Experimental Verification of a Control Algorithm for Nonlinear Systems

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NOMENCLATURE

f	Frequency, $[Hz]$
Z	Desired force amplitude
k	Number of harmonics to be controlled
n	Iteration counter
t	Time, $[\sec]$
V_N	Input voltage vector with sine and cosine components for each harmonic

ABSTRACT

When using electrodynamic vibration exciters to excite structures, the actual force applied to the structure under test is the reaction force between the exciter and the structure. The magnitude and phase of the reaction force is dependent upon the characteristics of the structure and exciter. Therefore the quality of the reaction force i.e. the force applied on the structure depends on the relationship between the exciter and structure under test.

Looking at the signal from the force transducer when exciting a structure with a sine wave, the signal will appear harmonically distorted within the regions of the resonance frequencies. This phenomenon is easily observed when performing tests on lightly damped structures. The harmonic distortion is a result of nonlinearities produced by the shaker when undergoing large-amplitude vibrations, at resonances.

When dealing with non-linear structures, it's of great importance to be able to keep a constant force level as well as a non-distorted sine wave in order to get reliable results within the regions of the resonance frequencies.

This paper presents the method and results from an experimental test creating a nondistorted excitation signal with constant force level.

1. INTRODUCTION

When estimating the properties of linear mechanical structures, there are several well established methods to use when performing the measurements. The structure is typically excited by using an impulse hammer or a shaker. These methods can be divided into different groups depending on the type of excitation signal used.

Measurements with an impulse hammer or a shaker with random noise signal is usually called broadband excitation, i.e. the excitation signal contains a wide range of frequencies covering the entire measurement range. An example of another type of excitation method is stepped-sine excitation, which instead of exciting a wide frequency band concentrates at a single frequency at a time. At each frequency all signals are measured, amplitude and phase relationships between each output and input are then determined.

When measurements on linear structures of structures with a small and negligible nonlinearity are done, random signals may be useful since this type of signals tend to smooth out the nonlinearity and give the best linear fit. However, if we want to identify the nonlinearity, random signals are less useful. By comparing transfer functions, of a nonlinear system, from measurements with both random and sine sweep excitation to a theoretical transfer function calculated using the harmonic balance method. This problem is further illustrated, see figure 1.1 and 1.2.

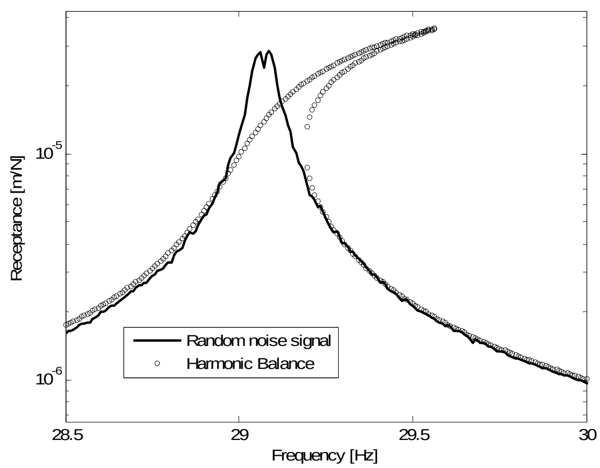

Figure 1.1. Transfer function of a nonlinear SDOF system obtained by random noise excitation and compared with a theoretical transfer function calculated with harmonic balance. The same RMS value of the force is used to calculate both transfer functions.

A shown in figure 1.1, the transfer function calculated using random excitation is unable to describe the true resonance peak. What we get instead is a rough approximation of the underlying linear system. By studying the disturbance of the transfer function as well as the coherence function it is possible to notice the presence of a nonlinearity, but difficult to describe it.

