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Saturated stabilization and tracking of a nonholonomic mobile robot ☆

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Abstract

This paper presents a framework to deal with the problem of global stabilization and global tracking control for the kinematic model of a wheeled mobile robot in the presence of input saturations. A model-based control design strategy is developed via a simple application of passivity and normalization. Saturated, Lipschitz continuous, time-varying feedback laws are obtained and illustrated in a number of compelling simulations. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In recent years, a lot of interest has been devoted to the stabilization and tracking of nonholonomic dynamical systems. A wheeled mobile robot under nonholonomic constraints, or its feedback equivalent chained form system, has served as a benchmark mechanical example in several papers – see, e.g., [1,2,4,6,7,9,14,16,22,24,25]. In addition to practical motivations, one of the technical reasons for this is, undoubtedly, that no continuous time-invariant stabilizing controller for this system exists, which is a corollary from the fact that Brockett's necessary condition for feedback stabilization [3] is not met. Many of the above references, as well as [5,6,8,12,18,23] therefore aim at developing suitable time-varying stabilizing (tracking) controllers for mobile robots or more general chained form nonholonomic systems.

In the present note, we want to study the stabilization and tracking problem for a wheeled mobile robot under saturation constraints on the inputs. The stabilization and tracking of nonholonomic systems with input saturations have been rarely addressed in the literature. In [18,19], the stabilization problem using bounded state-feedback was dealt with for a class of driftless controllable systems. The results of [18] are a direct application of ideas from passivity theory together with Pomet's time-varying method [23]. In this paper, instead of pursuing along the line of general schemes [23,18,1,19,10,11], we will exploit the physical structure of the mobile robot in an objective to design a simpler passivity-based, saturated, smooth

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(i.e., of class C^∞) feedback stabilizer for the kinematic model of a mobile robot. This complements the results in [2,6,9,14,24] where no saturation constraint was imposed on the inputs. We achieve our goal via passivity and normalization techniques known from adaptive control, see e.g. [13,17]. Our main contribution in this paper is to address the *saturated* global stabilization and tracking of mobile robots in one setting, whereas most of the past literature studied these two problems (without actuator saturation) separately. A similar framework has recently been developed in [2,6] for the regulation and tracking of a nonholonomic double integrator *in the absence of* input saturation. It should also be mentioned that prior *nonsmooth* stabilization algorithms in [1,4,16,19,20] as well as the smooth stabilization algorithms of [23,18] are unlikely to be extendible to the tracking case.

It should be mentioned that our work follows a different approach from those of recent papers [2,6] in that we base the control design on the physical model rather than its abstract, though mathematically feedback equivalent, three-dimensional chained form system or nonholonomic double integrator (see Remark 1 below).

The paper is organized as follows. In Section 2, the bounded state feedback stabilization problem for the wheeled mobile robot is addressed, while in Section 3 the bounded state feedback tracking problem is investigated. Section 4 contains the conclusions.

2. Stabilization via bounded state feedback

The purpose of this section is to show that, by exploiting the underlying physical structure, we can obtain an analytically simple, saturated, time-varying state-feedback stabilizer for the kinematic model of a wheeled mobile robot under actuator saturation.

2.1. Kinematic model

The benchmark wheeled mobile robot considered by many researchers (see, e.g., [16,4] and references therein) is described by the following kinematic model:

$$\begin{aligned}\dot{x}_c &= v\cos\theta,\\ \dot{y}_c &= v\sin\theta,\\ \dot{\theta} &= \omega,\end{aligned} \tag{1}$$

where v is the forward velocity, ω is the steering velocity, (x_c, y_c) is the position of the mass center of the robot moving in the plane and θ denotes its heading angle from the horizontal axis. Here, the velocities v and ω are taken as the inputs and are subject to the following constraints:

$$|\omega(t)| \leqslant \omega_{max}, \quad |v(t)| \leqslant v_{max} \ \ \forall t \geqslant 0, \tag{2}$$

where ω_{max} and v_{max} are given arbitrary positive constants.

The stabilization problem to be addressed, is to construct a time-varying state-feedback law of the form

$$\omega = \alpha_1(t, \theta, x_c, y_c), \quad v = \alpha_2(t, \theta, x_c, y_c) \tag{3}$$

in such a way that (2) holds and the zero solution of robot system (1) in closed loop with (3) is globally uniformly asymptotically stable (GUAS).

