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## Experimental testing of a large 9-story structure equipped with multiple nonlinear energy sinks subjected to an impulsive loading

N. E. Wierschem,<sup>1,a</sup> J. Luo,<sup>2,a</sup> S. Hubbard,<sup>3,b</sup> L. A. Fahnestock,<sup>4,a</sup>  
B. F. Spencer Jr.,<sup>5,a</sup> A. F. Vakakis,<sup>6,c</sup> and L. A. Bergman<sup>7,b</sup>

<sup>1</sup> Graduate Research Assistant, E-mail: [nwiersc2@illinois.edu](mailto:nwiersc2@illinois.edu)

<sup>2</sup> Graduate Research Assistant, E-mail: [jjluo2@illinois.edu](mailto:jjluo2@illinois.edu)

<sup>3</sup> Graduate Research Assistant, E-mail: [shubbar2@illinois.edu](mailto:shubbar2@illinois.edu)

<sup>4</sup> Assistant Professor, E-mail: [fhnstck@illinois.edu](mailto:fhnstck@illinois.edu)

<sup>5</sup> Nathan M. and Anne M. Newmark Endowed Chair, E-mail: [bfs@illinois.edu](mailto:bfs@illinois.edu)

<sup>6</sup> W. Grafton and Lillian B. Wilkins Professor, E-mail: [avakakis@illinois.edu](mailto:avakakis@illinois.edu)

<sup>7</sup> Professor, E-mail: [lbergman@illinois.edu](mailto:lbergman@illinois.edu)

<sup>a</sup> Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, 205 North Mathews Avenue, Urbana, IL 61801

<sup>b</sup> Department of Aerospace Engineering, University of Illinois at Urbana-Champaign, 104 South Wright Street, Urbana, IL 61801

<sup>c</sup> Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, 1206 West Green Street, Urbana, IL 61801

### ABSTRACT

Building structures can be critically affected by impulsive loads such as blasts, collisions, gusts, and pulse dominated earthquakes. The addition of nonlinear energy sinks (NESs) in buildings has been proposed as a means to rapidly and passively dissipate the energy in a system exposed to this type of loading. This rapid dissipation occurs because the essential nonlinearity of the NES allows it to resonate with any mode of the structure and engage in targeted energy transfer, the nearly one-way transfer of energy to the NES where it is locally dissipated. Additionally, the NES couples the modes of the structure and facilitates the transfer of energy from the lower modes of the structure to the higher modes, where it can be dissipated at a reduced time scale. In this study the experimental performance of a system of multiple NESs in a large 9-story test structure is discussed. Two different types of NESs are used, each of which employ a different type of restoring force; one type of NES utilizes a smooth restoring force that is roughly cubic, while the other utilizes a linear restoring force coupled with one-sided vibro-impacts. To load this system, an impulse-like ground motion is applied via a large shake table. The results of this study show that the system of NESs greatly improves the performance of the structure across a wide range of impulse amplitudes by reducing and very rapidly attenuating its response.

## INTRODUCTION

Impulsive events such as blasts, pulse dominated earthquakes, and wind gusts introduce large amounts of energy into building structures which can result in large structural demands. In order to maintain the integrity of the structure, this energy must be dissipated or otherwise dealt with. Passive mass dampers, such as tuned mass dampers have long been used redirect energy within a structure; however, these devices have proven to be inefficient during impulsive loadings due to their delayed response. In recent years nonlinear energy sinks (NESs) have been studied as an alternative supplemental damping device. In its simplest form these devices consists of a mass connected to the underlying system with an essentially nonlinear spring element. This essential nonlinearity means the response of the NES is not dominated by a single natural frequency and the device can response across a broad frequency range. As a result of this, the NES can vibrate with any mode of an underlying structure and participate in Targeted Energy Transfer (TET) (Kerschen et al. 2008), the transfer of energy from lower modes of vibration to higher ones where it is can be dissipated faster. Furthermore, energy transferred to the NES itself can be dissipated by damping in the NES.

Previous researchers have experimentally studied structures equipped with NESs; however, there have not been any large-scale experiments examining the performance of NESs and most previous work has utilized tabletop sized or small-scale specimens (Gourdon et al. 2007; McFarland et al. 2005; Nucera et al. 2008; D. D. Quinn et al. 2012; Wierschem, D.D. Quinn, et al. 2012). Additionally, little experimental work has been done on single-sided vibro-impact (SSVI) NESs (Al-Shudeifat et al. accepted for publication) or systems of multiple NESs (Vaurigaud et al. 2011), both of which are examined in this work.

