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WAVELET ANALYSIS OF THE 3-D WAKE TOPOLOGY FOR A SQUARE PRISM

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ABSTRACT

The three-dimensional flow topology of square prism intermediate wake flow based on multi-point hot-wire measurements is investigated using wavelet analysis.

A number of continuous wavelet techniques combining time and scale analysis are applied systematically to detect the primary and the secondary flow structures and to validate previously proposed topological models.

Our results suggest that a preferred alternate rib spanwise arrangement similar to the one proposed by Meiburg and Lasheras [1] is plausible for the vortex topology of high Reynolds number square prism intermediate wakes. Moreover, it seems that the wake topology has a preferred spanwise wavelength of approximately $2b = 2.4d$, which gives a relative spanwise/ streamwise wavelength ($2b/\lambda$) of 2/5.

Keywords: wake topology, wavelets, hot-wires

INTRODUCTION

Turbulent wake topologies

A flow topology incorporates the dominant flow structures. It is agreed that there are two types of dominant structures in the high Reynolds number intermediate wakes: primary structures represented by spanwise Karman vortices, also known as rolls, and secondary structures represented by streamwise vortices, also known as ribs [1-7]. It is also accepted that the ribs appear as pairs of counter-rotating vortices aligned with the diverging separatrix in the braid region between consecutive rolls. Despite numerous previous investigations [1-7], the rib-roll interaction and interconnection is still unclear. For example it is not clear if the ribs are interconnecting rolls with different vorticity signs or rolls with the same signs. Also it is unclear if there is a preferred spanwise spacing between pairs of ribs. These questions are addressed herein.

The first quantitative description of the intermediate wake topology is attributed to the Huston group [2,3]. Using a vorticity based eduction scheme they proposed two possible wake topologies for intermediate wakes with rolls distortions staggered and non- staggered with respect to each other in the spanwise direction. The roll distortions are outwards kinkings and the ribs are interconnecting rolls with the same vorticity signs.

Kiya and Masumura [4] using Fourier analysis found that the most significant contribution to the incoherent Reynolds shearing stress in the saddle region comes from fluctuations with frequencies around one-half of the shedding frequency. They concluded that from the two models proposed by Hussain and Hayakawa [2] only the model with staggered ribs and rollers is in accordance with their results.

Zhou and Antonia [5] based on critical point analysis applied to simultaneous cross hotwires data from two orthogonal planes showed that foci in the horizontal plane correspond to saddles in the vertical plane. This indicates that vortices, found in the horizontal plane between two consecutive rolls, correspond to rib-like structures

Based on pattern recognition analysis and using an ensemble averaged fine scale turbulence indicator function to correlate measurements from two orthogonal planes, Huang et all [6] found evidence of the counter-rotating rib structure in braid region of the intermediate wake of a mesh strip.

Relevant results are given by Lasheras and Choi [7] for mixing layer based on flow visualization and their numerical-experimental follow-up by Meiburg and Lasheras [1] for wake flow. They investigated the evolution of the perturbations in the wake generated by two spanwise distorted flat plates (one indented and the second one corrugated). The two configurations have two different wake topologies with a spanwise wavelength equal to the spanwise wavelength of the perturbations. Both topologies have ribs that interconnect rolls from both sides of the wake and are interestingly similar to the two natural modes (A and B) found in Williamson's [8] laminar circular cylinder wake flow visualizations and confirmed later

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by numerical (Barkley and Henderson [9]) and experimental (Brede et all [10]) investigations.

To our best knowledge the only published square prism wake topology analysis was done by Robichaux et all [11] for laminar wake. In their numerical investigation based on Floquet analysis they found the same natural modes as those corresponding to the circular cylinders that strengthen the idea of the similarity between the two wake configurations.

The objective of this paper, considering the importance of square prism in engineering applications, is the investigation of the three-dimensional topology model for square prism intermediate wake at high Reynolds number.

Wavelets analysis

The application of wavelet analysis in the field of fluid mechanics started with the work of Farge [12]. Since then numerous papers have been published indicating its success as an efficient fluid mechanics mathematical tool. The main advantage of wavelet analysis consists in its capability to perform a time-scale analysis. This approach reconciles the spectral and time- space turbulence theories previously considered as separate research investigations.

Broadly, two main directions can be identified in wavelet theory applied to turbulence publications: 1) Wavelet fundamental turbulence research, which includes the Navier-Stokes equations formulation in wavelet space and their numerical solutions. 2) Wavelet as a post-processing tool, which includes direct analysis of the turbulence characteristics from numerical or experimental data. The present investigation is part of the second direction. Previous applications of wavelet analysis in wakes have been mostly restricted to pure two-dimensional velocity data in vertical plane (Higuchi et all [13]). Based on these data, important primary large -scale vortex structures insights have been found but no information regarding the secondary structures were provided.

