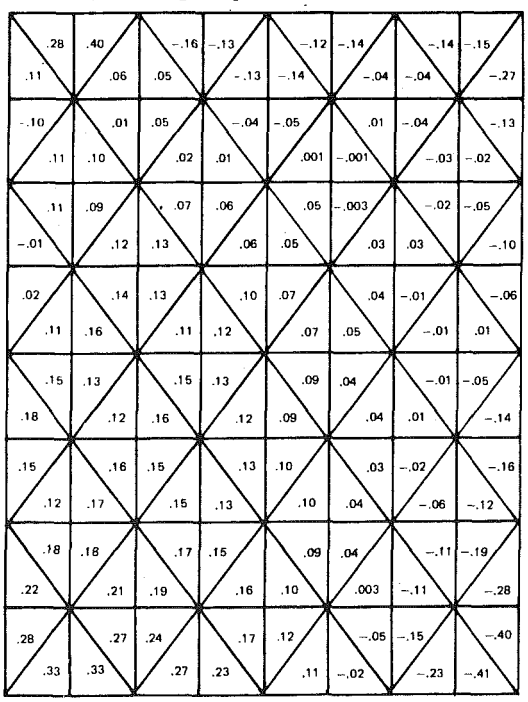

Fig. 2(*b*) New design for the 45 deg arc of a cylindrical shell

Table 3

Constraint	Design	Plate Results		Shell Results	
		NASTRAN	Perturbation	NASTRAN	Perturbation
Change in 1st Mode Frequency, $\Delta\omega_1/\omega_1$	-.049	-.027	-.037	-.042	-.041
Change in 2nd Mode Frequency, $\Delta\omega_2/\omega_2$	+.051	+.076	+.067	+.084	+.079
Admixture Coefficient 1st Mode to 2nd Mode C_{21}	Plate +.125	+.102	+.124		
	Shell -.125			-.112	-.118
Admixture Coefficient 3rd Mode to 2nd Mode C_{23}	-.025	-.021	-.026	-.027	-.028

quency of the seventh mode by 10 percent while containing the frequencies of the first six modes to be unchanged.

The first seven mode shapes for the blade, as provided by the Pratt and Whitney NASTRAN analysis, are presented in Fig. 5. The lines represents contours of constaint magnitude of vibration, and the resonant frequencies were: first mode-321 Hz, second mode-1,196 Hz, third mode-1259 Hz, fourth mode-2515 Hz, fifth mode-2639 Hz, sixth mode-3240 Hz, and seventh mode-3,813 Hz. No constraints were placed on the shape of the vibration modes, nor on the frequencies of the modes higher than the seventh.

Two passes were run on the compressor blade. That is, we used the original NASTRAN analysis of the blade and the CHANGES program to design a new blade (new thicknesses), and the NASTRAN analysis of the new blade, using the new thicknesses, was again run through the CHANGES program. On the first pass, as shown in Fig. 6, our design goal was a zero frequency change for the first six modes and a 10 percent increase in the seventh mode. The results of the NASTRAN analysis of this first redesign indicated that the greatest frequency change among the first six modes was 1.6 percent (first mode) while the seventh mode frequency was increased by 6.1 percent.

On the second pass, our design goals were to return the first six frequencies to their original values, and to increase the frequency of the seventh mode by the remaining 3.9 percent. The results of this design change provided a total frequency change of plus 8.7 percent of the seventh mode and no change, from the original frequencies, greater than 2 percent (shown in the third mode) for the first six modes.