In order to achieve this control objective we apply the change of coordinates

$$\begin{aligned}x_1 &= x_c\sin\theta - y_c\cos\theta,\\ x_2 &= x_c\cos\theta + y_c\sin\theta,\\ x_3 &= \theta\end{aligned} \tag{4}$$

that preserves the origin and transforms our system (1) into

$$\begin{aligned}\dot{x}_1 &= \omega x_2,\\ \dot{x}_2 &= -\omega x_1 + v,\\ \dot{x}_3 &= \omega.\end{aligned} \tag{5}$$

We will employ an important passivity property, associated with the (x_1, x_2)-subsystem, to design a desired time-varying state-feedback controller. Then, we follow [8, Proposition 2] to complete the stability analysis. First of all, define a set $\mathscr{BF}_r$ of continuous and bounded functions indexed by a parameter $r > 0$, i.e.

$$\mathscr{BF}_r = \{\phi : \mathbb{R} \to \mathbb{R} \mid \phi \text{ is continuous and } -r \leqslant \phi(x) \leqslant r \ \forall x \in \mathbb{R}\} \tag{6}$$

and a corresponding set of saturation functions $\mathscr{S}_r$, i.e.

$$\mathscr{S}_r = \{\phi : \mathbb{R} \to \mathbb{R} \mid \phi \in \mathscr{BF}_r,\ s\phi(s) > 0 \text{ for all } s \neq 0\}. \tag{7}$$

Examples of nontrivial functions in $\mathscr{S}_r$ include, for instance,

$$\phi(x) = \frac{2rx}{1+x^2}, \quad \phi(x) = \frac{2r}{\pi}\arctan(x), \quad \phi(x) = r\tanh(x). \tag{8}$$

To be more specific on the passivity property of the (x_1, x_2)-subsystem, let us consider the storage function

$$V_1(x_1, x_2) = \tfrac{1}{2}x_1^2 + \tfrac{1}{2}x_2^2. \tag{9}$$

Then, the time derivative of V_1 satisfies

$$\dot{V}_1 = x_2 v \tag{10}$$

which implies the passivity (or more precisely, losslessness) of the (x_1, x_2)-subsystem with input v and output x_2. Note that this passivity property is irrespective of the choice of ω.

Therefore, we are led to choose the following control law

$$v = -\phi_1(x_2), \tag{11}$$

where ϕ_1 belongs to $\mathscr{S}_{\varepsilon_1}$ for any ε_1 in $(0, v_{\max}]$. As a result, (10) implies

$$\dot{V}_1 = -x_2\phi_1(x_2) \leqslant 0. \tag{12}$$

Inspired by the iterative time-varying design in our earlier work [8], it is shown that the choice of the following time-varying state-feedback control law together with (11) achieves the proposed control objective:

$$\omega = -\phi_2(x_3) + \phi_3(x_1)\sin t, \tag{13}$$

where $\phi_2 \in \mathscr{S}_{\varepsilon_2}$ and $\phi_3 \in \mathscr{S}_{\varepsilon_3}$ for any pair of positive constants $(\varepsilon_2, \varepsilon_3)$ such that $\varepsilon_2 > \varepsilon_3$ and $\varepsilon_2 + \varepsilon_3 \leqslant \omega_{\max}$.

We are ready to state and prove the first main result of this paper.

Proposition 1. *The equilibrium* $(x_c, y_c, \theta) = (0, 0, 0)$ *of closed-loop system* (1), (11) *and* (13) *is globally uniformly asymptotically stable* (*GUAS*). *In particular*, *given any saturation levels* $\omega_{\max} > 0$, $v_{\max} > 0$ *as in* (2), *we can always tune the design parameters* $\varepsilon_1, \varepsilon_2$ *and* ε_3 *so that* (2) *holds while* $(x_c, y_c, \theta) = (0, 0, 0)$ *is GUAS.*

Proof. It follows along similar lines of [8, Proof of Proposition 2] using LaSalle's invariance principle.

Since the mapping defined in (4) is a global diffeomorphism preserving the origin, we will work with the new x-coordinates. We first demonstrate that the solutions of the closed-loop system (5), (11) and (13) are well-defined for every $t \geqslant 0$ and are bounded. From (12) and the fact that V_1 is positive-definite (with respect to (x_1, x_2), not (x_1, x_2, x_3)!) and radially unbounded (w.r.t. (x_1, x_2) only), it follows that $(x_1(t), x_2(t))$ are bounded on the maximal interval $[0, t_c)$, with $0 < t_c \leqslant \infty$. Going back to the x_3-equation in closed loop with (13), i.e.,

$$\dot{x}_3 = -\phi_2(x_3) + \phi_3(x_1)\sin t, \tag{14}$$

we conclude that $x_3(t)$ is also bounded on $[0, t_c)$. Therefore, $t_c = \infty$. The uniform stability of the trivial solution $(x_c(t), y_c(t), \theta(t)) = (0, 0, 0)$ follows from the definition of $x = (x_1, x_2, x_3)$ in (4), the periodicity of the closed-loop system, and (12) and (14).