Herein, the square prism wake topology is investigated based on wavelet analysis applied to experimental hot-wire three-dimensional velocity field data.

EXPERIMENTAL CONFIGURATIONS

Experiments were carried out in the large close-return wind tunnel (BLWT II) at the Boundary Layer Wind Tunnel Laboratory, the University of Western Ontario. A square prism with section 30mm by 30mm was horizontally installed approximately at the wind tunnel mid-plane between two end plates situated 120 cm apart. This gave a negligible blockage of 2% and an aspect ratio (radio of square prism length to its side length) of 40. The upstream velocity was 11 m/s and the longitudinal, lateral and span-wise turbulence intensities were about 0.17%, 0.24%, and 0.35% respectively. The corresponding Reynolds number based on square edge and on free stream velocity was 22,000 (see Fig. 1).

Velocity measurements were taken using multi- hotwires in the intermediate wake of the square prism at 26 diameters downstream of the wake generator where the turbulence intensity was below 20%. The hot- wire probes were connected to DANTEC 90C10 constant temperature anemometers. The probes were calibrated via Dantec calibration system 90H10.

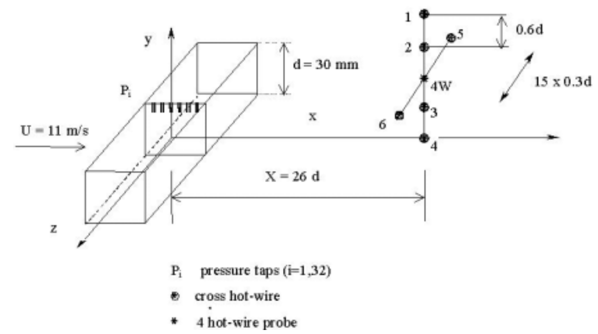

Figure 1: The experimental setup C

Three experimental set-ups have been used. For all configurations signals were sampled at 6 kHz gaving approximately 120 points per vortex shedding period and allowed recording of at least 2,500 periods of vortex shedding.

For the first experimental set-up (A), vertical (y) velocity profiles were taken using two cross- hot wire probes, one measuring u and v velocity components and the other measuring u and w velocity components.

The second experimental set-up (B), designated for cross correlation calculations, consisted of two cross-hot wire probes. Both wires were placed in the horizontal mid wake plane; one probe was kept fixed, while the other probe was traversed in the spanwise (z) direction.

In the third experimental setup (C) (Fig. 1) a hot wire cross-holder with a four-wire probe (4W) in the centre and other 6 cross-wire probes distributed in the lateral (Oy) and in spanwise (Oz) directions (Fig.1) has been employed. Measurements were taken at 15 spanwise stations with a step $0.3d$. The cross hot-wire probes 1, 2, 3, 4 measured the longitudinal and lateral velocity components (u and v) while the cross hot-wires 5 and 6 measured the longitudinal and spanwise velocity components (u and w). The 4-hot-wire probe provided all three-velocity components. The distance between probes in both the vertical and the horizontal directions was $0.6d$.

RESULTS

A systematic wavelet based investigation of wake topology is proposed. The flowchart of the investigation starts with the choice for a proper wavelet, considering the importance of this step in the wavelet analysis. Afterwards the most energetic scales are determined knowing that they are associated with the most dominant coherent structures. The lateral extent of these most energetic scales is estimated and the identification of the local events that are responsible for the general properties found

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in the previous steps is performed. At the end, a three-dimensional flow topology is proposed and the agreement with other proposed wake topologies is discussed.

Wavelet choice

In this work one -dimensional Marr wavelet, also known as Mexican-hat, continuous and real-valued wavelet-type is used. The formula of the mother Marr wavelet is:

$$\Psi(x) = (1 - x^2) \cdot e^{-\frac{x^2}{2}} \qquad (1)$$

The wavelet transform is defined as

$$\tilde{f}(l,t) = \frac{1}{\sqrt{l}} \cdot \int_{-\infty}^{+\infty} f(t') \cdot \Psi\left(\frac{t'-t}{l}\right) \cdot dt' \qquad (2)$$

where l is the wavelet scale and t is the time.

In this paper wavelet coefficients are divided by the maximum wavelet map coefficient, thus:

$$\bar{\tilde{f}}(l,t) = \frac{\tilde{f}(l,t)}{\max\limits_{l,t}\left|\tilde{f}(l,t)\right|} \qquad (3)$$

The correspondence between wavelet scale (l) and classical Fourier frequency (f_F) is:

$$f_F = \frac{\omega_0}{l} \cdot f_{sampl} \qquad (4)$$

where, ω_0 is a constant depending on the wavelet function and f_{sampl} is the sample frequency.