In this study a large 9 story structure with a system of NESs subjected to an impulse-like ground motion is examined. The system of NESs used in this structure consists of combination of Type I NES and SSVI NESs. These two types of NESs each employ a different type of essentially nonlinear restoring force; the Type I NES utilizes a smooth restoring force that is roughly cubic, while the SSVI utilizes a linear restoring force coupled with one-sided vibro-impacts. The results of shake table tests with this system are evaluated through comparisons with tests of the system when the NESs are disabled.

## TEST STRUCTURE

The test structure used in this experimental study was developed specifically for the large scale validation of a system of NESs. This test structure, which is shown in Figure 1, is 5.13 m tall and has a mass of approximately 11,000 kg. This structure is currently the world's largest test bed for NES technology and substantially larger in both height and mass than the previous largest test bed (Wierschem, Luo, et al. 2012). The structure consists of nine 2.74 m by 1.22 m steel plate floors. The bottom 7

floors are solid 3.81 cm thick plate and the top two floors are 4.44 cm thick with cutouts in them to accommodate the NESs within the floors.

An identical column layout is used for each floor of the test structure. As shown in Figure 1, this layout consists of 8 columns that are arranged such that there is one bay in the short direction of the plate and 3 bays in the long direction. The columns on the bottom floor have a rectangular 19.05 cm by 1.43 cm cross-section and are 66.04 cm tall, while the columns used on every other floor have a 13.97 cm by 1.43 cm cross-section and are 50.80 cm tall. As a result of this oblong shape, the bending stiffness of the columns in one direction is much larger than the other. With this in mind the columns are orientated so that their weak direction is along the short length of the plates. Additionally, the columns are manufactured using high strength steel to allow the building to elastically accommodate relatively large deformations.



**Figure 1. 9 story test structure**

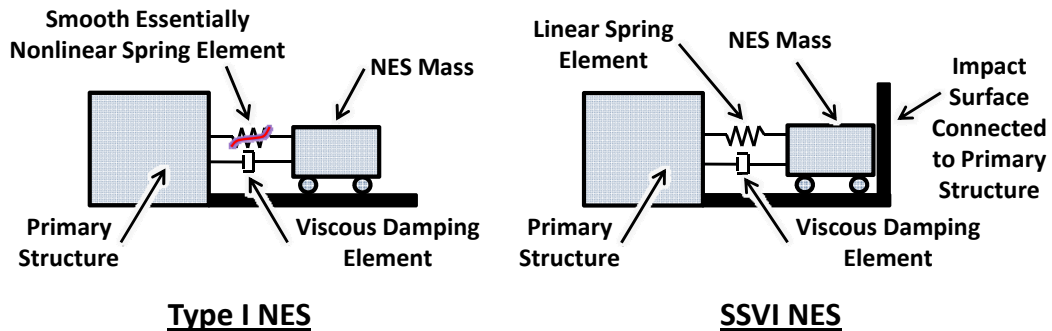
Covering one of the long sides of the test structure is cladding made out of 0.13 cm thick steel plates. This cladding is connected to the structure via bolts into the side of the floor slab at each floor level. The purpose of this cladding is to serve as a pressure face for another set of tests with this structure. The relatively small mass of the cladding and its flexibility in the weak translational direction of the structure mean that its effect on the weak directional translational natural frequencies is minor. However, larger stiffness to torsion and strong direction motion mean that this cladding serves to increase the torsional and strong direction translational natural frequencies compared to the structure without cladding.

To determine the as-constructed modal parameters of the structure, instrumented hammer testing and sine sweeps, via shake table excitation, were performed. For this testing the NESs were locked and the structure can be considered to be linear. As a result of this testing, the natural frequencies of the structure in the

weak direction were determined to be 1.74, 5.37, 9.10, 12.72, 15.96, 18.95, 21.63, 23.92, 25.48 Hz. Additionally, the torsional natural frequencies of the structure less than 30 Hz were determined to be 5.81, 18.16, and 29.36 Hz.

