# A New Approach to a FSM Implementation in Embedded Memory Blocks of FPGA Devices with Use of the Functional Decomposition 

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#### Abstract

Since modern FPLD devices have very complex structure there is necessity to develop new methods that would allow fully exploiting the possibilities these devices offer. The paper presents a general method for the synthesis targeted to ROM-based implementation of sequential circuits using embedded memory blocks of programmable devices. The method is based on the serial decomposition concept and relies on decomposing the memory block into two blocks: a combinational address modifier and a smaller memory block. This new approach allows reducing the required memory size at the cost of additional logic cells for address modifier implementation. This makes possible implementation of FSMs that exceed available memory by using embedded memory blocks and additional programmable logic.


## 1. Introduction ${ }^{1}$

Current methodologies and productivity improvements are failing to keep pace with the rapid and ongoing increase in complexity and technology improvements. Nowadays, by taking advantage of the opportunities the modern microelectronic technology provides us with, we are in a position to build very complex digital circuits and systems at relatively low cost. There is a large variety of logic building blocks that can be exploited. The library of elements contains various types of gates; a lot of complex gates can be generated in (semi)custom CMOS design; and the field programmable logic families include different types of (C)PLDs and FPGAs. However, the opportunities created by modern microelectronic technology are not fully exploited because of weaknesses in traditional logic design methods. According to International Technology Roadmap for Semiconductors (1997) annual growth rate in complexity of digital devices is equal to

[^0]$58 \%$ while annual growth rate in productivity is only $21 \%$.

Among other synthesis methods decomposition has become an important tool in the analysis and design of digital systems. It is fundamental to many fields in modern engineering and science [3], [6], [9], [17], [18], [19]. Functional decomposition is based on well known "divide and conquer" paradigm and relies on breaking down a complex system into a network of smaller and relatively independent co-operating sub-systems, in such a way that the original system's behavior is preserved. A system is decomposed into a set of smaller subsystems, such that each of them is easier to analyze, understand and synthesize.

Recently, new methods of logic synthesis based on functional decomposition have been developed [1], [4], [7], [11], [15]. Unfortunately decomposition-based methods are considered as methods suitable mainly for implementation of combinational functions.

Modern FPLD devices have very complex structure. They combine PLA like structures as well as FPGA and even memory-based structures. In many cases, designers can not utilize all possibilities, such complex architectures provide due to the lack of appropriate synthesis methods. Embedded memory blocks make possible an implementation of memory based blocks, such as shift registers or RAM blocks. These memory resources make up considerably large part of the device, i.e. EP20K1500E devices provide 51,840 logic elements and 442 Kbits of SRAM. Taking under consideration conversion factors of logic elements and memory bits to logic gates ( 12 gates/logic element and 4 gates/memory bit) it turns out that memory blocks make up over 70\% of all logic resources. In many cases, though, these resources are not utilized due to the fact that the designer does not need to implement such memory parts like RAM or large registers. However, such memory blocks allow implementation of sequential machines in a way that requires less logic cells than the traditional flipflop based implementation. This may be used to implement "non-vital" sequential parts of the design, saving logic cell resources for more important sections. Since the size of embedded memory blocks is limited, such an implementation may require more memory than
available in a device. To reduce memory usage in ROMbased sequential machine implementations, decomposition-based methods can be successfully used [10].

In this paper, basic information is introduced first. Secondly, application of decomposition in the implementation of sequential machines is presented. Subsequently, some experimental results, obtained with a prototype tool that implements functional decomposition, are discussed.

The experimental results demonstrate that decomposition is capable of constructing solutions (utilizing embedded memory blocks) of comparable or even better quality than the methods implemented in commercial systems.

## 2. Basic notions

### 2.1 Functional decomposition

Let $A$ and $B$ be two subsets of $X$ such that $A \cup B=X$. Assume that the variables $x_{1}, \ldots, x_{n}$ have been relabeled in such way that:
$A=\left\{x_{1}, \ldots, x_{r}\right\}$ and
$B=\left\{x_{n-s+1}, \ldots, x_{n}\right\}$.
Consequently, for an n-tuple $x$, the first $r$ components are denoted by $x^{A}$ and the last $s$ components by $x^{B}$.