The vibration mode shape of the seventh mode was not constrained and did change with each redesign, as can be noted by observing line number 3 in each of the three diagrams illustrated in Fig. 7. Figure 8 shows the polarity of the thickness changes for the blade elements; the blackened elements are those that were thinned by the design process, and the white elements are those that were thickened. Thus, it can be seen that there will be a change in the airfoil shape. To examine this effect, we looked at the airfoil shape at the tip and at 70 percent span which were chosen not because they are typical, but because they are the spans with the largest changes in thickness. To get a rough plot of how the airfoil changed, we straightened out the cord and made the elements of equal size. Thickness changes were symmetrically distributed between the two surfaces, and are plotted in Figs. 9 (tip) and 10 (70 percent span) using an exaggerated scale of 0.1 in. of thickness (vertical), to several in. in length (horizontal). The distribution of the centers of the elements (black dots) lie along two lines. Therefore, we have drawn a profile for each line. The upper row of centers corresponds to the upper airfoil shape, and the lower row of centers corresponds to the lower shape. At the tip, the airfoil has been thinned at the leading edge, and then thickened, by approximately 25 mils, farther to the left. At the 70 percent span point, the new shape is also irregular, but here the new blade is thicker where

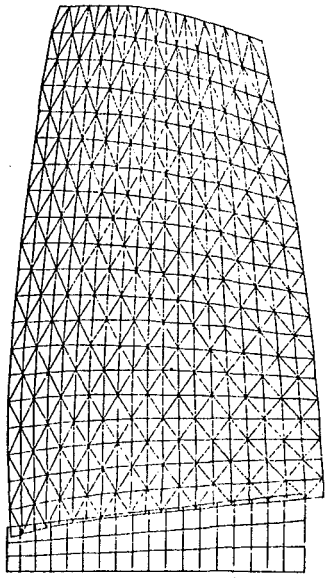

Fig. 3 Element break-up for a compressor blade

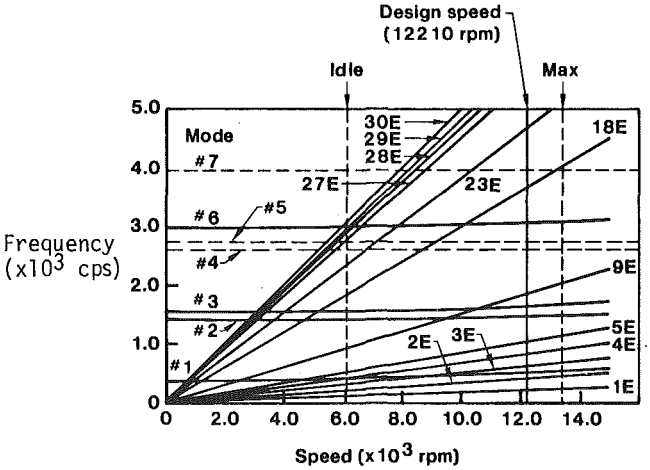

Fig. 4 Resonance diagram for a disk with a set of blades

the tip airfoil was thinned. It is our feeling that the individual variation in thickness is not as important as the overall envelope of thickness, and we are quite confident that these element thicknesses could be smoothed without changing the new mode properties.

Discussion

The technique of inverse perturbation has been shown to work with finite element vibration analyses as inputs. The results obtained with

