Development of hybrid one-dimensional finite element/analytical method for analysis of Lamb wave propagation characteristics in composite panels

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The objective is to develop a hybrid one-dimensional finite element (FE)/analytical method to analyze the Lamb Wave propagation characteristics in a composite panel for the purpose of detecting the specific structural defects in the composite panel. Here, it is proposed to use a composite panel using a finite element approximation in the thickness direction while wave solutions are assumed in the plane directions. Thus, it makes possible to use the small number of finite elements even for ultrasonic frequency analysis in computationally efficient manner. By solving the eigenvalue problem of the resulting hybrid FE/analytical matrix equation, the dispersion relations of the Lamb waves propagating in composite panels can be calculated in wave number/frequency domain. The resulting dispersion relations can be then used to determine the frequency, wavelength, and wave speed of a specific Lamb wave. In order to validate the proposed method, the Lamb wave speeds of a aluminum panel are calculated by using a conventional, analytical method and the proposed method. It is shown that the Lamb wave speeds obtained by using the proposed method agree well with the speeds calculated by using the analytical method.

1   INTRODUCTION

Multi-layered, composite panels have been widely used in structural applications due to their superior mechanical properties. However, it is difficult to predict their structural failures since structural defects in the composite panels, e.g., the breakages of reinforced fibers or the delaminations between composite layers, are not easily detected by conventional, structural health monitoring (SHM) methods that have been developed for metal structures.

Recently, SHM methods based on Lamb waves have been extensively studied by many researchers since a Lamb wave is propagating a long distance with a small spatial decay rate. The Lamb waves also make it possible to identify the types of structural defects: note that a Lamb wave with a specific mode shape in the thickness direction can be exclusively sensitive to a specific structural defect in a composite panel.

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Thus, in order to identify the locations, sizes, and types of structural defects in a composite panel, it is important to identify the characteristics of the Lamb waves propagating in the composite panel. In particular, the wave speed of a Lamb wave is one of the most important information required for SHM methods based on the Lamb waves.

Here, it is proposed to use a hybrid one-dimensional finite element/analytical method to analyze the Lamb wave propagation characteristics in composite panels. The proposed method uses a finite element (FE) approximation in the thickness direction while wave solutions are assumed in the plane direction. Thus, the proposed method makes it possible to easily model the complicated multi-layers of composite panels by using the one-dimensional finite elements. The model size is also much smaller than that of the corresponding two-dimensional or three-dimensional models.

2 THEORY

2.1 Hybrid One-Dimensional Finite Element/Analytical Method

A hybrid one-dimensional finite element/analytical method was first proposed to analyze the sound transmission characteristics of composite panels by Kim[1]: i.e.,

$$\mathbf{L}\{\mathbf{u}(x, y, t)\} + \mathbf{M}\frac{\partial^2 \mathbf{u}(x, y, t)}{\partial t^2} = \mathbf{f}(\mathbf{x}).$$ (1)

where $\mathbf{M}$ is the mass matrix. In Eq. (1), the linear operator, $\mathbf{L}$ is defined as

$$\mathbf{L} = \mathbf{K}_{xx}\frac{\partial^2}{\partial x^2} + \mathbf{K}_{xy}\frac{\partial^2}{\partial x\partial y} + \mathbf{K}_{yy}\frac{\partial^2}{\partial y^2} + \mathbf{K}_{xz}\frac{\partial}{\partial x} + \mathbf{K}_{yz}\frac{\partial}{\partial y} + \mathbf{K}_{zz}$$ (2)

where $\mathbf{K}$ represents a stiffness matrix. In Eq. (1), the nodal displacements are represented as

$$\mathbf{u}(x, y, t) = \begin{bmatrix} u_1 & v_1 & w_1 & \cdots & u_N & v_N & w_N \end{bmatrix}^{\mathrm{T}}$$ (3)

where u_n, v_n, w_n are the n-th nodal displacements in the x-, y-, and z-directions, respectively. Note that the nodal displacements are the functions of x, y, and t.

2.2 Lamb Wave Propagation in Composite Panels

For analyzing of the Lamb wave propagation characteristics of a composite panel, consider a plane strain case by neglecting the y-direction nodal displacements: i.e., $v_n = 0$ where $n = 1, 2, \ldots, N$. It is also considered that there is no external forces: i.e., $\mathbf{f} = \mathbf{0}$ in Eq. (1). The spatial derivative with respect to the y-direction can be also neglected in Eq. (2). Then, the displacement vector can be assumed as

$$\bar{\mathbf{u}}(x, y, t) = \bar{\mathbf{U}}\exp(ikx - i\omega t)$$ (4)

where the complex amplitude vector is defined as

$$\bar{\mathbf{U}} = \begin{bmatrix} U_1 & W_1 & U_2 & W_2 & \cdots & U_N & W_N \end{bmatrix}^{\mathrm{T}}.$$ (5)

From Eqs. (1) – (5), the resulting eigenvalue problem can be written as

$$\left(-\bar{\mathbf{K}}_{xx}k^2 + i\bar{\mathbf{K}}_{xz}k + \bar{\mathbf{K}}_{zz} - \omega^2\bar{\mathbf{M}}\right)\bar{\mathbf{U}} = \mathbf{0}.$$ (6)

For a nontrivial displacement vector, the determinant of the first term should be equal to zero: i.e.,

$$\det\left(-\bar{\mathbf{K}}_{xx}k^2 + i\bar{\mathbf{K}}_{xz}k + \bar{\mathbf{K}}_{zz} - \omega^2\mathbf{M}\right) = 0. \tag{7}$$