Let $F$ be a Boolean function, with $n>0$ inputs and $m>0$ outputs. Let $(A, B)$ be as defined above. Assume that $F$ is specified by a set $\mathbf{F}$ of the function's cubes. Let $G$ be a function with $s$ inputs and $p$ outputs; let $H$ be a function with $r+p$ inputs and $m$ outputs. The pair $(G, H)$ represents a serial decomposition of $F$ with respect to $(A, B)$, if for every minterm $b$ relevant to $F$, $\mathrm{G}\left(b^{B}\right)$ is defined, $G\left(b^{B}\right) \in\{0,1\}^{p}$, and $F(b)=H\left(b^{A}\right.$, $\left.G\left(b^{B}\right)\right) . G$ and $H$ are called blocks of the decomposition.

Partition-based representation of Boolean functions can be used to describe functional decomposition algorithms [3], [9], [12], [13], [16].

If there exists a r-partition $\Pi_{G}$ on $\mathbf{F}$ such that $P(B) \leq \Pi_{G}$, and $P(A) \bullet \Pi_{G} \leq P_{F}$, then $F$ has a serial decomposition with respect to $(A, B)$.


## Fig. 1. Schematic representation of the serial decomposition

The serial decomposition process consists of the following steps: an input support selection (the most time consuming part of the process), calculation of
partitions $P(A), P(B)$ and $P_{F}$, construction of partition $\Pi_{G}$, and creation of functions $H$ and $G$ [12].

### 2.2. Finite state machine

Let $A=\langle V, S, \delta\rangle$ be an FSM (completely or incompletely specified) with no outputs (outputs are omitted as they have insignificant impact on the method), where:
$V$ - set of input symbols,
$S$ - set of internal states,
$\delta$ - state transition function,
and $m=\left\lceil\log _{2} \mid \emptyset\right\rceil, n=\left\lceil\log _{2}|S|\right\rceil$ denote the number of input and state variables respectively.

To describe logic dependencies in such an FSM special partition description [3] and special partition algebra [6] are employed.

Let $\boldsymbol{K}$ be a one-to-one correspondence between the domain $D_{\delta}$ of transition function and $K=\{1, \ldots, p\}$, where $p=\left|D_{\delta}\right|$. The characteristic partition $P_{c}$ of an FSM is defined in the following way:
$\left(k_{1}, k_{2}\right) \in B_{P c}$ iff $\delta\left(K^{1}\left(k_{1}\right)\right)=\delta\left(K^{1}\left(k_{2}\right)\right)$
Thus, each block $B_{P c}$ of the characteristic partition includes these elements from $K$ which correspond to pairs ( $v, s$ ) from the domain $D_{\delta}$ such that the transition function $\delta(v, s)=s$, maps them onto the same next state $s$ '.

A partition $P$ on $K$ is compatible with partition $\pi$ on $S$ iff for any inputs $v_{a}, v_{b}$ the condition that $s_{i}, s_{j}$ belong to one block of partition $\pi$ implies that the elements from $K$ corresponding to pairs $\left(v_{a}, s_{i}\right)$ and $\left(v_{b}, s_{j}\right)$ belong to one block of the partition $P$.

A partition $P$ on $K$ is compatible with partition $\theta$ on $V$ iff for any state $s_{a}, s_{b}$ the condition that $v_{i}, v_{j}$ belong to one block of partition $\theta$ implies that the elements from $K$ corresponding to pairs $\left(v_{i}, s_{a}\right)$ and ( $v_{j}, s_{b}$ ) belong to one block of the partition $P$.

In particular, a partition $P$ on $K$ is compatible with the set $\{\pi, \theta\}$ if it is compatible with both $\pi$ and $\theta$, while it is compatible with set $\left\{\pi_{1}, \ldots, \pi_{\alpha}\right\}$ of partitions on $S$ (or set $\left\{\theta_{1}, \ldots, \theta_{\alpha}\right\}$ of partitions on $V$ ) iff it is compatible with $\pi=\pi_{1} \bullet \pi_{2} \bullet \pi_{3} \bullet \ldots \bullet \pi_{\alpha}\left(\theta=\theta_{1} \bullet \theta_{2} \bullet \theta_{3} \bullet \ldots \bullet \theta_{\alpha}\right)$.