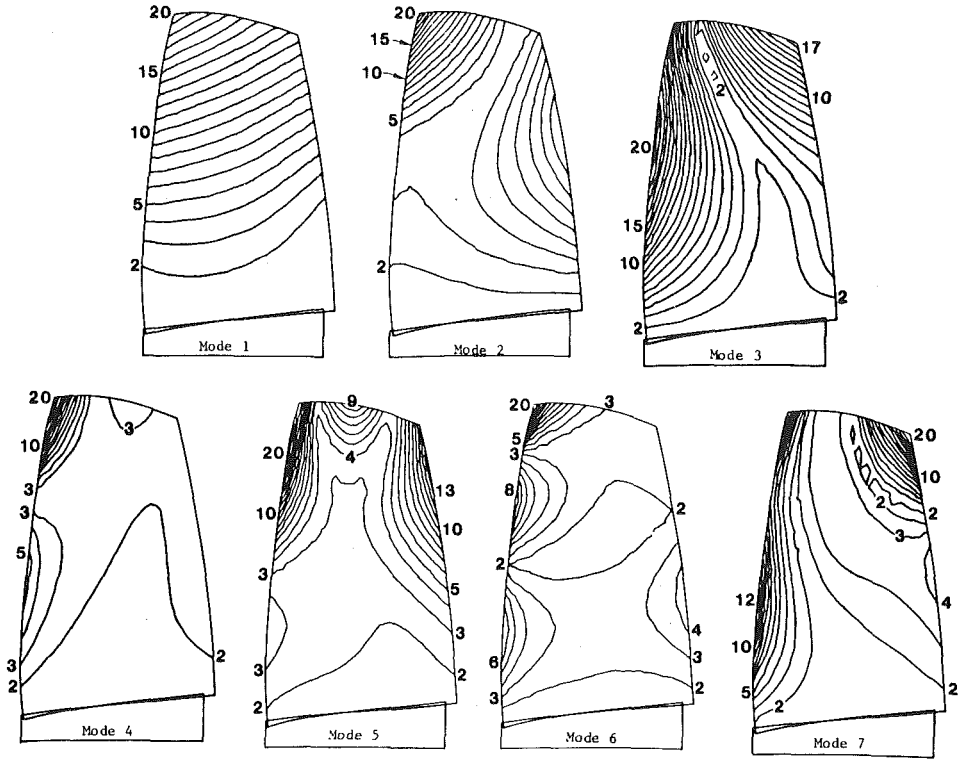

Fig. 5 The first seven modes of the blade modeled in Fig. 3. Lines are contours of constant vibration magnitude

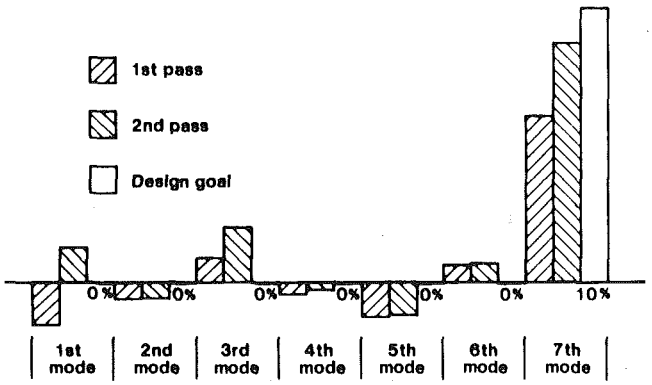

Fig. 6 Graph of the result of inverse perturbation design. The third bar in each section is the design goal, the first bar is the result of the first redesign, and the middle bar is the result of second redesign starting from the results of the first.

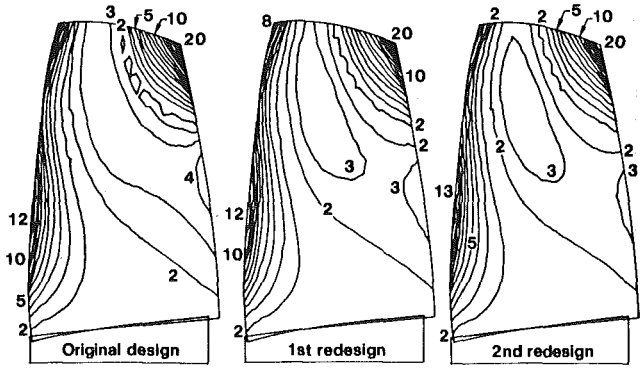

Fig. 7 Changes in the seventh vibration mode shape

the compressor blade illustrate a number of important features of the program. First, the program was executed only with the 800 elements that model the blade and not with the 54 elements that model the root. Consequently, the root was not changed in the redesigning of the blade, and it seems resonable that other critical regions of the blade could also be withheld from the process in the same way. It is also interesting to note that, as shown in Figs. 9 and 10, this program generates smaller changes in thickness where the blade is already thin. These areas make smaller contributions to the perturbation functions and, therefore, receive smaller changes. This fact makes it desirable to use the program iteratively when a large change in the modal frequencies or shapes is attempted. A large change, applied in one step, can generate negative thicknesses, whereas its application in several small changes would avoid this. If an area became thin as the result of the first step in the design, it would not be thinned so much in the successive steps because of the reformulation of the perturbation functions.

