ESTIMATION OF VARIANCE OF TIME TO RECRUITMENT FOR A TWO GRADE MANPOWER SYSTEM WITH TWO SOURCES OF DEPLETION AND TWO TYPES OF POLICY DECISIONS WHEN THE BREAKDOWN THRESHOLD HAS THREE COMPONENTS

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Abstract: A marketing organization consisting of two grades with policy and transfer decisions forming two different sources of depletion is considered. The policy decisions are classified into two different types according to the intensity of attrition. In this paper the problem of time to recruitment for this organization is analysed when the breakdown threshold, a level of maximum allowable manpower depletion, has three components namely normal threshold of depletion of manpower, threshold of frequent breaks of existing workers and threshold of backup or reservation of manpower sources. A mathematical model is constructed and using a univariate policy of recruitment based on shock model approach, analytical results are obtained to estimate the variance of the time to recruitment by considering different forms of the breakdown threshold for the cumulative loss of manpower in the organization. The results are numerically illustrated by assuming specific distributions and relevant findings are presented.

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Key Words: two grade manpower system, two sources of depletion of manpower, two types of policy decisions, breakdown threshold of depletion with three components, shock model approach, univariate policy of recruitment and performance measures

1. Introduction

In any organization depletion of manpower is quite common when a policy decision is announced. Two sources of depletion of manpower are considered here. Decisions on pay, perquisites and work target form the first source and transfer of personal to sister organizations form the second source of depletion. As immediate recruitment after depletion of each personal is not advisable due to cost and time consumption, recruitment is postponed to a point of time beyond which normal activities cannot be continued due to shortage of manpower. This level of allowable manpower depletion is called breakdown threshold. Elangovan et.al [6] have initiated the study on recruitment problem for a single grade manpower system with two sources of depletion and obtained the variance of time to recruitment using univariate CUM policy of recruitment when the loss of man power in the organization due to the two sources of depletion, inter-policy decision times, inter-transfer decision times and threshold for the loss of man power in the organization, each forming a sequence of independent and identically exponential random variables with different means. Sathiya and Srinivasan [8] have studied the work of Elangovan when the loss of manpower follow poisson (Geometric) distribution. Later Suganya and Srinivasan [9] have extended the work of sathiya and Srinivasan for two grade manpower system. Recently, Dhivya and Srinivasan [2],[3],[4] and [5] have extended the work of Elangovan et.al for a two grade manpower system under different conditions on the inter-policy decisions, inter-transfer decisions and thresholds for the cumulative loss of manpower in each grade. Arivazhagan et.al [1] have determined the mean time to recruitment for a single manpower system with policy decisions forming the only one source of depletion when the threshold for the cumulative loss of manpower has three components. In all the above cited works, it is assumed that all policy decisions have same attrition rate. This assumption on policy decisions is not realistic since policy decisions can have high or low intensity of attrition. This aspect of policy decisions having high or low intensity of attrition is taken into account here. The objective of the present paper is to extend the work of Arivazhagan et.al [1] for a two grade manpower system with two sources of depletion when the policy decisions are classified into two

types according to the intensity of attrition.