## 3. ROM implementation of finite state machines

FSM can be implemented using ROM Read Only Memory) [10]. Figure 2 shows the general architecture of such an implementation. State and input variables $\left(q_{1}, q_{2}, \ldots, q_{n}\right.$ and $\left.x_{1}, x_{2}, \ldots, x_{m}\right)$ constitute ROM address variables $\left(a_{1}, a_{2}, \ldots, a_{m+n}\right)$. The ROM would consist of words, each storing the encoded present state (control field) and output values (information field). The next state would be determined by the input values and the present-state information feedback from memory.

This kind of implementation requires much fewer logic cells than the traditional flip-flop implementation (or does not require them at all, if memory can be
controlled by clock signal - no register required); therefore, it can be used to implement "non-vital" FSMs of the design, saving LC resources for more important sections of the design. However, a large FSM may require too much of buried memory resources.


## Fig. 2. Implementation of FSM using memory blocks

The size of the memory needed for such an implementation depends on the lengths of the address and memory word.

Let $m$ be the number of inputs, $n$ be the number of state encoding bits and $y$ be the number of output functions of FSM. The size of memory needed for implementation of such an FSM can be expressed by the following formula:
$M=2^{(m+n)} \times(n+y)$,
where $m+n$ is the size of the address, and $n+y$ is the size of the memory word.

Since modern programmable devices contain embedded memory blocks, there exists a possibility to implement FSM using these blocks. The size of the memory blocks available in programmable devices is limited, though. For example, Altera's FLEX family EAB (Embedded Array Block) has 2048 bits of memory and the device FLEX10K10 consists of 3 such EAB's. Functional decomposition can be used to implement FSMs that exceeded that size.

### 3.1. Address modifier

Any FSM, say $A$, defined by a given transition table can be implemented as in Fig. 3 using an address modifier.

If $\pi_{1}, \ldots, \pi_{n}$ are partitions on $S, \theta_{1}, \ldots, \theta_{m}$ are partitions on $V$, and $P_{k}$ is partition on $K$ compatible with either $\pi_{i}$ or $\theta_{j}$ then $P=\left\{P_{1}, \ldots, P_{m+n}\right\}$ is the set of all partitions compatible with $\left\{\pi_{1}, \ldots, \pi_{n}, \theta_{1}, \ldots, \theta_{m}\right\}$. Partitions $\pi_{1}, \ldots, \pi_{n}$ correspond to state variables and $\theta_{1}, \ldots, \theta_{m}$ correspond to input variables. To achieve unambiguous encoding of address variables, and at the same time maintaining the consistency relation $\boldsymbol{K}$ with the transition function $\delta$, partitions $P_{1}, . ., P_{w}$ have to be found, such that:
$P_{1} \bullet P_{2} \bullet P_{3} \bullet \ldots \bullet P_{w} \leq P_{c}$.

This is the necessary and sufficient condition for $\left\{P_{1}, \ldots, P_{w}\right\}$ to determine the address variables. This is because each memory cell is associated with a single block of $P_{c}$, i.e. with those elements from K which map the corresponding $(v, s)$ pairs onto the same next state.


Fig. 3. Implementation of FSM using an address modifier

The selection of $w(w<n+m)$ partitions from the set $\left\{P_{1}, \ldots, P_{m+n}\right\}$ is made such that they produce the simplest addressing unit. Such a selection is possible thanks to the notion of r -admissibility [9].

A set $\left\{P_{1}, \ldots, P_{k}\right\}$ is r -admissible in relation to partition $P$ iff there is a set $\left\{P_{k+1}, \ldots, P_{r}\right\}$ of two block partitions such that the following condition holds:
$P_{1} \bullet P_{2} \bullet P_{3} \bullet \ldots \bullet P_{k} \bullet P_{k+1} \bullet \ldots \bullet P_{r} \leq P_{c}$,
and no set of $r-k-1$ partitions exist which meets this requirement.