With regard to Fig. 6, the question may be asked as to whether a third iteration could be expected to achieve a design closer to the goal. This seems doubtful for the following reason. The resulting changes in the first six modal frequencies indicate an error level in the computations. This error level appears constant between the two itera-

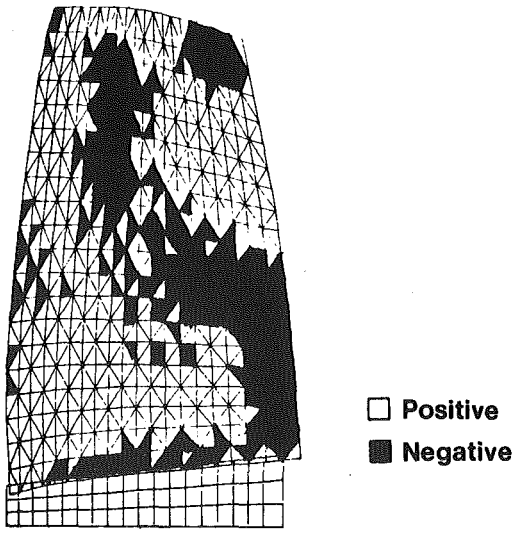

Fig. 8 Polarity of design changes of element thicknesses

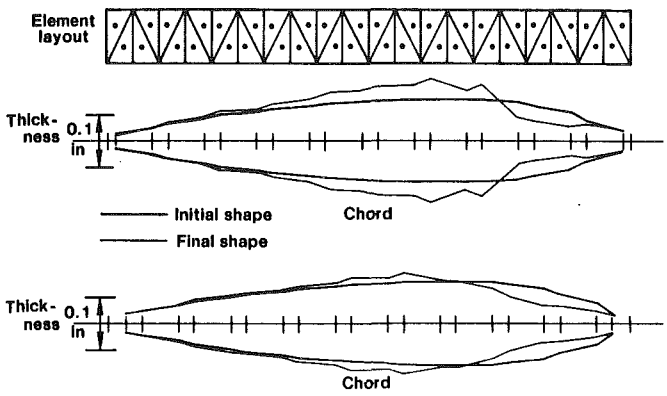

Fig. 9 Simplified representation of change in airfoil at tip

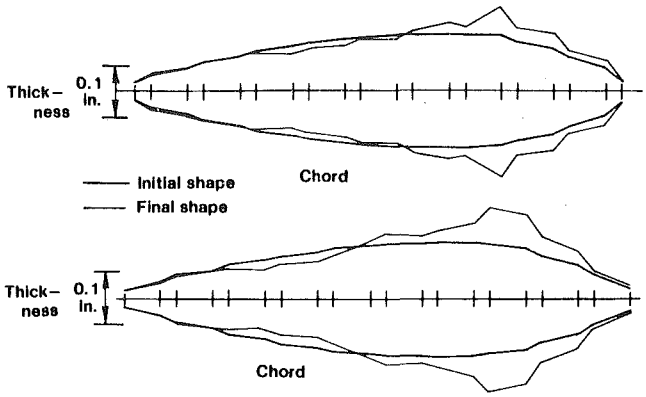

Fig. 10 Simplified representation of change in airfoil at 70 percent of span

tions, and the remaining increase desired for the seventh mode after the second iteration is of the same magnitude as the error level. Thus, a third iteration may have only a random chance of moving toward the design goal. The source of this error level has not been established, but there is good likelihood that it is inherent in the NASTRAN analyses, which are approximate.

Finally, it is of interest to know how this redesign affected the entire weight of the blade. This may be computed by summing the change in masses of all the elements and dividing by the total mass. The result is that the blade increased in weight by 4.25 percent. The product of element mass times radius from the center of the disk, when summed, gives a parameter, which multipled by the square of rotation speed, gives the centrifugal loading of the blade. Similarly, the sum of the product of element radius times change in element mass normalized by the total mass-radius product gives the change in centrifugal loading. This is an increase of 4.48 percent.

In conclusion, it must be kept in mind that this program does not, as yet, allow for design of such usual parameters as cord length, aspect ratio, twist, camber, etc. It is hoped that further work will permit modification of this design routine to make it more general.

References

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