For Marr wavelet, ω_0 is:

$$\omega_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{5}{2}} \qquad (5)$$

Since the analyzed data are time series, one-dimensional wavelet functions are used. Due to the poor spatial resolution imposed by the experimental limitations, the use of higher order wavelet analysis even for simultaneous multi hotwire data is inadequate.

The choice of a continuous wavelet is due to its unfolding of the flow information on the space-scale complete grid, as opposed to the dyadic grid used for orthogonal wavelets (Farge [12]), therefore allowing for a better event definition.

The use of the real-valued wavelets as opposed to the complex-valued wavelets is debatable in turbulence wavelet analysis literature. Some researchers highly recommend the use of complex- valued wavelets (Farge [12]) to avoid spurious oscillations of wavelet coefficients that will make their graphical representation difficult to interpret. However, this is valid especially for higher dimension analysis, where the amount of data can be significantly high. In the present investigation, one-dimensional wavelets are applied and the use of real-valued data is not likely to produce difficulties in interpreting the data. Another debate in choosing real or complex valued wavelets refers to the advantage of using complex valued wavelets due to the importance of information contained in the phase of complex valued wavelet. Though, Meneveau [14] indicated that the phase information is revealed also in the case of real valued wavelet by the translation of the wavelet itself.

Moreover, Marr real wavelet has been successfully applied in fluid mechanics due to its high capabilities to localize flow related events associated with signal maxima and minima [15,16].

Energy dominant scales. Wavelet power spectrum.

The dominant flow scales are associated with peaks in the power spectra density function. Wavelet- mean power spectra are evaluated for all three-velocity components along the half wake width based on experimental setup A. An advantage of the wavelet power spectra over the Fourier one in the identification of the most energetic scales in the flow is that the former is free of harmonics and free of oscillations that could be misleading in identifying the peak or the slope of the power spectra [13,17].

The expression for the wavelet mean power spectra at coordinate y is:

$$E(l,y) = f_F \cdot \frac{1}{2 \cdot c_\Psi} \int_0^T \left|\tilde{f}(l.t,y)\right|^2 dt \qquad (6)$$

where T is the time size of the signal and c_Ψ is a wavelet dependent energy constant that should be finite for the wavelet to be admissible and f_F is given by Eq.4.

The distribution of the most energetic scales along the wake width can be visualized easily if the wavelet- mean power spectrum is divided at each coordinate y by the energy of the most energetic scale at that position:

$$\bar{E}(l,y) = \frac{E(l,y)}{\max\limits_{l}(E(l,y))} \qquad (7)$$

The 1% most energetic scales given by Eq. 7 are evaluated for all velocity components (u, v, w) along the half- wake width. The most energetic scales for u and v velocity components are found to correspond to the frequency of vortex shedding for all y coordinates. This is a clear signature of the primary structures, which are the most energetic flow events in the vertical plane.

Figure 2 shows a very interesting result for the w spanwise velocity. In this figure the 1% most energetic scales at the wake mid- plane are delimited by the two curves shown. They correspond to approximately double the shedding frequency indicating a shift towards the larger scales for higher y coordinates and a tendency to move back towards lower scales close to the wake upper edge. (Note that f_s is the shedding frequency and f is evaluated based on Eq. 4.)

The most energetic scale associated to the double of shedding frequency towards the wake mid-plane can be

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explained by the presence of the secondary structures (ribs) -the only y oriented vortices at this location. If this phenomenon would be a roller- type signature, it would also be seen in the u wavelet mean power spectrum. This is however not the case and no particular changes close to the wake mid- plane have been observed. We assume that the shift towards the lower scales at the position corresponding to the roll is the signature of the roll distortions that gives a lateral (y) vorticity component. The second shift of the most energetic scales close to the wake edge needs further clarifications.

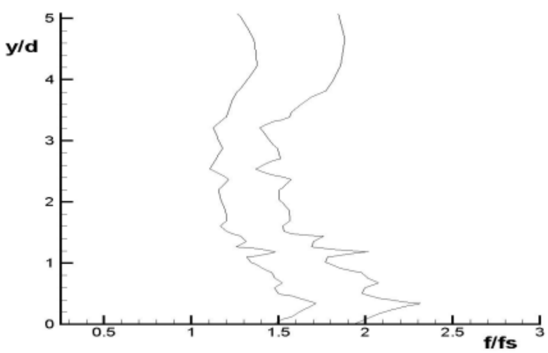

Figure 2: The distribution of the most energetic 1% scales of the w velocity component (area bordered by the two energy lines) along the wake width

It is worth noting that Kiya and Matsumura [4] also found a most energetic double frequency of shedding scale in the w velocity component in a normal flat plate wake, in agreement with our results.