For partition $\rho \leq \sigma$ let $\sigma \mid \rho$ denote the quotient partition and $\varepsilon(\sigma \mid \rho)$ the number of elements in the largest block of $\sigma \mid \rho$. Let $e(\sigma \mid \rho)$ be the smallest integer equal to or larger than $\log _{2} \varepsilon(\sigma \rho)$ (i.e. $\left.e(\sigma \mid \rho)=\left\lceil\log _{2} \varepsilon(\sigma \mid \rho)\right\rceil\right)$. Then the r -admissibility of $\left\{P_{1}, \ldots, P_{\mathrm{k}}\right\}$ is $r=k+e\left(\pi \mid \pi_{\mathrm{f}}\right)$,
where $\pi$ is the product of $P_{1}, \ldots, P_{\mathrm{k}}$ and $\pi_{\mathrm{f}}$ is the product of $\pi$ and $P$.

If $\boldsymbol{P}=\left\{P_{1}, \ldots, P_{\mathrm{k}}\right\}$ is r -admissible in relation to $P$ then each subset of $\boldsymbol{P}$ is $\mathrm{r}^{\prime}$-admissible, where $r^{\prime} \leq r$.

The smallest partition i.e. one where each element is a separate block, will be denoted as $P(0), \pi(0), \theta(0)$, etc.

### 3.2. Input/state encoding

The source of the complexity of the address modifier is in the address variables which depend on more than one input/state variables. Therefore it is important to choose such an encoding of input and internal state symbols that we could obtain maximal set of partitions $P$ (compatible with $\pi$ or $\theta$ ) whose r -admissibility in relation to $P_{\mathrm{c}}$ is $w$, where $w$ is the number of address bits of the given ROM block.

Appropriate encoding will be determined by generating partitions with the knowledge that $r$ -
admissibility of a partition $P$ compatible with partition $\pi=\left(B_{1} ; \ldots ; B_{i} ; \ldots ; B_{\alpha}\right)$ or compatible with partition $\theta=\left(B_{1} ; \ldots ; B_{\mathrm{i}} ; \ldots ; B_{\alpha}\right)$ is:
$r=\left\lceil\log _{2} \alpha\right\rceil+\left\lceil\log _{2} \max \mid \delta\left(B_{\mathrm{i}}\right)\right\rceil$,
where $B_{\mathrm{i}}$ is a block of partition $\pi$ or $\theta, \delta$ is the transition function, max $\left\lceil\left(B_{\mathrm{i}}\right)\right\rceil$ denotes the number of elements in the most populous set $\delta\left(B_{i}\right), i \in\{1 \ldots \alpha\}$. Because of the one-to-one correspondence between partitions $P$ and $\pi$ or $\theta, \mathrm{r}$-admissibility of $\pi, \theta$ or $\{\pi, \theta\}$ in relation to $P_{\mathrm{c}}$ can be considered.

For a given $w$, the necessary encoding that allows the implementation of the FSM with the use of address modifier can be found in the following way:

1. find $r_{1}=\mathrm{r}$-admissibility of $\theta(0)$; find $r_{2}=\mathrm{r}$ admissibility of $\pi(0)$,
2. if $r_{1}=w$ (or $r_{2}=w$ ) then $a_{1}=x_{1}, \ldots, a_{\mathrm{x}}=x_{\mathrm{x}}$ (or $a_{1}=$ $q_{1}, \ldots, a_{\mathrm{x}}=q_{q}$ ) and further encoding partition are searched among $\pi(0)$ (or $\theta(0)$ ),
3. if both $\left.r_{1}\right\rangle w$ and $\left.r_{2}\right\rangle w$ then for subsequent steps $\theta$ if $|V|<|S|$ or $\pi$ if $|V|>|S|$ is taken,
4. assume that $\theta$ was chosen in the previous step; for $i=1,2, \ldots$ and $\alpha=2^{\mathrm{m}-\mathrm{i}}$ find $\theta=\left(B_{1} ; \ldots ; B_{\alpha}\right)$ so that $\left|B_{1}\right|+\left|B_{2}\right|+\ldots+\left|B_{\alpha}\right|=|V|$ and whose radmissibility equals $w$.
In a similar way $\pi=\left(D_{1} ; \ldots ; B_{\beta}\right), \beta=2^{2^{\mathrm{nj}}, j=1,2, \ldots}$ are found. The set $\{\pi, \theta\}$ must have r-admissibility of $w$. Partitions $\pi$ and $\theta$ can be represented as follows:
```
\pi= \mp@subsup{\pi}{1}{}\bullet\mp@subsup{\pi}{2}{}\bullet\ldots\bullet\mp@subsup{\pi}{\textrm{k}}{},
0=\mp@subsup{0}{1}{}\bullet\mp@subsup{0}{2}{}\bullet\ldots\bullet
```

where
$k=\left\lceil\log _{2} \beta\right\rceil$,
$l=\left\lceil\log _{2} \alpha\right\rceil$.
The encoding of the remaining input and state variables can be obtained from the following rules:
$\pi_{1} \bullet \pi_{2} \bullet \ldots \bullet \pi_{k} \bullet \pi^{\prime}=\pi(0)$,
$\theta_{1} \bullet \theta_{2} \bullet \ldots \bullet \theta_{l} \bullet \theta^{\prime}=\theta(0)$,
where $\pi$ ' and $\theta^{\prime}$ represent partitions induced by those variables.

After all the variables are encoded the process may be considered as a decomposition of the memory block into two blocks: a combinational address modifier and a smaller memory block. Decomposition is computed for partitions:

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\(P(A)=\pi \bullet \theta=\pi_{1} \bullet \pi_{2} \bullet \ldots \bullet \pi_{k} \bullet \theta_{1} \bullet \theta_{2} \bullet \ldots \bullet \theta_{l}\),
\(P(B)=\pi^{\prime} \bullet \theta^{\prime}\).
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Appropriately chosen strategy of decomposition may allow reducing required memory size at the cost of additional logic cells for address modifier implementation. This makes possible implementation of large FSMs that need more than available memory by making use of the embedded memory blocks and additional programmable logic.

Example: FSM implementation with concept of address modifier.

Let us consider FSM described in Table 1a. This FSM can be implemented using ROM memory with 5
addressing bits. This would require memory of size of 32 words. In order to implement this FSM machine in ROM with 4 addressing bits, the address modifier is required.

## Table 1. a) FSM table, b) transition mapping $K$

| a) | $v$ |  |  |  |  | $x_{1} x_{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | b) | 00 | 01 | 10 | 11 | $q_{2} q_{3}$ | $q_{1}$ |
|  |  | $v_{2}$ | $v_{3}$ | $v_{4}$ |  | $v_{1}$ | $v_{2}$ | $\nu_{3}$ | $v_{4}$ |  |  |
| $s_{1}$ | $S_{1}$ | $s_{2}$ | $s_{4}$ | - | $s_{1}$ | 1 | 2 | 3 | - | 00 |  |
| $s_{2}$ | - | - | $S_{5}$ | $s_{4}$ | $s_{2}$ | - | - | 4 | 5 | 01 | 0 |
| $s_{3}$ | $s_{3}$ | $s_{2}$ | $s_{1}$ | $s_{3}$ | $s_{4}$ | 6 | - | 7 | 8 | 10 |  |
| $s_{4}$ | $S_{2}$ | - | $s_{4}$ | $s_{1}$ | $s_{3}$ | 9 | 10 | 11 | 12 | 11 |  |
| $S_{5}$ | $S_{3}$ | $s_{1}$ | $s_{4}$ | $S_{2}$ | $s_{5}$ | 13 | 14 | 15 | 16 | 01 | 1 |