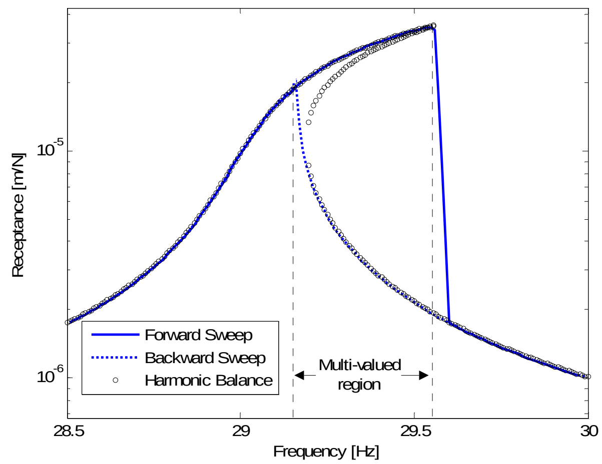

Figure 1.2. Transfer functions from both forward and backward sine sweeps compared to the theoretical transfer function obtained with the harmonic balance method.

In figure 1.2 the transfer function from both a forward and a backward sine sweep excitation is shown. Within the multi-valued region the transfer function has three real solutions. This is why the transfer function of the forward sweep differs from the transfer function of the backward sweep and also the reason for the sudden jumps seen in figure 1.2.

Based on the forward and backward sweep it's possible to get a good description of the transfer function of the nonlinear system. Another benefit when using sinusoidal excitation signals are that the force amplitude can be set in a more controlled manner. This is important since the effects of the nonlinearity are force dependent. Therefore it's preferred to use sinusoidal excitation when dealing with nonlinear systems. Also, most of the methods developed for identifying and characterizing nonlinearities in complex systems are based on sinusoidal excitation.

In simulations there's no problem to get a perfect sinusoidal excitation signal. But when measurements are made on real structures, the excitation signal gets distorted for different reasons. If the structure has a strong nonlinear behavior, the response signal will contain higher harmonics. These harmonics are also transferred to the force signal, since the force applied to the structure under test is the reaction force between the exciter and the structure itself. This problem is further amplified due to the shakers nonlinear behavior at large displacements, when the shakers coil is moving in the nonlinear parts of the magnetic field. Therefore, higher harmonics can be observed in the force signal even when exciting linear systems, particularly lightly damped systems experiencing large-amplitude vibrations around its resonance frequencies. Figure 1.3 shows the layout of a typical measurement system, the system consists of a signal generator, amplifier, shaker and a force sensor.

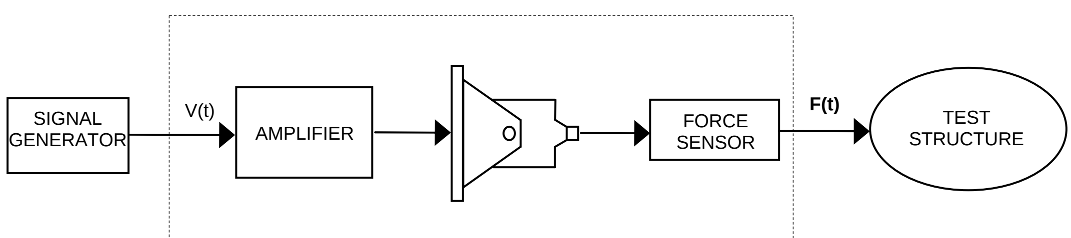

Figure 1.3 Layout of a typical measurement system, an amplified voltage signal is sent to the shaker and a sensor is used to measure the force applied on the structure.

A principal model which shows what happens when dealing with nonlinear systems is shown in figure 1.4, a nondistorted sinusoidal signal is sent through an unknown nonlinear system and the output becomes a distorted signal containing higher harmonics. The unknown nonlinearity depends on the shaker, the interaction between the shaker and the structure as well as the nonlinearity in the structure.

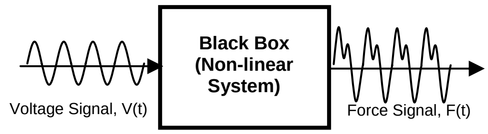

Figure 1.4. The dotted frame in figure 1.3 is treated as a black box. A harmonic voltage signal excites the nonlinear system, the response is a distorted force signal.

This paper will describe a control algorithm for sinusoidal excitation of nonlinear systems and the results of measurements done using this algorithm.

2. THE FORCE CONTROL ALGORITHM

The basic concept used in this work, based on methods developed by Bucher, Ewins et al [1,2,3], is to send a distorted voltage signal into the system. The phase and amplitude of the frequency content in the voltage signal should be chosen in a way so that the resultant force, the output of the nonlinear system, only consists of the desired fundamental frequency. Since there is an unknown nonlinear relationship between the input and output signal, see figure 1.4, an iterative procedure is required to find the desired input signal.