From Eq. (7), the dispersion relations (i.e., the relations between k and ω) of the Lamb waves propagating in the composite panel can be calculated. Then, the phase speeds of the Lamb waves can be obtained from the following equation:

$$c_p = \frac{\omega}{k}. \tag{8}$$

3   VALIDATION CASE

In order to validate the proposed hybrid method, the wave speeds of the Lamb waves propagating in an aluminum plate with the thickness of 2 mm is calculated by using both an analytical method and the proposed method. The analytical solutions are obtained that satisfy the following equations[2]:

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2 pq}{(q^2 - k^2)^2} \quad \text{for symmetric modes} \tag{9}$$

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{(q^2 - k^2)^2}{4k^2 pq} \quad \text{for antisymmetric modes} \tag{10}$$

where

$$p^2 = \left(\frac{\omega}{c_L}\right)^2 - k^2, \; q^2 = \left(\frac{\omega}{c_T}\right)^2 - k^2, \; c_L^2 = \frac{\lambda + 2G}{\rho}, \; c_T^2 = \frac{G}{\rho}, \; \text{and} \; \lambda = \frac{2Gv}{1 - 2v}.$$

In these equations, G is the shear modulus and v is the Poisson's ratio.

Table 1 shows the material properties of the aluminum panel. Figure 1 shows the resulting analytical solutions of Eqs. (9) and (10) where each black line represents the wave speed of a Lamb wave with a specific mode shape as the function of frequency, f multiplied by the thickness, d of the aluminum panel. Note that the left plot in Fig. 1 shows the wave speeds of symmetric modes in the thickness direction while the right plot represents the wave speeds of antisymmetric modes. Figure 2 shows the wave speeds of the Lamb waves that are calculated by using the proposed method. When the wave speeds of the symmetric and antisymmetric modes in Fig. 1 are compared with the wave speeds in Fig. 2, the wave speeds of the two methods agree well with each other.

4   CONCLUSIONS

In this article, a hybrid one-dimensional finite element/analytical method is introduced to analyze the Lamb wave propagation characteristics in composite panels. Through the comparison between the Lamb wave speeds of an aluminum panel calculated by using an analytical method and the proposed method, it is shown that the Lamb wave speeds calculated by using the proposed method agree well with the analytical ones. In the near future, the Lamb wave propagation characteristics of various composite panels will be analyzed by using the hybrid one-dimensional/analytical models and the resulting characteristics will be compared with experimentally measured characteristics.

REFERENCES
1. Yong-Joe Kim, "Identification of sound transmission characteristics of honeycomb sandwich panels using hybrid analytical/one-dimensional finite element method," Proceedings of Inter-Noise 2006, Honolulu, Hawaii, 2006.
2. Joseph L. Rose, *Ultrasonic Waves in Solid Media*, Cambridge University Press, 1999.

Table 1 - Material properties of aluminum panel.

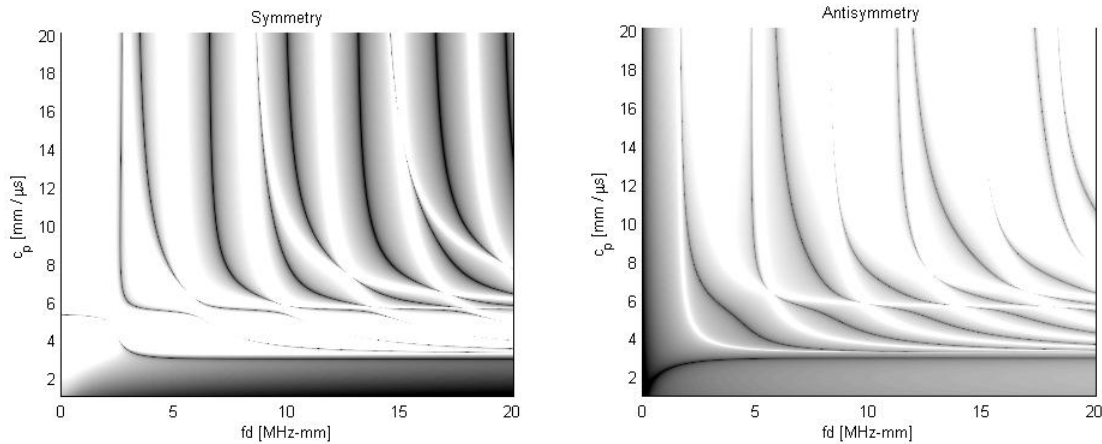| Material Properties	Values
Young's modulus, E (Pa)	7.1×10^{10}
Density, ρ (kg/m^3)	2700.0
Poisson's ratio, v	0.2396

Fig. 1 - *Analytical Lamb wave speeds (in black lines) calculated from Equations (9) and (10): wave speeds, c_p of symmetric modes (left) and wave speeds, c_p of antisymmetric modes (right) represented as the function of frequency, f multiplied by the thickness, d of aluminum plate.*
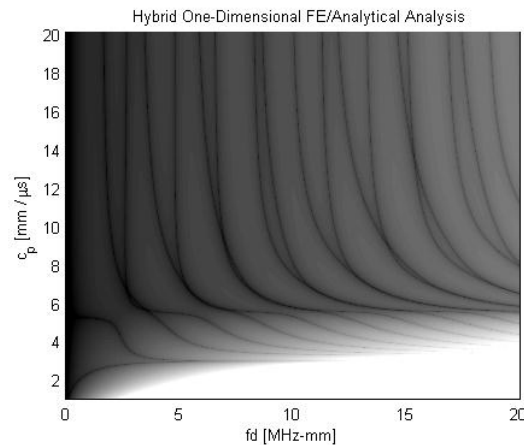

Fig. 2 - *Lamb wave speeds (in black lines) calculated by using the proposed method: wave speeds of both symmetric and antisymmetric modes are shown in the same figure as the function of frequency, f multiplied by the thickness, d of aluminum plate.*