Let us implement given FSM in a structure shown on Fig. 3 with 3 free variable $(a=3)$ and one output variable from address modifier ( $c=1$ ). To find the appropriate state/input encoding and partitioning the FSM's state transition table (the next states are numbered with numbers) is divided into 8 subtables (encoded by free variables), each of them having no more then two different next states, that can be encoded with one variable - address modifier output variable (to achieve that, rows $s_{3}$ and $s_{4}$ changed places with each other). Next the appropriate state and input encoding is introduced (Tab. 1b). The partition based description of this process is given below.

$$
\begin{aligned}
& \text { Let } A=\left\{x_{1}, x_{2}, q_{1}\right\}, B=\left\{q_{2}, q_{3}\right\} \\
& P(A) \mid P_{F}=((1)(6) ;(2) ;(3,7)(4) ;(5)(8) ;(9,13) ; \\
& (10)(14) ;(11)(15) ;(12)(16)) \\
& P(B)=\overline{1,2,3} ; \overline{4,5,13,14,15,16} ; \overline{6,7,8} ; \overline{9,10,11,1}
\end{aligned}
$$

Following the serial functional decomposition method the partition $\Pi_{G}$ has to be computed. Detailed descriptions of the process can be found in [12].


The decomposition can not be constructed without adding to set $B$ variable $x_{1}$ that separates symbol 3 and allows constructing partition $\Pi_{G}$ that satisfied decomposition condition.
$B^{\prime}=\left\{x_{1}, q_{2}, q_{3}\right\}$
$P\left(B^{\prime}\right)=(\overline{1,2} ; \overline{3} ; \overline{13,14} ; \overline{4,5,15,16} ; \overline{6} ; \overline{7,8} ; \overline{9,10} ; \overline{11,12})$
Finally partition $\Pi_{G}$ is as follows:

$$
\Pi_{G}=(\overline{1,2,3,6,7,8,9,10,11,12} ; \overline{4,5,13,14,15,16})
$$

This allows to compute the truth table of address modifier. The structure of constructed implementation of given FSM is shown in Fig. 4.


Fig. 4. Implementation of FSM from table 1b using an address modifier

Such an implementation requires memory of size of 16 words and additional logic to implement address modifier.

## 4. Experimental results

The proposed method was applied to implement several examples from standard benchmark set in FLEX10K10 devices using ALTERA MAX+PlusII system. In Table 2 a comparison of different FSM implementation techniques are presented. In the column named ROM Implementation, the number of bits required to implement a given FSM using ROM is presented. FLEX10K10 device is equipped only with 3 EAB memory blocks each consisting of 2048 bits. Most of the presented FSM examples cannot be implemented in this device, because their implementations require much more memory resources than available. In the column called FF Implementation, the number of logic cells required to implement the given FSM in the "traditional" way using flip-flops is given. To describe the FSM for this kind of implementation, a special AHDL (Altera Hardware Description Language) construction was used. In the column under $A M$ implementation, the results of implementation of the given FSM using the concept of address modifier are presented. In this approach, the address modifier was implemented using logic cell resources and ROM was implemented in EAB blocks (Fig. 3). The number of logic cells and the number of memory bits are given in the table as results. It can be easily noticed that the application of decomposition improves the quality of ROM as well as flip -flop implementation.

The application of address modifier concept allows implementing FSM in such a way that only about $37 \%$ of logic cell resources required in flip-flop implementation and about $27 \%$ of memory resources required in ROM implementation is used. Application of address modifier concept allows implementing all the presented FSMs using available memory and additional parts (address modifier) implemented in CLBs.

In Table 3 results of implementation of several "real life" FSMs are presented. Following examples were used in the experiments:

- DESaut - the state machine used in DES algorithm implementation,
- 5B6B - the 5B-6B coder,
- count4 - 4 bit counter with COUNT UP, COUNT DOWN, HOLD, CLEAR and LOAD.
Each sequential machine was described by a transition table. The results for each method of implementation are presented using the number of logic cells and memory bits required (i.e. area of the circuit) and the maximal frequency of clock signal (i.e. speed of the circuit). The columns under the $F F_{-} M A X+$ PlusII heading present results obtained by the Altera MAX+PlusII system for the classical flip-flop implementation of FSM. The ROM columns provide the results of ROM implementation; the columns under $A M_{-} R O M$ present the results of ROM implementation with the use of address modifier. Especially interesting is the implementation of the 4 -bit counter. Its description with a transition table leads to a strongly non-optimal implementation. On the other hand, its description using a special AHDL construct produces very good results. The ROM implementation of this example requires too many memory bits (the size of required memory block exceeds the available memory), thus it cannot be implemented in the given structure. Application of the address modifier concept allows reducing the necessary size of memory, and that makes the implementation possible. The performance of the FSMs implemented with the use of address modifier concept is not significantly degraded.