## NONLINEAR ENERGY SINKS

The system of NESs studied in this work consists of a combination of Type-I NESs and single-sided vibro-impact NESs (SSVI NESs). Phenomenological models of these NESs are shown in Figure 2. As this figure shows, the idealized representations of both types of NESs are quite simple and consist largely of masses, springs, and dampers. The Type I NES is composed of a mass connected to a primary structure with a viscous damping element and a smooth (no discontinuities in the restoring force) essentially nonlinear spring element. The smooth essential nonlinearity of this spring element means that the response of the NES is not dominated by a single natural frequency and the device can resonate with and participate in the transfer of energy with any mode of the primary structure (Sapsis et al. 2012). Additionally, as shown in Figure 2, the SSVI NES consists of a mass attached to a primary structure with a viscous damping element and a linear spring element; for this type of NES the relative displacement of the mass is limited on one side due to an impact surface that is connected to the primary structure. Due to the discontinuity in restoring force, these impacts are broadband event and because of them energy is scatter throughout the structure, including to its higher modes (Al-Shudeifat et al. accepted for publication). The transfer of energy to higher modes that occurs as a result of both types of NESs is beneficial because at these higher frequencies the energy can be naturally dissipated faster due to the reduced time scale. Furthermore, the NESs also help quickly eliminate the response of the test structure due to the energy dissipated by their relative motion.



**Figure 2: Phenomenological models of Type I and SSVI NESs**

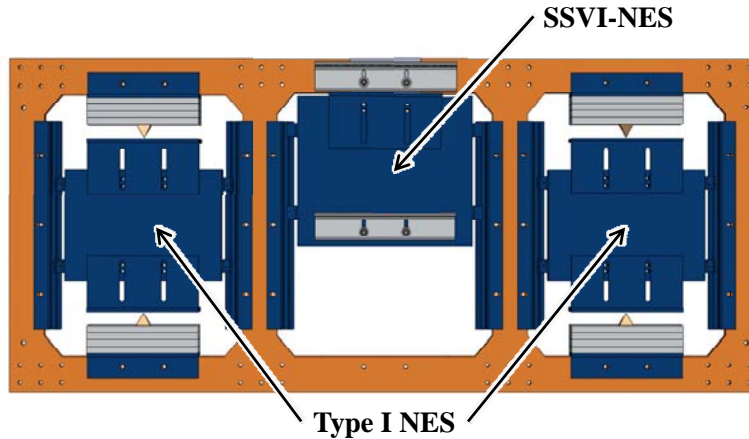
As mentioned previously, the NESs in the test structure are positioned within cutouts built into the floor plates of the 8<sup>th</sup> and 9<sup>th</sup> floors. These cutouts and NES masses, shown in Figure 3, are the same for both floors. Additionally, both these floors contain three separate NESs. These NESs all consist of solid steel masses that have Thomson SSUPB012 pillow blocks containing linear bearings mounted on their

sides. These linear bearing allow the masses to move on sets of 1.91 cm round rail that are attached to the floor plate inside each cutout. The rails are positioned such that the masses move uniaxially in the short direction of the floor plate. The two NESs on each side of these floors are the Type I NESs. These NESs are coupled to the structure using specially shaped elastomeric bumpers that are mounted on each side of the NES and are put into compression when the NES moves. The special shape of these bumpers provides an essentially nonlinear, and nearly cubic, restoring force to these NES masses. Each Type I NES on the 8<sup>th</sup> floor has the same set of bumpers, but the Type I NESs on the 9<sup>th</sup> floor have a different set of bumpers. In the center of the 8<sup>th</sup> and 9<sup>th</sup> floors are SSVI NESs. Each SSVI-NES is coupled to the floor plate with a set of elastic cords. These elastic cords are positioned such that, at rest, the NES mass is in contact with one side of the floor plate. While unintended, there is a small amount of pretension in the elastic cords; the result of this is that the NES mass is secured against the side of the floor plate under very low loading conditions. When the motion of the floor is large enough to exceed this small pretensioning force the SSVI-NES will move relative to the floor. When this happens the elastic cords connecting the NES to the floor plate will stretch and produce a restoring force that results in the NES mass being forced to collide with the floor with significant velocity. The result of this steel on steel collision is energy dissipation, the amount of which related to the coefficient of restitution, and high frequency scattering of the remaining amount of energy. Each of the NESs in this system has its own locking mechanism; consequently, tests can be performed with all of the NESs free to move, all the NESs locked, or part of the NESs locked and unlocked.