Cross correlation analysis

The scale analysis based on the one point measurements such as those performed for the experimental setup A indicates certain time characteristics of the flow or, if the Taylor hypothesis is considered, streamwise (x) characteristics of the flow topology but they offer no information in regard to the lateral extend of these characteristics. If the vertical extend of primary vortices it is known to be comparable to the half-wake width, no information is available for the spanwise extend of secondary structures. To overcome this problem a spanwise correlation analysis is performed based on experimental setup B. A classical cross correlation analysis is applied to the u and w velocity component and the spanwise integral length scale is evaluated by integrating the cross correlation function. This is found to be approximately 1.2d.

Wavelet pattern recognition

The most energetic events of a lateral extent estimated in previous sections are analyzed based on the three-dimensional simultaneous velocity field measurements provided by the experimental setup C in two orthogonal planes.

The identification process is based on the Wavelet Pattern Recognition (WPR) technique developed by Hangan [18]. This technique that performs pattern recognition in wavelet space has the advantage of an objective initial template (based on maxima lines)and it allows for the detection of multi-scale flow structures. When the method is applied to the extraction of the flow structures with a predominant scale (frequency), as in the present work, the method becomes similar to the phase averaging technique with the advantage of having a build-in local wavelet filtering. It is known that a non-localized Fourier filter that is applied to the signal prior to the application of phase averaging technique may introduce phase shift problems for significant non-stationary signals.

The WPR is applied in both the vertical and the horizontal planes using as triggers the lateral v and the spanwise w velocity components respectively provided by the 4-wire probe (see Fig. 1).

The initial wavelet templates are shown in fig. 3a and 4a for the vertical (xy) and horizontal (xz) planes respectively. It can be noticed that in the vertical plane (Fig. 3a) the wavelet coefficients are centred at the shedding frequency, while in horizontal plane they are centred at the double shedding frequency in accordance with the results from previous sections. The widths in x/d of the two templates of the two templates are equal to the shedding period allowing us to identify the “topology cell “ of the flow. The initial templates were chosen after careful investigations of the instantaneous wavelet maps.

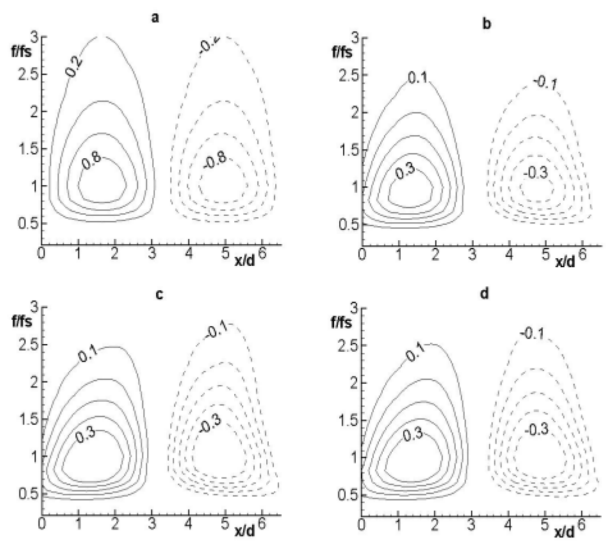

Figure 3: WPR templates in wavelet space for the vertical plane based on v signal a) initial template b) final template for probe 1 c) final template for probe 4W d) final template for probe 4

The WPR threshold is set to 1.5 of the standard deviation of the correlation function between template and the signal. The final templates shown in fig 3 and 4 are the result of window averaging of 37% of the flow in horizontal plane and 33% of

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the flow in the vertical plane. The relative high probability of appearance of investigated events combined with the use of a relative high threshold indicates that the analyzed events are statistically significant for the wake flow. It is also interesting that the secondary structures seen in the horizontal plane have almost the same probability of appearance as the primary structures from the vertical plane. Therefore, the two structures could play an equally important role in the wake topology.

For a better physical interpretation, the ensemble- averaged events are also presented for the physical (velocity) field. In the vertical plane (Fig. 5 upper) the alternation of rolls as well as position of foci and saddle points are in agreement with other experimental results (Zhou and Antonia [19]). An interestingly similar picture is obtained in the horizontal plane (Fig. 5 lower) but with structures having approximate half the scale of those from the vertical plane also in agreement with the results of Hayakawa and Hussain [3].