It rests to prove the attractivity. In other words, we only need to establish the asymptotic convergence of the closed-loop solutions $x(t)$. Like in [23,8], we invoke the well-known LaSalle's invariance principle. Since the closed-loop system comprised of (5), (11) and (13) is a 2π-periodic time-varying system, it can be considered as a time-*invariant* system evolving on $S^1 \times \mathbb{R}^3$ (see, e.g., [23, p. 156])

$$\begin{aligned} \dot{\tau} &= 1, \\ \dot{x}_1 &= (-\phi_2(x_3) + \phi_3(x_1)\sin\tau)x_2, \\ \dot{x}_2 &= (\phi_2(x_3) - \phi_3(x_1)\sin\tau)x_1 - \phi_1(x_2), \\ \dot{x}_3 &= -\phi_2(x_3) + \phi_3(x_1)\sin\tau, \end{aligned} \tag{15}$$

where $\tau \in \mathscr{S}^1$ denoting the circle $\mathbb{R}/2\pi\mathbb{Z}$.

According to LaSalle's invariance principle, any bounded trajectory $(\tau(t), x(t))$ goes to the largest invariant set E contained in $\dot{V}_1 = 0$. Clearly, on the set where $\dot{V}_1 = 0$, (12) yields that $x_2 = 0$. We claim that $E = \{(\tau, x) \in \mathscr{S}^1 \times \mathbb{R}^3 \mid x_1 = x_2 = 0\}$. Once this claim is established, from (14), it follows that $x_3(t)$ tends to 0 as $t \to \infty$.

We prove our claim by contradiction. If $E = \{(\tau, x) \in \mathscr{S}^1 \times \mathbb{R}^3 \mid x_1 = x_2 = 0\}$ is not the largest invariant set, then there exists a trajectory $(\tau(t), x(t))$ such that $x_2(t) = 0\ \forall t \geqslant 0$ but $x_1(t) \neq 0$ for each t in an open subset I_0 of $[0, \infty)$. On I_0, from (15), $\dot{x}_1 = 0$ and therefore $x_1(t)$ is a *nonzero* constant $x_1^\star$. This together with the (x_2, x_3)-subsystem of (15) implies that $x_3(t)$ is constant on I_0, say $x_3(t) = x_3^\star\ \forall t \in I_0$, and that

$$\phi_2(x_3^\star) - \phi_3(x_1^\star)\sin\tau(t) = 0 \quad \forall t \in I_0. \tag{16}$$

This leads to a contradiction. Therefore, all closed-loop solutions (which have been shown bounded) converge to the set $E = \{(\tau, x) \in \mathscr{S}^1 \times \mathbb{R}^3 \mid x_1 = x_2 = 0\}$. Consequently, $x_3(t)$ goes to 0 as $t \to \infty$. Finally, the proof of Proposition 1 is completed. □

Remark 1. It is of interest to note that we have based our *saturated* controller design on system (5) that is *not* in the standard chained form [21]. An additional change of feedback of the type $v = \omega x_1 + \tilde{v}$ is required

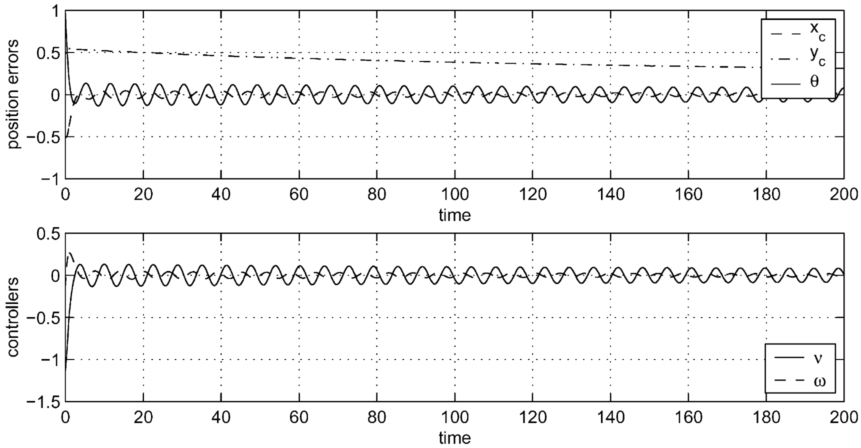

Fig. 1. Stabilization of the kinematic model with initial conditions $[x_c(0), y_c(0), \theta(0)]^T = [-0.5, 0.5, 1]^T$.

to bring (5) into a standard chained form adopted by several authors – see, e.g., [21,1,4,8,10,16,25]. However, this feedback transformation violates the input-saturation requirement (2).