2. Model Description

Consider an organization with two grades taking policy and transfer decisions at random epochs in $(0, \infty)$. The policy decisions are classified into two types depending upon their intensity of attrition. Let p be proportion of policy decisions having high attrition rate μ_h and $(1-p)$ be the proportion of policy decisions having low attrition rate μ_l. The most suitable distribution of inter-policy decision times is hyper exponential with distribution $F(.)$, probability density function $f(.)$. Let the inter-transfer decision times be independent and identically distributed exponential random variables with distribution $W(.)$, probability density function $w(.)$ and mean $\frac{1}{\mu_2}(\mu_2 > 0)$. It is assumed that the two sources of depletion are independent. Let $f_m(.)$ be the m-fold convolution of $f(.)$ with itself and $w_n(.)$ be the n-fold convolution of $w(.)$ with itself. At every policy decision making epoch a random number of persons quit the organizations and at every transfer decision making epoch a random number of persons are transferred to the sister organizations. There is an associated loss of manpower in each grade, if a person quits or transferred. It is assumed that the loss of manpower is linear and cumulative. For $i = 1, 2, 3...$, let X_{Ai} and X_{Bi} be the continuous random variables representing the amount of depletion of manpower(loss of man hours) in grades A and B respectively caused due to the ith policy decision. It is assumed that X_{Ai} and X_{Bi} are independent for each i and each form a sequence of independent and identically distributed random variables with distributions $G_A(.)$ and $G_B(.)$ and probability density functions $g_A(.)$ and $g_B(.)$ respectively. Let $\overline{X}_{Am}$ and $\overline{X}_{Bm}$ be the cumulative depletion of manpower in the first m policy decisions in grades A and B respectively. Let $\overline{X}_m$ be the cumulative depletion of manpower in the organization due to the first m policy decisions. For $j = 1, 2...$, let Y_{Aj} and Y_{Bj} be the continuous random variables representing the amount of depletion of manpower in grades A and B respectively caused due to the jth transfer decision. It is assumed that Y_{Aj} and Y_{Bj} are independent for each j and each form a sequence of independent and identically distributed random variables with probability density functions $h_A(.)$ and $h_B(.)$ respectively. Let $\overline{Y}_{An}$ and $\overline{Y}_{Bn}$ be the cumulative depletion of manpower in the first n transfer decisions in grades A and B respectively. Let $\overline{Y}_n$ be the cumulative depletion of manpower in the organization due to the first n transfer decisions. For each i and j X_{Ai}, X_{Bi}, Y_{Aj} and Y_{Bj} are statistically independent. Let $\overline{g}_A(.)$ and $\overline{g}_B(.)$ be

the Laplace transforms of $g_A(.)$ and $g_B(.)$ respectively. Let Z be the breakdown threshold level for the cumulative loss of manpower in the organization. For grade A, let Z_{A1} be the normal exponential threshold of depletion of manpower with mean $\frac{1}{\theta_{A1}}(\theta_{A1} > 0)$, Z_{A2} be the exponential threshold of frequent breaks of existing workers with mean $\frac{1}{\theta_{A2}}(\theta_{A2} > 0)$ and Z_{A3} be the threshold of backup or reservation of manpower sources with mean $\frac{1}{\theta_{A3}}(\theta_{A3} > 0)$. For grade B, let Z_{B1}, Z_{B2} and Z_{B3} be the normal exponential threshold of depletion of manpower, exponential threshold of frequent breaks of existing workers and exponential threshold of backup or reservation of manpower sources with means $\frac{1}{\theta_{B1}}, \frac{1}{\theta_{B2}}$ and $\frac{1}{\theta_{B3}}(\theta_{B1}, \theta_{B2}$ and $\theta_{B3} > 0)$ respectively. Let $k(.)$ be the probability density function of Z. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold.

Let T be the random variable denoting the time to recruitment with distribution $L(.)$, probability density function $l(.)$, Laplace transform $\bar{l}(s)$, mean $E(T)$ and variance $V(T)$. Let $N_P(T)$ be the number of policy decisions required to make recruitment at T and $N_{Trans.}(T)$ be the number of transfer decisions required to make recruitment at T. Let $\overline{X}_{N_P(T)}$ and $\overline{Y}_{N_{Trans.}(T)}$ be the respective total loss of manpower in $N_P(T)$ and $N_{Trans.}(T)$ decisions incurred by the organization during $(0, T)$.