Using a basis ground motion, which is discussed in a subsequent section, as the loading on the structure for numerical analyses, the parameters of these NESs were determined using an optimization analysis. In this optimization analysis the goal of the objective function was to maximum a measure of the apparent damping in the 1<sup>st</sup> mode known as the effective damping (Sapsis et al. 2012). The stiffness parameters for the NESs that resulted from this analysis, as well as the mass percentage of each NES, can be found in Table 1. These stiffness parameters were used to design the elastomeric bumpers and elastic cords for the NESs.

	Mass (% of Structure Total Mass)	Design Stiffness Coefficient (N/m)	Design Stiffness Exponent
<b>8th Floor Type I NES</b>	1.5	$7.56 \times 10^8$	3
<b>9th Floor Type I NES</b>	1.5	$1.07 \times 10^8$	3
<b>8th Floor SSVI NES</b>	3.5	14546	1
<b>9th Floor SSVI NES</b>	3.5	12219	1

**Table 1. Design values of mass and stiffness for NESs**

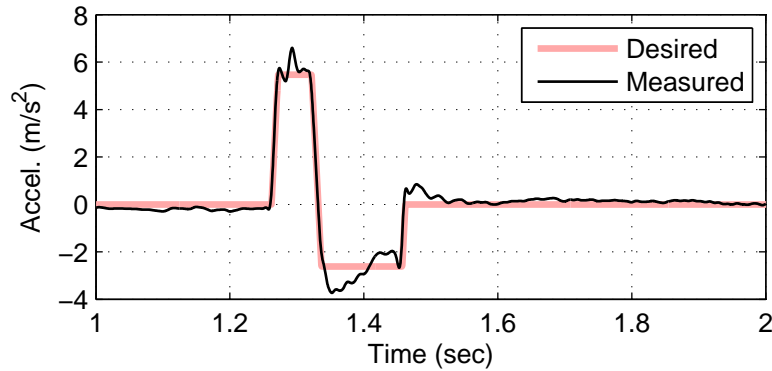


**Figure 3. 8<sup>th</sup> and 9<sup>th</sup> Floors**

### **SHAKE TABLE GROUND MOTION**

Testing of the structure outlined in the previous section was performed using the Triaxial Earthquake and Shock Simulator (TESS) shake table at the US Army Corps of Engineering Construction Engineering Research Laboratory in Champaign, IL (U.S. Army Engineer Research and Development Center 2008). This large 3.66 m by 3.66 m shake table can support payloads in excess of 50000 kg. Additionally, due to its design with shock loading in mind, TESS has large force capabilities and frequency range. Before performing the primary testing with this shake table, a routine developed by the manufacture of the shake table control system designed to improve the tracking of the shake table was performed. In this routine measurements of the shake table response were taken when individual sine sweeps were performed in all six degree of freedom. From these measurements, the routine developed a compensation matrix that was automatically applied to a desired ground motion when inputted into the system.

As the purpose of this study is to investigate the effectiveness of the system of NESs at mitigating the effects of impulsive loads on the test structure, a suitable ground motion for this task was required. To develop this ground motion a numerical model of the structure, updated based on the identification discussed in a previous section, was used. With this model, the response of the structure when subjected to a uniform impulsive load in the weak direction of the structure was simulated. From this response and further simulations a ground motion was developed to match the impulsive load response in both amplitude and modal energy distribution. The resulting basis ground motion used as an input for the experiments in this study can be seen in Figure 4. In this figure the desired and an example of the achieved ground acceleration from the basis shake table motion is shown. The effectiveness of the compensation routine described above can be seen in this figure as the desired and achieved motions match up reasonable well. To investigate the response of the structure at different load levels, this basis desired ground motion can be scaled accordingly.