2.2. Simulations

To support our results, we simulated with MATLAB™ the wheeled mobile robot (1) in closed-loop with the controller (11, 13) with $\varepsilon_1 = 1$, $\varepsilon_2 = 1.5$ and $\varepsilon_3 = 0.5$, and $\phi_i(s) = \varepsilon_i \tanh(s)$, which guarantees that $|\omega(t)| \leqslant \omega_{\max} = 2$ and $|v(t)| \leqslant v_{\max} = 1$ for all $t \geqslant 0$. The resulting performance is depicted in Fig. 1.

From the plots in Fig. 1, we observe a slow convergence to the origin, which is a quite well-known effect when using smooth time-varying controllers (cf. [20]).

3. Tracking via bounded state feedback

3.1. Kinematic model

In this section, we address the tracking problem for robot (1) under a constraint on the velocities. To quantify the saturation level, it is assumed that the reference trajectory (x_r, y_r, θ_r) satisfies

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r, \\ \dot{y}_r &= v_r \sin\theta_r, \\ \dot{\theta}_r &= \omega_r, \end{aligned} \tag{17}$$

where ω_r and v_r are bounded reference velocities.

The objective is to find time-varying state-feedback controllers of the form

$$\omega = \omega^*(t, \theta, x_c, y_c), \quad v = v^*(t, \theta, x_c, y_c) \tag{18}$$

such that $x_c(t) - x_r(t)$, $y_c(t) - y_r(t)$ and $\theta(t) - \theta_r(t)$ tend to zero as $t \to +\infty$ while guaranteeing the following property:

$$|\omega(t)| \leqslant \omega_{\max}, \quad |v(t)| \leqslant v_{\max} \ \ \forall t \geqslant 0, \tag{19}$$

where $\omega_{\max} > \sup_{t\geqslant 0} |\omega_r(t)|$ and $v_{\max} > \sup_{t\geqslant 0} |v_r(t)|$ are arbitrary constants.

As in [9] (see also [14]), consider the following tracking errors:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_c \\ y_r - y_c \\ \theta_r - \theta \end{bmatrix}. \tag{20}$$

Obviously, for any value of θ, $(x_e, y_e, \theta_e) = 0$ if and only if $(x_c, y_c, \theta) = (x_r, y_r, \theta_r)$.

As it can be directly checked, the tracking error dynamics of the robot satisfy

$$\begin{aligned} \dot{x}_e &= \omega y_e - v + v_r \cos\theta_e, \\ \dot{y}_e &= -\omega x_e + v_r \sin\theta_e, \\ \dot{\theta}_e &= \omega_r - \omega. \end{aligned} \tag{21}$$

We show next that the following control laws solve our tracking problem:

$$\omega = \omega_r + \frac{\lambda_1 v_r y_e}{1 + x_e^2 + y_e^2} \frac{\sin\theta_e}{\theta_e} + h_{\lambda_2}(\theta_e), \tag{22}$$

$$v = v_r \cos\theta_e + h_{\lambda_3}(x_e), \tag{23}$$

where $\lambda_1, \lambda_2, \lambda_3$ are positive design parameters and $h_{\lambda_2} \in \mathscr{S}_{\lambda_2}$, $h_{\lambda_3} \in \mathscr{S}_{\lambda_3}$. Note that $\sin(\theta_e)/\theta_e = \int_0^1 \cos(s\theta_e)\,ds$ is a smooth function in θ_e.