3. Main Results

The survivor function of T is

$$P(T > t) = P(\overline{X}_{N_P(T)} + \overline{Y}_{N_{Trans.}(T)} < Z). \tag{1}$$

Since $\{N_P(T)\}$ and $\{N_{Trans.}(T)\}$ form two independent ordinary renewal processes, from Renewal theory [7] and (1) we find that

$$P(T > t) = \sum_{m=0}\sum_{n=0}[F_m(t) - F_{m+1}(t)][W_n(t) - W_{n+1}(t)]P(\overline{X}_m + \overline{Y}_n \le Z), \tag{2}$$

where

$$P(\overline{X}_m + \overline{Y}_n \le Z) = \int_0 P(\overline{X}_m + \overline{Y}_n < z)k(z)\mathrm{d}z. \tag{3}$$

We now estimate the mean and variance of T for different cases:

Case (i).

$$Z = \min (Z_{A1} + Z_{A2} + Z_{A3}, Z_{B1} + Z_{B2} + Z_{B3}).$$

Since

$$P(Z > z) = \{P(Z_{A1} + Z_{A2} + Z_{A3} > z)P(Z_{B1} + Z_{B2} + Z_{B3} > z)\},$$

from the hypothesis and on simplification it can be shown that

$$
\begin{aligned}
k(z) = {} & C_1(\theta_{A1} + \theta_{B1})e^{-(\theta_{A1}+\theta_{B1})z} + C_2(\theta_{A2} + \theta_{B2})e^{-(\theta_{A2}+\theta_{B2})z} + \\
& C_3(\theta_{A3} + \theta_{B3})e^{-(\theta_{A3}+\theta_{B3})z} - C_4(\theta_{A1} + \theta_{B2})e^{-(\theta_{A1}+\theta_{B2})z} + \\
& C_5(\theta_{A1} + \theta_{B3})e^{-(\theta_{A1}+\theta_{B3})z} - C_6(\theta_{A2} + \theta_{B1})e^{-(\theta_{A2}+\theta_{B1})z} - \\
& C_7(\theta_{A2} + \theta_{B3})e^{-(\theta_{A2}+\theta_{B3})z} + C_8(\theta_{A3} + \theta_{B1})e^{-(\theta_{A3}+\theta_{B1})z} - \\
& C_9(\theta_{A3} + \theta_{B2})e^{-(\theta_{A3}+\theta_{B2})z},
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
C_1 &= \frac{\theta_{A2}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})}, \\
C_2 &= \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})}, \\
C_3 &= \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})}, \\
C_4 &= \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})}, \\
C_5 &= \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})}, \\
C_6 &= \frac{\theta_{A1}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})}, \\
C_7 &= \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})}, \\
C_8 &= \frac{\theta_{A1}\theta_{A2}\theta_{B2}\theta_{B3}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})}, \\
C_9 &= \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B3}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})}
\end{aligned}
\tag{5}
$$

From (2), (3) and (4) we get

$$
\begin{aligned}
P(T > t) = {} & C_1 D_{\theta_{A1},\theta_{B1}}(t) + C_2 D_{\theta_{A2},\theta_{B2}}(t) + C_3 D_{\theta_{A3},\theta_{B3}}(t) \\
& - C_4 D_{\theta_{A1},\theta_{B2}}(t) + C_5 D_{\theta_{A1},\theta_{B3}}(t) - C_6 D_{\theta_{A2},\theta_{B1}}(t) \\
& - C_7 D_{\theta_{A2},\theta_{B3}}(t) + C_8 D_{\theta_{A3},\theta_{B1}}(t) - C_9 D_{\theta_{A3},\theta_{B2}}(t)
\end{aligned}
\tag{6}
$$

where $\quad D_{\alpha,\beta}(t) = (\sum_{m=0} [F_m(t) - F_{m+1}(t)][\overline{g}_A(\alpha + \beta)\overline{g}_B(\alpha + \beta)]^m)$

$$(\sum_{n=0} [W_m(t) - W_{m+1}(t)][\overline{h}_A(\alpha + \beta)\overline{h}_B(\alpha + \beta)]^n)$$

Therefore from (6)

$$L(t) = 1 - P(T > t) = 1 - \sum_{i=1}^{9} p_i(t),$$

$$l(t) = -\sum_{i=1}^{9} p_i(t) \text{ and } \overline{l}(s) = -\sum_{i=1}^{9} \overline{p_i}(s) \tag{7}$$

where $p_i(t)$ is the ith term in the right side of (6) and $p_i(t) = \frac{d}{dt} p_i(t), i = 1, 2, ...9$.