**Figure 4. Basis ground motion**

## INSTRUMENTATION

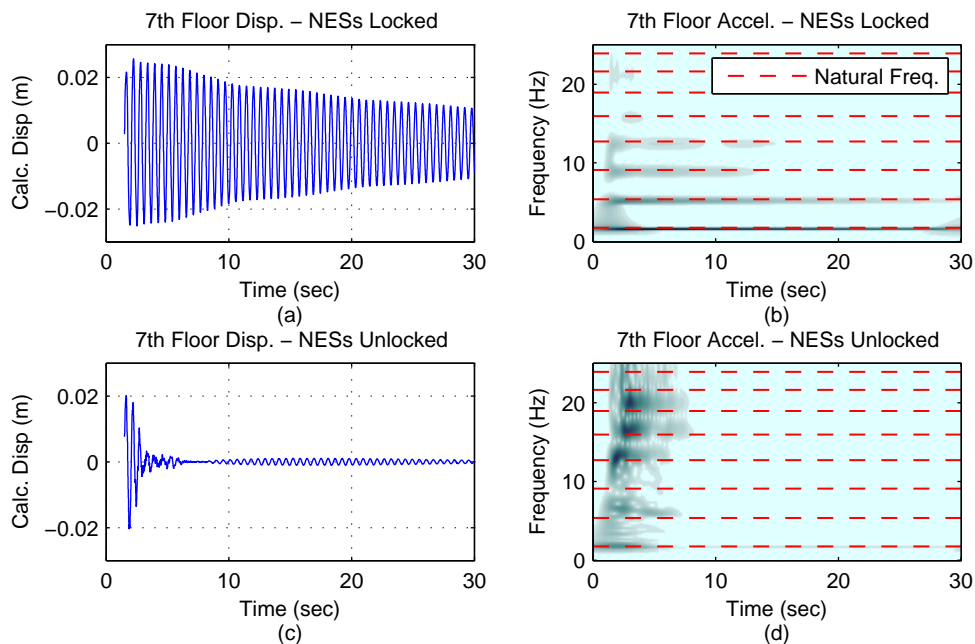
In order to measure the response of the structure due the ground motion, the structure and the NESs were instrumented with accelerometers. Accelerometers orientated in the direction of the table motion were placed on each NES and two were placed at each floor. With the two acceleration measurements on each floor and at different locations on the floor, the torsional response of each floor can be calculated. Moreover, with this information and the mode shapes of the structure, this acceleration data is sufficient to calculate the translational and torsional response of the structure in modal coordinates. A multitude of accelerometers models were used for this testing, include PCB models 353B33, 353B01, 353B03, 353A, and 3701G3FA3G as well as Endevco models 7290A and 7490E. To best capture the response of the structure, the higher acceleration capacity models were positioned where the highest accelerations where anticipated, which include the SSVI-NESs and the floors with NESs.

In addition to acceleration measurements, the strain in the 1<sup>st</sup> floor columns was also measured. The strain gages used for this measurement are the YEFLA-5-5LT gages manufactured by the Tokyo Sokki Kenkyujo Co. These unidirectional strain gages were placed approximately 7.6 cm from the end of the column and at its mid-width where the surface of the columns had been prepared. Multiple strain gages positioned at different locations allow the strain due to both translational and torsional motion to be measured. With these strain measurements the real structural demand on the 1<sup>st</sup> floor, which in many cases is the critical point in a structure, can be measured and compared.

## EXPERIMENTAL RESULTS

Using the TESS shake table, the test structure was subjected to the ground motion shown in Figure 4. To evaluate the effectiveness of the system of NESs, tests using this ground motion were performed with the NESs both unlocked (free to move) and locked. The calculated displacement response of the 7<sup>th</sup> floor relative to

the shake table is shown in Figure 5a and c. The displacement records shown in this figure were calculated from the acceleration response using a combination of numerical integration and low and high pass filtering to suppress noise in the data and prevent unrealistic drift in the signal (Boore and Bommer 2005). Figure 5a and c shows that when the NESs are unlocked, the response of the structure is quickly eliminated, with the unlocked response diminished to small fraction of the response when the NESs are locked after only a few second. Careful inspection of the unlocked response shows that after a couple cycles and the response of the structure is nearly completely diminished, the higher frequency content of the response is more prevalent. This phenomenon can also be seen in the wavelet spectrum of the acceleration response of the 7<sup>th</sup> floor, which is shown in Figure 5b and d. The wavelet spectra is an advantageous way of examining these signal as the wavelet spectra can show how the frequency content of a signal varies with time (Labat 2005). These figures show that, in the case when the NESs are locked, the acceleration response is dominated by the first mode and contains little additional higher mode behavior; however, when the NESs are unlocked the response contains some quickly eliminated 1<sup>st</sup> mode behavior, but is dominated by high mode behavior