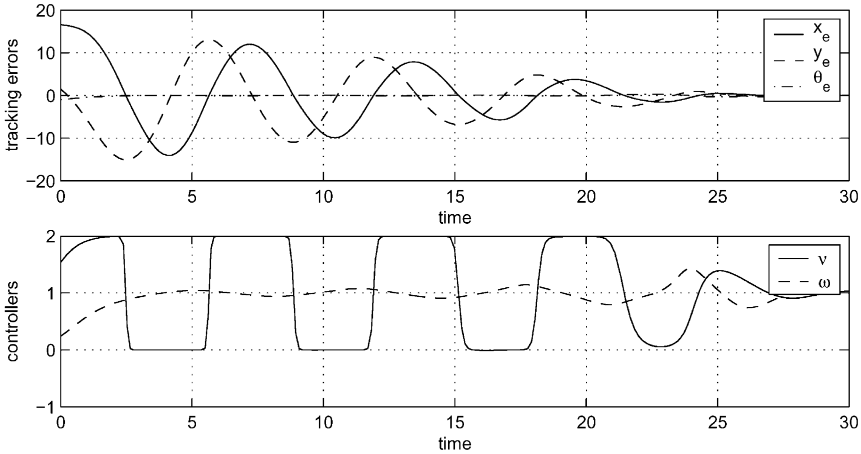

Fig. 2. Tracking of the kinematic model with initial errors $[x_e(0), y_e(0), \theta(0)]^T = [16.6, 1.5, -1]^T$.

Proposition 2. *Assume that ω_r and v_r are bounded and uniformly continuous over* $[0, \infty)$. *If either $\omega_r(t)$ or $v_r(t)$ does not converge to zero, then the zero equilibrium of closed-loop system* (21)–(23) *is globally asymptotically stable. In particular, given any $\omega_{max} > \sup_{t \geqslant 0} |\omega_r(t)|$ and $v_{max} > \sup_{t \geqslant 0} |v_r(t)|$, we can always tune our design parameters λ_1, λ_2 and λ_3 so that the condition* (19) *is met for the controllers* (22), (23).

Proof. Motivated by [13], consider the positive definite and proper Lyapunov function candidate

$$V_2(x_e, y_e, \theta_e) = \frac{\lambda_1}{2} \log(1 + x_e^2 + y_e^2) + \frac{1}{2}\theta_e^2. \qquad (24)$$

Differentiating V_2 along the solutions of closed-loop system (21)–(23) yields

$$\dot{V}_2(x_e, y_e, \theta_e) = -\frac{\lambda_1 x_e h_{\lambda_3}(x_e)}{1 + x_e^2 + y_e^2} - \theta_e h_{\lambda_2}(\theta_e) \leqslant 0. \qquad (25)$$

Therefore, the trajectories $(x_e(t), y_e(t), \theta_e(t))$ are uniformly bounded on $[0, \infty)$. It follows, as in [9], by direct application of Barbălat's lemma [15] that

$$\lim_{t \to \infty} [x_e(t) h_{\lambda_3}(x_e(t)) + \theta_e(t) h_{\lambda_2}(\theta_e(t))] = 0, \qquad (26)$$

which, in turn, gives

$$\lim_{t \to \infty} (|x_e(t)| + |\theta_e(t)|) = 0. \qquad (27)$$

It remains to prove that $y_e(t)$ goes to zero as $t \to \infty$. Indeed, this fact can be established by mimicking the arguments used in the proof of [9, Proposition 2].

The last statement of Proposition 2 is more or less direct. □

Remark 2. Proposition 2 complements earlier tracking results in [14,24,6,9], where the saturation issue is not addressed. Our approach relies on a full exploitation of the "triangular" structure in (21), along with the use of normalization technique found in adaptive control (see, e.g., [13]).

3.2. Simulations

To support our results, we simulated closed-loop system (21)–(23). The desired trajectory has been given to be $\omega_r(t) = 1$, $v_r(t) = 1$, i.e. a circle. Using $\lambda_1 = 1$ and $h_{\lambda_2}(s) = h_{\lambda_3} = \tanh(s)$, which guarantees us that $|\omega(t)| \leqslant \omega_{max} = 3$ and $|v(t)| \leqslant v_{max} = 2$ for all $t \geqslant 0$, we obtained starting from the initial condition $[x_e(0), y_e(0), \theta_e(0)]^T = [16.6, 1.5, -1]^T$ the performance as depicted in Fig. 2.

We see that the control inputs obviously remain within their bounds and yield a quick convergence to the desired trajectory.

4. Conclusions

Global solutions to the stabilization and tracking problem for the kinematic model of a wheeled mobile robot with input saturations are derived. The contributions of this paper include having developed a new framework for saturated stabilization and tracking of mobile robots, and having obtained analytically simple, saturated, time-varying state-feedback controllers. We are currently working on extending the obtained results to a simplified dynamic model of the mobile robot and other underactuated mechanical systems.

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