The mean and variance of time to recruitment can be computed from (7) and from the result

$$E(T) = -[\frac{d}{ds}[\overline{l}(s)]]_{s=0} = \sum_{i=1}^{9} [\frac{d}{ds}[\overline{p_i}(s)]]_{s=0}, \quad E(T^2) = [\frac{d^2}{ds^2}[\overline{l}(s)]]_{s=0}$$

$$= \sum_{i=1}^{9} [\frac{d^2}{ds^2}[\overline{p_i}(s)]]_{s=0} \tag{8}$$

We now compute $[\frac{d}{ds}[\overline{p_i}(s)]]_{s=0}$ and $[\frac{d^2}{ds^2}[\overline{p_i}(s)]]_{s=0}$ for $i = 1$. The computation for i=2,3,...,9 is similar.

On simplification, the first term of right side of (6) is found to be

$P_1(t) = C_1 D_{\theta_{A1},\theta_{B1}}(t)$

$$= C_1\{1 - [1 - \overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})] \sum_{m=1}^{\infty} F_m(t)[\overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})]^{m-1} +$$

$$[1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})] \sum_{n=1}^{\infty} W_n(t)[\overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})]^{n-1}$$

$$- [1 - \overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})][1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})]$$

$$\sum_{m=1}^{\infty} F_m(t)[\overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})]^{m-1} \sum_{n=1}^{\infty} W_n(t)[\overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})]^{n-1}\}$$

$$\tag{9}$$

Since $w_n(t) = \frac{\mu_2^n e^{-\mu_2 t} t^{n-1}}{(n-1)!}$ by hypothesis, we find that

$$[1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})] \sum_{n=1} W_n(t)[\overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})]^{n-1}$$

$$= \mu_2[1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})]e^{-\mu_2[1-\overline{h}_A(\theta_{A1}+\theta_{B1})\overline{h}_B(\theta_{A1}+\theta_{B1})]t}$$

(10)

From (9) and (10) and on simplification we get

$$[\frac{d}{ds}[\overline{p_1'}(s)]]_{s=0} = \frac{C_1[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})))]}{\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1}))[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1}))) \atop \times \overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})]}$$

(11)

$$[\frac{d^2}{ds^2}[\overline{p_1}(s)]]_{s=0} = \frac{\begin{array}{c} -C_1[2\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})) \\ (1 - \overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})) \\ \overline{f}\,(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})))+ \\ 2[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1}))) \\ \overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})] \\ {[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})))]]} \end{array}}{\begin{array}{c} \mu_2^2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1}))^2 \\ {[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})))} \\ \overline{g}_A(\theta_{A1} + \theta_{B1})\overline{g}_B(\theta_{A1} + \theta_{B1})]^2 \end{array}}$$

(12)

where

$$\overline{f}(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})))$$
$$= \frac{p\mu_h}{\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})) + \mu_h}$$
$$+ \frac{(1 - p)\mu_l}{\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})) + \mu_l}$$

and

$$\overline{f}\,(\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})))$$
$$= \frac{-p\mu_h}{[\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})) + \mu_h]^2}$$
$$- \frac{(1 - p)\mu_l}{[\mu_2(1 - \overline{h}_A(\theta_{A1} + \theta_{B1})\overline{h}_B(\theta_{A1} + \theta_{B1})) + \mu_l]^2}$$

by hypothesis.