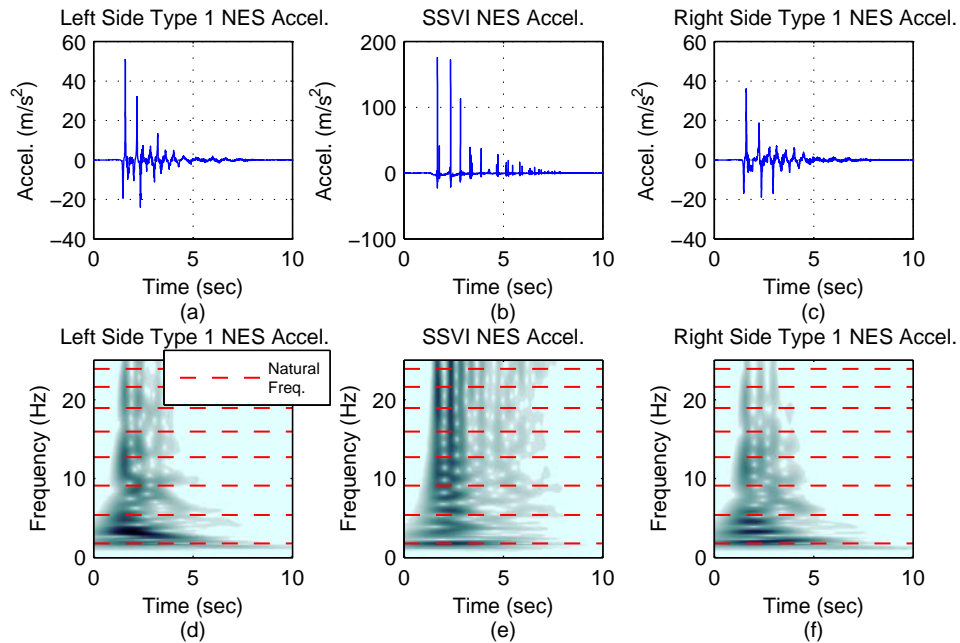


**Figure 5. Response of the 7<sup>th</sup> floor of the test structure a) Locked case, calculated displacement, b) Locked case, acceleration wavelet spectra, c) Unlocked case, calculated displacement, and d) Unlocked case, acceleration wavelet spectra**

Figure 6 presents the time history and wavelet spectra of the acceleration response of the 9<sup>th</sup> floor NESs when the structure is subject to the basis ground motion. This figure shows that the NESs experience a high level of acceleration during the test. This is particularly true for the SSVI NES, which is shown to experience large asymmetrical accelerations due to the impacts in one direction only. Examining the wavelet spectra reveals the broadband nature of the NESs which

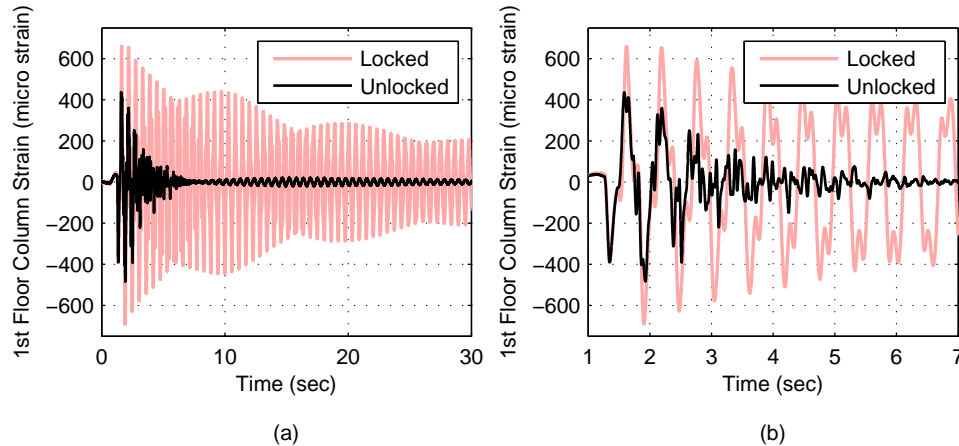


allows them to interact with multiple widely spaced modes of the structure. In the case of the SSVI NES, the broad frequency response of the NES occurs in bands that correspond to the impacts seen in the acceleration time history. For the Type I NESs, the broad frequency response is more continuous due to their smooth nonlinearity. Both the high acceleration and broadband behavior of the NESs only last for a few seconds in this test; after this point the response of the structure has been diminished enough that the NESs are no longer excited.