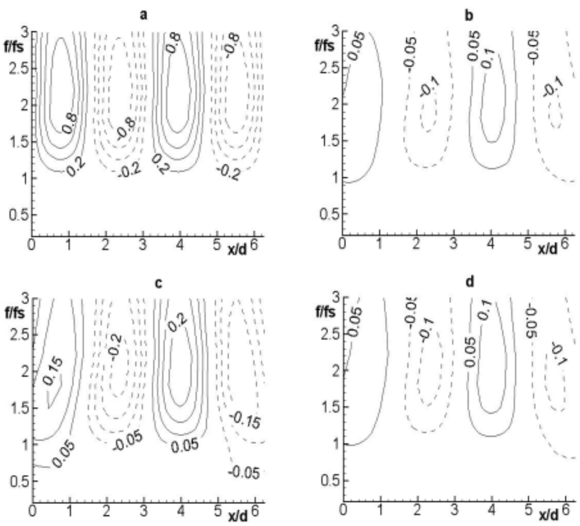

Figure 4: WPR templates in wavelet space for the horizontal plane based on w signal a) initial template b) final template for probe 5 c) final template for probe 4W d) final template for probe 6

Based on the above results we propose a three-dimensional vortex topology model that includes the most energetic scales from both planes (Fig 5). (Note that the thick horizontal lines represent the intersection line of the two planes of measurement.) This model has Karman rolls aligned in spanwise direction and ribs that are interconnecting rolls from both wake sides. In the horizontal (xz) plane the ribs appear as alternating vortices similar to the Karman vortices from the vertical plane with the distance between row vortices of different signs of approximately 1.2d. Similar features are found for all 15 spanwise positions analyzed. It may be inferred that a spanwise wavelength of $2.4d$ which gives a relative spanwise wavelength ($2b/\lambda$) of $2/5$ characterizes the wake topology. Two ribs are cutting the horizontal mid-wake plane in one shedding period explaining the double frequency of vortex shedding most energetic scale in w velocity component. This topology is therefore in agreement with our results regarding the most energetic scales, correlation analysis and the wavelet pattern recognition.

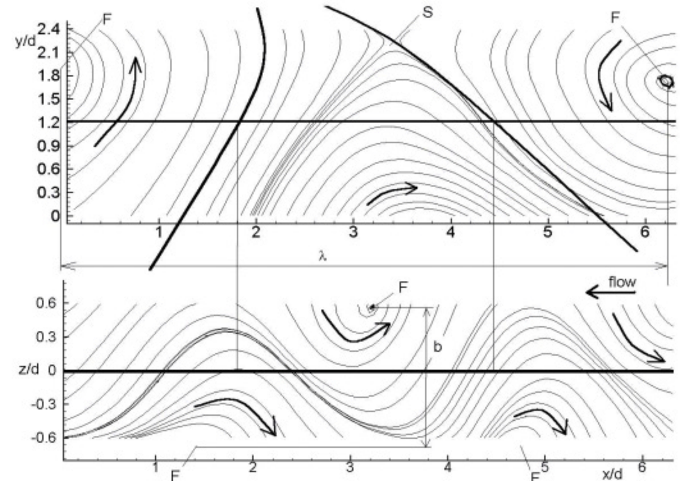

Figure 5: The final WPR templates in physical space and the proposed vortex topology

The present model is in good agreement with the wake model proposed by Meiburg and Lasheras [1] for the case of an indented flat plate. The only difference is the alternation of the ribs in streamwise direction. Same comments are valid for the comparison with the Williamson [8] and Robichaux [11] mode A topology since this is similar to the indented flat plate topology of Meiburg and Lasheras [1] as already indicated. The spanwise wavelength found in our investigation (2.4d) is nearly half of the one found by Robichaux et all (5.22d) [11] for the laminar near wake mode A.

CONCLUSIONS

Wavelet analysis is applied to simultaneous multipoint hotwire experiments to provide better 3-D understanding of the intermediate high Reynolds number square prism wake topology.

Based on a systematic wavelet analysis it may be inferred that an arrangement of ribs that interconnect consecutive rollers with opposite vorticity sign is plausible for the intermediate high Reynolds number square prism wake. The proposed wake topology model is similar to the Meiburg and Lasheras [1] wake topology model for the indented flat plate and also to the Williamson's mode A [8] laminar wake flow topology. Though, in the horizontal plane our results indicate that vortices are alternating as in the Karman vortex street.

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The relative spanwise wavelength ($2b/\lambda = 2/5$) is found to be lower than the spanwise wavelength of the Robicheaux [11] square prism laminar wake mode A.

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