From (8) ,(11) and (12) we get

$$\begin{aligned}
E(T) = &\ C_1 E_{\theta_{A1},\theta_{B1}} + C_2 E_{\theta_{A2},\theta_{B2}} + C_3 E_{\theta_{A3},\theta_{B3}} \\
&- C_4 E_{\theta_{A1},\theta_{B2}} + C_5 E_{\theta_{A1},\theta_{B3}} - C_6 E_{\theta_{A2},\theta_{B1}} \\
&- C_7 E_{\theta_{A2},\theta_{B3}} + C_8 E_{\theta_{A3},\theta_{B1}} - C_9 E_{\theta_{A3},\theta_{B2}}
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
E(T^2) = &\ C_1 E^2_{\theta_{A1},\theta_{B1}} + C_2 E^2_{\theta_{A2},\theta_{B2}} + C_3 E^2_{\theta_{A3},\theta_{B3}} \\
&- C_4 E^2_{\theta_{A1},\theta_{B2}} + C_5 E^2_{\theta_{A1},\theta_{B3}} - C_6 E^2_{\theta_{A2},\theta_{B1}} \\
&- C_7 E^2_{\theta_{A2},\theta_{B3}} + C_8 E^2_{\theta_{A3},\theta_{B1}} - C_9 E^2_{\theta_{A3},\theta_{B2}}
\end{aligned} \tag{14}$$

(13) and (14) give the variance of the time to recruitment for this case.

Case (ii)

$$Z = \max (Z_{A1} + Z_{A2} + Z_{A3}, Z_{B1} + Z_{B2} + Z_{B3}).$$

Since

$$P(Z \le z) = \{P(Z_{A1} + Z_{A2} + Z_{A3} \le z)P(Z_{B1} + Z_{B2} + Z_{B3} \le z)\},$$

from the hypothesis and on simplification it can be shown that

$$\begin{aligned}
k(z) = &- C_1(\theta_{A1} + \theta_{B1})e^{-(\theta_{A1}+\theta_{B1})z} - C_2(\theta_{A2} + \theta_{B2})e^{-(\theta_{A2}+\theta_{B2})z} - \\
&\ C_3(\theta_{A3} + \theta_{B3})e^{-(\theta_{A3}+\theta_{B3})z} + C_4(\theta_{A1} + \theta_{B2})e^{-(\theta_{A1}+\theta_{B2})z} - \\
&\ C_5(\theta_{A1} + \theta_{B3})e^{-(\theta_{A1}+\theta_{B3})z} + C_6(\theta_{A2} + \theta_{B1})e^{-(\theta_{A2}+\theta_{B1})z} + \\
&\ C_7(\theta_{A2} + \theta_{B3})e^{-(\theta_{A2}+\theta_{B3})z} - C_8(\theta_{A3} + \theta_{B1})e^{-(\theta_{A3}+\theta_{B1})z} + \\
&\ C_9(\theta_{A3} + \theta_{B2})e^{-(\theta_{A3}+\theta_{B2})z} + C_{10}\theta_{A1}e^{-\theta_{A1}z} - C_{11}\theta_{A2}e^{-\theta_{A2}z} + \\
&\ C_{12}\theta_{A3}e^{-\theta_{A3}z} + C_{13}\theta_{B1}e^{-\theta_{B1}z} - C_{14}\theta_{B2}e^{-\theta_{B2}z} + C_{15}\theta_{B3}e^{-\theta_{B3}z},
\end{aligned}$$

where $C_i, i = 1......9$ are given by (5) and

$$C_{10} = \frac{\theta_{A2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})}, \quad C_{11} = \frac{\theta_{A1}\theta_{A3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})},$$

$$C_{12} = \frac{\theta_{A1}\theta_{A2}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})}, \quad C_{13} = \frac{\theta_{B2}\theta_{B3}}{(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})},$$

$$C_{14} = \frac{\theta_{B1}\theta_{B3}}{(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B2})}, \quad C_{15} = \frac{\theta_{B1}\theta_{B2}}{(\theta_{B3} - \theta_{B2})(\theta_{B3} - \theta_{B1})}.$$