In this case two iteration schemes have been used to solve the problem.

- A standard Newton-Raphson solver.
- A quasi-newton solver, Broyden's method.

The main difference between these two methods is that the quasi-newton method avoids the direct computation of the Jacobian matrix. On the other hand, the quasi-newton methods only have linear convergence towards the correct solution. Both these methods need an initial guess close to the correct solution in order to converge. Since it's not always sure that the initial guess is close enough for convergence and measured signals can be contaminated with noise, a global convergence strategy is implemented in the solution process. The idea of this strategy is to reduce the step size to make sure that every step taken is in the right direction.

The procedure which the control algorithm follows is explained below and a flowchart of the control algorithm is shown in figure 2.1.

Step 1: Select the desired force amplitude and starting frequency of the force signal, Z_d .

Set initial amplitude on the voltage signal, V_{start}

Step 2: Find V that will give $Z_d \approx Z_M$

Step 3: Find relevant peaks (k) in the force spectrum. Define the input voltage vector V_I .

Step 4: Determine the Jacobian matrix.

Step 5: Make an iteration to calculate a new voltage vector V_I .

Step 6: Repeat step 4 and 5 until the solution has converged (F_{CRIT} is fulfilled).

Step 7: Check for new harmonics, if new harmonics has become relevant, repeat step 3 to 5.

Step 8: Increment the frequency and start from step 2.