## 5. Conclusions

Despite an acknowledged design productivity gap in which the number of available transistors grows faster than the ability to design them meaningfully, there are methods such as functional decomposition that allow to manage complex digital designs by applying well know paradigm "divide and conquer".

Balanced decomposition is recognized by most designers as very useful method for implementation of combinational functions using FPGA-based architectures. However, results presented in this paper show that functional decomposition can be efficiently and effectively applied beyond the implementation of combinational circuits. Decomposition can be applied to implement large FSM in an alternate way - using ROM. This kind of implementation requires much fewer logic cells than the traditional flip-flop implementation; therefore, it can be used to implement "non-vital" FSMs of the design, saving LC resources for more important sections of the design. However, large FSMs may require too much buried memory resources. With the concept of address modifier, memory usage can be significantly reduced. The experimental results shown in this paper demonstrate that the synthesis method based on functional decomposition can help in implementing sequential machines using ROM memory.

Table 2. Comparison of FSM implementation results of standard benchmarks in FPGA architecture (EPF10K10LC84-3 device ). ${ }^{\text {1) }}$ Implementation not possible - not enough memory resources, ${ }^{2}$ implementation not possible - not enough CLB resources

| Benchmark | ROM Implementation \#bits |  | FF implementation \#LCs |  | AM implementation <br> \#LCs <br> \#bits |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| bbtas | 160 |  | 10 |  | 7 | 80 |
| beecount | 448 |  | 32 |  | 14 | 112 |
| d14 | 512 |  | 60 |  | 21 | 256 |
| mc | 224 |  | 14 |  | 2 | 56 |
| lion9 | 320 |  | 24 |  | 1 | 80 |
| train11 | 320 |  | 25 |  | 15 | 8 |
| bbsse | 22528 | 1) | 52 |  | 3 | 5632 |
| cse | 22528 | 1) | 92 |  | 2 | 5632 |
| ex4 | 13312 | 1) | 28 |  | 2 | 3328 |
| mark1 | 10240 | 1) | 40 |  | 2 | 5120 |
| s1 | 24576 | 1) | 137 |  | 96 | 5632 |
| sse | 22528 | 1) | 52 |  | 3 | 5632 |
| tbk | 16384 | 1) | 759 | 2) | 333 | 4093 |
| s389 | 22528 | 1) | 64 |  | 9 | 5632 |
| $\Sigma$ | 156608 |  | 1389 |  | 510 | 41293 |
| \% | 100 | \% | 100 | \% | 36.7\% | 26.4\% |

Table 3. Comparison of FSM implementation results in FPGA architecture (EPF10K10LC84-3 device). ${ }^{\text {1) }}$ FSM described with special AHDL construction; ${ }^{2}$ decomposition not possible; ${ }^{3)}$ not enough memory bits to implement the project

| Example | FF_MAX+PlusII |  | ROM |  | AM_ROM |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LCs / Bits | Speed <br> $[M H z]$ | LCs / Bits | Speed <br> [MHz] | LCs / Bits | Speed <br> $[M H z]$ |
| DESaut | $46 / 0$ | 41,1 | $8 / 1792$ | 47,8 | $7 / 896$ | 47,1 |
| 5B6B | $93 / 0$ | 48,7 | $6 / 448$ | 48,0 | $-2)$ | $-2)$ |
| count4 | $72 / 0$ | $44,2^{2}$ | $16 / 16384$ | $-{ }^{3)}$ | $12 / 1024$ | 39,5 |

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