As proceeding in case (i), we get

$$
\begin{aligned}
E(T) &= C_{10}E_{\theta_{A1}} - C_{11}E_{\theta_{A2}} + C_{12}E_{\theta_{A3}} + C_{13}E_{\theta_{B1}} - C_{14}E_{\theta_{B2}} \\
&+ C_{15}E_{\theta_{B3}} - C_1 E_{\theta_{A1},\theta_{B1}} - C_2 E_{\theta_{A2},\theta_{B2}} - C_3 E_{\theta_{A3},\theta_{B3}} \\
&+ C_4 E_{\theta_{A1},\theta_{B2}} - C_5 E_{\theta_{A1},\theta_{B3}} \\
&+ C_6 E_{\theta_{A2},\theta_{B1}} + C_7 E_{\theta_{A2},\theta_{B3}} - C_8 E_{\theta_{A3},\theta_{B1}} + C_9 E_{\theta_{A3},\theta_{B2}}
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
E(T^2) &= C_{10}E^2_{\theta_{A1}} - C_{11}E^2_{\theta_{A2}} + C_{12}E^2_{\theta_{A3}} + C_{13}E^2_{\theta_{B1}} - C_{14}E^2_{\theta_{B2}} \\
&+ C_{15}E^2_{\theta_{B3}} - C_1 E^2_{\theta_{A1},\theta_{B1}} - C_2 E^2_{\theta_{A2},\theta_{B2}} - C_3 E^2_{\theta_{A3},\theta_{B3}} \\
&+ C_4 E^2_{\theta_{A1},\theta_{B2}} - C_5 E^2_{\theta_{A1},\theta_{B3}} + C_6 E^2_{\theta_{A2},\theta_{B1}} \\
&+ C_7 E^2_{\theta_{A2},\theta_{B3}} - C_8 E^2_{\theta_{A3},\theta_{B1}} + C_9 E^2_{\theta_{A3},\theta_{B2}}.
\end{aligned}
\tag{16}
$$

(15) and (16) give the variance of the time to recruitment for this case.

Case (iii)

$$
Z = Z_{A1} + Z_{A2} + Z_{A3} + Z_{B1} + Z_{B2} + Z_{B3}
$$

In this case it can be shown that

$$
\begin{aligned}
k(z) &= C_{16}\theta_{A1}e^{-\theta_{A1}z} - C_{17}\theta_{A2}e^{-\theta_{A2}z} + C_{18}\theta_{A3}e^{-\theta_{A3}z} + C_{19}\theta_{B1}e^{-\theta_{B1}z} \\
&- C_{20}\theta_{B2}e^{-\theta_{B2}z} + C_{21}\theta_{B3}e^{-\theta_{B3}z},
\end{aligned}
$$

where

$$
C_{16} = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})(\theta_{B1}-\theta_{A1})(\theta_{B2}-\theta_{A1})(\theta_{B3}-\theta_{A1})},
$$

$$
C_{17} = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})(\theta_{B1}-\theta_{A2})(\theta_{B2}-\theta_{A2})(\theta_{B3}-\theta_{A2})},
$$

$$
C_{18} = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})(\theta_{B1}-\theta_{A3})(\theta_{B2}-\theta_{A3})(\theta_{B3}-\theta_{A3})},
$$

$$
C_{19} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{B1}-\theta_{A1})(\theta_{B1}-\theta_{A2})(\theta_{B1}-\theta_{A3})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})},
$$

$$
C_{20} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{B2}-\theta_{A1})(\theta_{B2}-\theta_{A2})(\theta_{B2}-\theta_{A3})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})},
$$

$$
C_{21} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{B3}-\theta_{A1})(\theta_{B3}-\theta_{A2})(\theta_{B3}-\theta_{A3})(\theta_{B3}-\theta_{B1})(\theta_{B3}-\theta_{B2})}
$$

$$
E(T) = C_{16}E_{\theta_{A1}} - C_{17}E_{\theta_{A2}} + C_{18}E_{\theta_{A3}} + C_{19}E_{\theta_{B1}} - C_{20}E_{\theta_{B2}} + C_{21}E_{\theta_{B3}} \tag{17}
$$

and

$$E(T^2) = C_{16}E^2_{\theta_{A1}} - C_{17}E^2_{\theta_{A2}} + C_{18}E^2_{\theta_{A3}}$$
$$+ C_{19}E^2_{\theta_{B1}} - C_{20}E^2_{\theta_{B2}} + C_{21}E^2_{\theta_{B3}}. \quad (18)$$