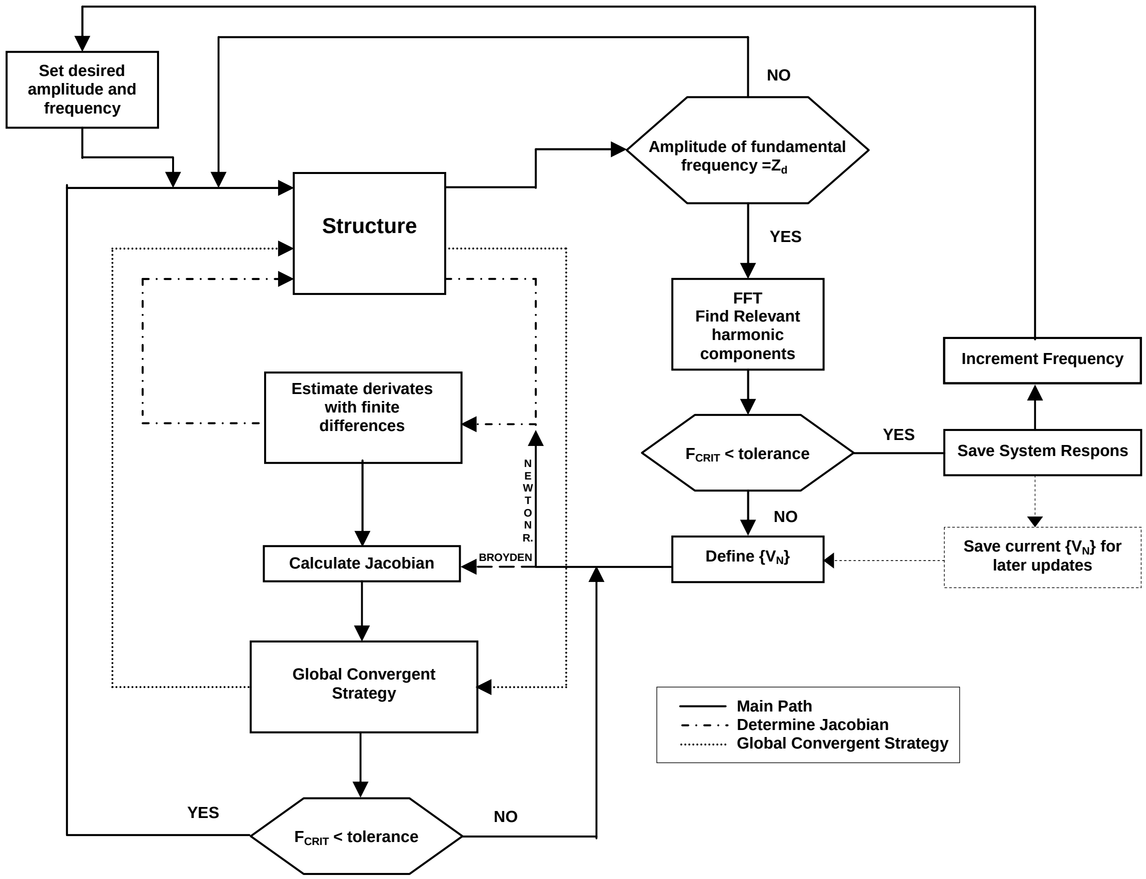

Figure 2.1 Flowchart of the force control algorithm.

For a more detailed description of the force control algorithm, see Josefsson, Magnevall [4].

3. MEASUREMENTS

The force control algorithm described in the previous chapter has shown good results in simulations. In this chapter the algorithm will be tested in real measurements, both the Newton-Raphson and the Broyden solver has been tested.

The DAQ hardware used is National Instruments DAQ 6062 E, which is a 12 bit PCMCIA card. The software used for data acquisition is the Data Acquisition Toolbox in MATLAB® [5]. Acceleration and Force are measured on the test object. Figure 3.1 shows a schematic picture of the measurement system setup.

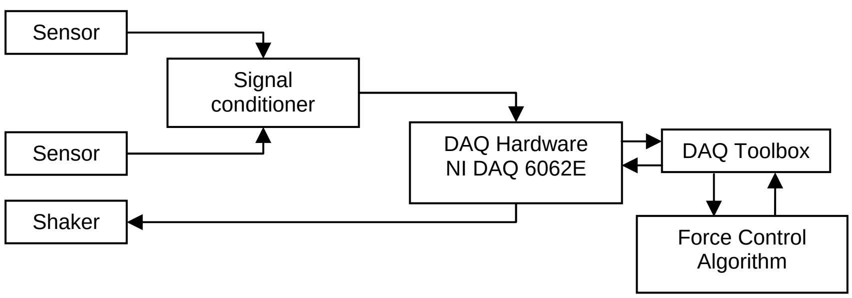

Figure 3.1. A schematic picture of the measurement system setup.

The test object chosen is a lightly damped linear structure. As explained in the introduction, nonlinearities also appear in linear structures experiencing large-amplitude vibrations. This mainly occurs around the systems resonance frequencies. The nonlinearities in these measurements are mainly due to the fact that the coil of the shaker is moving in the nonlinear region of the magnetic field and because of the shakers inability to keep a constant force level due to the large vibrations in the regions of the resonance frequencies, so called force dropouts. As an initial test of the force control algorithm, this type of nonlinear behavior was chosen to be controlled. A picture of the test setup is shown in figure 3.2.

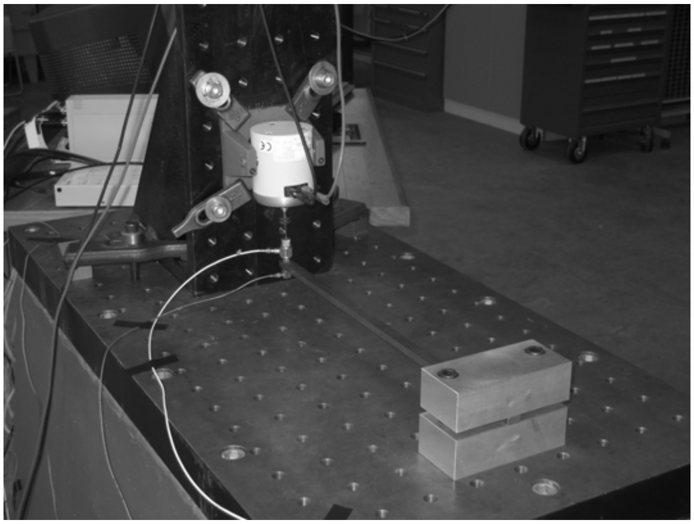

Figure 3.2. The test setup used in the experiments, a cantilever beam. Point accelerance is measured in its free end.

Measurements with sine sweep and random excitation signals were made to get an estimate of where the first resonance frequency was located. To avoid force dropouts in the area around the resonance frequency more energy were added to the sine sweep signal in that area. Figure 3.3 shows the envelope of a typical sine sweep signal used in the measurements.