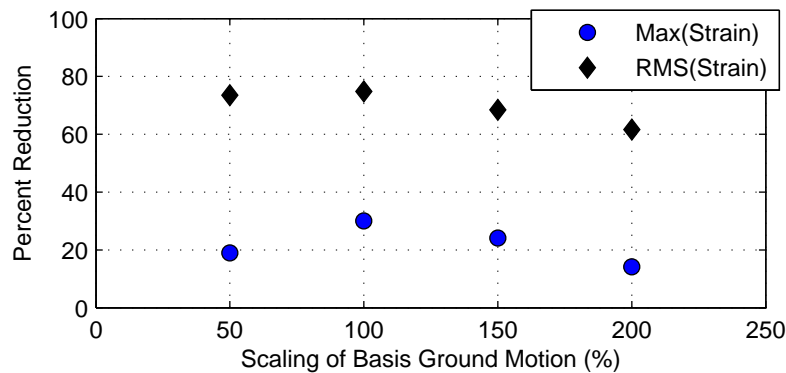
**Figure 6. Acceleration response of the 9<sup>th</sup> floor NESs (a) Left side Type I NES time history, (b) SSVI NES time history, (c) Right side Type I NES time history, (d) Left side Type I NES wavelet spectra, (e) SSVI NES wavelet spectra, and (f) Right side Type I NES wavelet spectra**

Examining the acceleration and displacement time histories can give meaningful insight into the response of the structure; however, these measures do not directly correspond with the structural demand on the system. Unlike other response measures, examining the strain in the 1<sup>st</sup> floor columns does give insight into the demands on the structure due to the proportionality of this strain with the column moment demand. Accordingly, the measured strain of one of the interior 1<sup>st</sup> floor columns during tests with the NESs locked and unlocked is shown in Figure 7. As shown in this figure, like the displacement and acceleration response, when the NESs are unlocked the strain in the 1<sup>st</sup> floor column is rapidly reduced compared to the case with the NESs locked. Furthermore, this figure also shows the transition with the NESs unlocked of the structure's response from 1<sup>st</sup> mode dominated to predominately higher mode behavior. Perhaps most importantly, Figure 7 also shows that with the NESs unlocked a substantial reduction in maximum strain occurs.



**Figure 7. 1<sup>st</sup> floor column strain (a) Full view (b) Zoomed view**

Figure 5 thru Figure 7 show the capability of the system of NESs to reduce and quickly eliminate the response of structure at the particular load level provide by the basis ground motion. In order to show the NESs are robust to changes in the amplitude level, additional tests were performed across a range of load levels. These different load levels correspond to a scaling of the basis ground motion, shown in Figure 4, and vary from a 50% to 200% scaling. Using the strain response to these ground motions, two performance measures were calculated comparing the case when the NESs are unlocked to case when the NESs are locked. The first measure is the reduction in peak 1<sup>st</sup> floor column strain; the second is the reduction in the RMS of the strain time history calculated using thirty seconds of data. The resulting measures over the range of ground motions tested are shown in Figure 8. As shown by this figure, a large reduction in peak and RMS strain is observed over the entire range of ground motions. Additionally, this figure shows that the maximum reduction in both these measures occurs at the 100% ground motion level, which is the load level the system was designed at. At this level, with the NESs unlocked, the peak and RMS 1<sup>st</sup> floor column strains are reduced by 30.0% and 74.8%, respectively.



**Figure 8. Percent reduction in maximum and RMS 1<sup>st</sup> floor column strain across a range of scaled base motion for NESs unlocked case compared to case with NESs locked**

## CONCLUSION

This paper presents an experimental study of a large scale 9 story structure equipped with a system of multiple passive nonlinear energy sinks (NESs) that is subjected to a ground motion designed to simulate the effects of an impulsive load on the structure. The system of nonlinear energy sinks integrated into this structure includes four Type I – NESs, which utilize a smooth essential nonlinearity provided by specially shaped elastomeric bumpers. Also included in the system of NESs are two single-sided vibro-impact NESs, which produce an essential nonlinearity through steel to steel impacts. Using comparisons with tests when the NESs are disabled, the results of this study show that this system of NESs is capable of quickly eliminating the response of the structure. By examining the response of the NESs and the wavelet spectra of the structure's response, it was shown that this quick response elimination corresponds with the large broad-band response of the NESs and the transfer of energy to the higher modes of the structure. Additionally, the mitigation of the base structure's response by these passive devices includes a significant reduction in the peak demand on the structure, as measured by the strain in the 1st floor columns. The results from tests at different scaling factors of the ground motion demonstrated that the system of NESs is capable of reducing and quickly eliminating the response of the structure across a wide range of load levels.

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