In the equations (13), (15) and (17)

$$E_{\alpha,\beta} = \frac{C_1[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta)))]}{\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))) \\ \overline{g}_A(\alpha+\beta)\overline{g}_B(\alpha+\beta)]}$$

and

$$E_\alpha = \frac{C_1[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha)))]}{\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha))[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha)))\overline{g}_A(\alpha)\overline{g}_B(\alpha)]}.$$

In the equation (14), (16) and (18)

$$E^2_{\alpha,\beta} = \frac{\begin{aligned} &C_1[2\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))(1 - \overline{g}_A(\alpha+\beta)\overline{g}_B(\alpha+\beta)) \\ &\overline{f}\,(\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))) + 2[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))) \\ &\overline{g}_A(\alpha+\beta)\overline{g}_B(\alpha+\beta)][1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta)))]] \end{aligned}}{\begin{aligned} &\mu_2^2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))^2[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha+\beta)\overline{h}_B(\alpha+\beta))) \\ &\overline{g}_A(\alpha+\beta)\overline{g}_B(\alpha+\beta)]^2 \end{aligned}}$$

and

$$E^2_\alpha = \frac{\begin{aligned} &C_1[2\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha))(1 - \overline{g}_A(\alpha)\overline{g}_B(\alpha))\overline{f}\,(\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha))) \\ &+2[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha)))\overline{g}_A(\alpha)\overline{g}_B(\alpha)][1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha)))]] \end{aligned}}{\mu_2^2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha))^2[1 - \overline{f}(\mu_2(1 - \overline{h}_A(\alpha)\overline{h}_B(\alpha)))\overline{g}_A(\alpha)\overline{g}_B(\alpha)]^2}.$$

(17) and (18) give the variance of the time to recruitment for this case.

Special case. Suppose X_{Ai}, X_{Bi}, Y_{Aj} and Y_{Bj} follow exponential distribution with parameters $\alpha_{1A}, \alpha_{1B}, \alpha_{2A}$ and α_{2B}, respectively.

In this case

$$\bar{g}_A(\theta) = \frac{\alpha_{1A}}{\alpha_{1A} + \theta}, \quad \bar{g}_B(\theta) = \frac{\alpha_{1B}}{\alpha_{1B} + \theta},$$
$$\bar{h}_A(\theta) = \frac{\alpha_{2A}}{\alpha_{2A} + \theta}, \quad \bar{h}_A(\theta) = \frac{\alpha_{2B}}{\alpha_{2B} + \theta}. \quad (19)$$

Using (19) in (13), (15), (17), (14),(16) and (18), we get the analytical results for the mean and variance of the time to recruitment for all the cases in closed form.

Note. The results for the performance measures corresponding to cases (i), (ii) and (iii) when the inter-policy decisions times are independent and identically distributed exponential random variables can be deduced from our results for the respective cases by taking $p = 1$.

4. Numerical Illustration

The mean and variance of time to recruitment for all the cases are numerically illustrated by varying one parameter and keeping all the other parameters fixed. The effect of the nodal parameters $\alpha_{1A}, \alpha_{1B}, \alpha_{2A}, \alpha_{2B}, \mu_h, \mu_l, \mu_2$ and p on the mean and variance of time to recruitment are shown in the following table.

Table 1: $\mu_h = 2$; $\mu_l = 4$; $\mu_2 = 2$; $p = 0.07$; $\theta_{A1} = 0.002$;
$\theta_{A2} = 0.01$; $\theta_{A3} = 0.03$; $\theta_{B1} = 0.07$; $\theta_{B2} = 0.04$; $\theta_{B3} = 0.05$

α_{1A}	α_{1B}	α_{2A}	α_{2B}	case(i)		case(ii)		case(iii)	
				E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	0.2	0.1	0.5	0.844	0.453	8.063	42.84	9.181	36.80
0.5	0.2	0.1	0.5	1.286	0.928	12.79	107.4	14.52	93.28
0.9	0.2	0.1	0.5	1.378	1.058	13.69	123.2	15.55	106.9
0.1	0.1	0.2	0.5	0.785	0.417	7.420	33.66	8.329	30.09
0.1	0.5	0.2	0.5	1.118	0.740	11.11	74.84	12.42	67.98
0.1	0.9	0.2	0.5	1.181	0.818	11.77	83.86	13.16	76.33
0.2	0.1	0.1	0.5	0.844	0.453	8.063	42.84	9.181	36.80
0.2	0.1	0.5	0.5	1.047	0.684	10.36	61.03	11.45	57.51
0.2	0.1	0.9	0.5	1.080	0.732	10.72	63.49	11.78	60.85
0.2	0.1	0.5	0.1	1.022	0.641	9.474	67.58	11.10	53.84
0.2	0.1	0.5	0.5	1.286	0.928	12.79	107.4	14.52	93.28
0.2	0.1	0.5	0.9	1.330	0.986	13.31	114.02	15.05	100.2

Table 2: $\alpha_{1A} = 0.5; \alpha_{1B} = 0.5; \alpha_{2A} = 0.2; \alpha_{2B} = 0.1;\ \theta_{A1} = 0.002;$
$\theta_{A2} = 0.01;\ \theta_{A3} = 0.03;\ \theta_{B1} = 0.07;\ \theta_{B2} = 0.04;\ \theta_{B3} = 0.05$

μ_h	μ_l	μ_2	p	case(i)		case(ii)		case(iii)	
				E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	1.5	2.0	0.07	1.924	2.302	18.30	281.7	21.85	218.7
0.5	1.5	2.0	0.07	1.889	2.148	17.49	244.8	20.78	189.3
0.9	1.5	2.0	0.07	1.872	2.091	17.31	237.1	20.52	184.1
0.5	1.1	2.0	0.07	1.970	2.350	17.88	270.5	21.54	202.7
0.5	1.5	2.0	0.07	1.889	2.148	17.49	244.8	20.78	189.3
0.5	1.9	2.0	0.07	1.818	1.986	17.08	226.1	20.12	178.5
1.5	1.2	0.1	0.7	2.998	5.202	30.25	496.3	33.14	481.8
1.5	1.2	0.5	0.7	2.372	3.315	23.04	341.27	26.09	297.9
1.5	1.2	0.9	0.7	1.963	2.295	18.31	255.5	21.51	202.3
0.7	1.5	2.0	0.01	1.861	2.063	17.23	233.8	20.41	181.9
0.7	1.5	2.0	0.05	1.873	2.097	17.33	238.1	20.55	184.7
0.7	1.5	2.0	0.09	1.885	2.130	17.42	242.1	20.68	187.2

From the above tables the following inference are presented which agree with reality:

1. When α_{1A} increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.

2. When α_{1B} increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.

3. When α_{2A} increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.

4. When α_{2B} increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.

5. When μ_h increases and keeping all the other parameters fixed, the mean and variance of time to recruitment decrease for all the three cases.

6. When μ_l increases and keeping all the other parameters fixed, the mean and variance of time to recruitment decrease for all the three cases.

7. When μ_2 increases and keeping all the other parameters fixed, the mean and variance of time to recruitment decrease for all the three cases.

8. When p increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.

5. Conclusion

The manpower model developed in this paper accommodates an important nodal parameter namely attrition rate with respective proportion and this aspect makes the manpower problem more realistic compared to the stochastic models in the literature confined to the present study. The findings reported in this paper will be very useful in planning recruitments in future for those marketing organizations with depletion of manpower due to attrition. The goodness of fit for the distributions assumed in this paper can be examined by collecting relevant data and using simulation the applicability of the developed model can be studied.

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