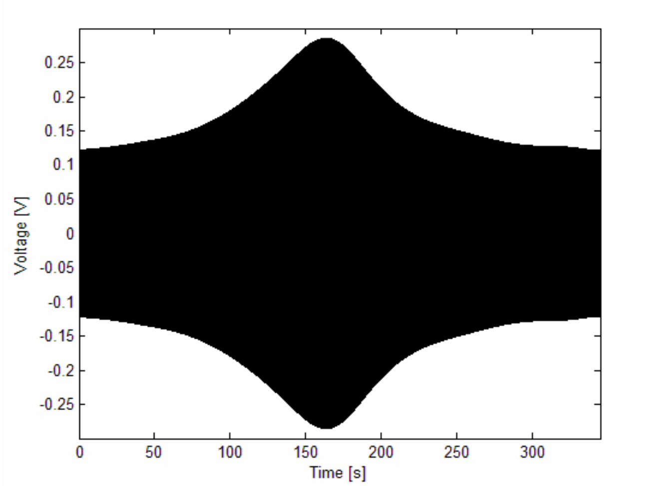

Figure 3.3. An example of a sine sweep signal used in the tests.

Figure 3.4 shows the FRFs obtained from random and sine sweep measurements of different force amplitudes.

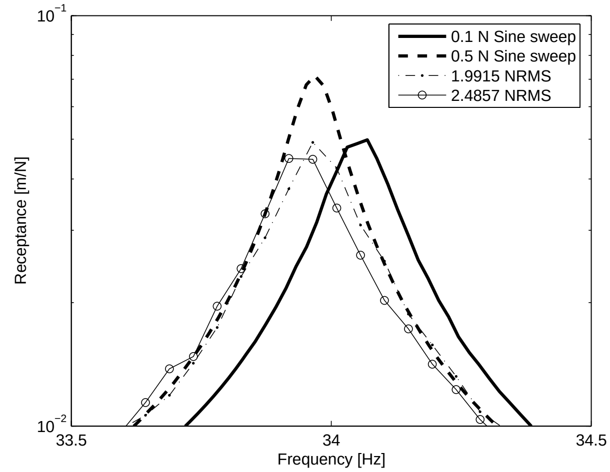

Figure 3.4. FRFs obtained from random and sine sweep measurements.

As can be seen in figure 3.4, the transfer functions differ a bit from each other depending on the force level and the type of excitation signal used. The difference in amplitude between the sine sweep signals is approximately 20 percent. A slight difference in the estimation of the resonance frequency can also be noticed. This could indicate that there still is a problem with force dropouts at low force levels.

Keeping the results from the initial measurements in mind, the force amplitude was set to 0.05 N and the frequency span to sweep over was set to 25-40 Hz. The frequency increment far away from the resonance was set to 0.5 Hz and close to the resonance the frequency increments were set to 0.05 Hz, 0.02 Hz and 0.01 Hz.

An example of an iteration process using Broyden's method is shown in figure 3.5. Here the fundamental frequency has already been adjusted to the desired force level. The force signal contains four higher harmonics which are controlled by the algorithm. Every iteration consists of five measurements of eight seconds each.

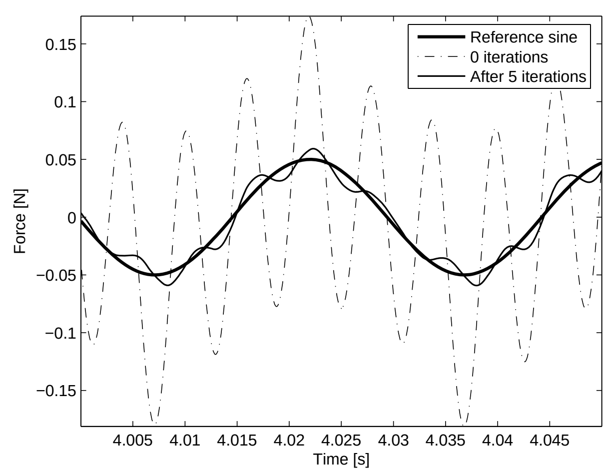

Figure 3.5. The Broyden solver used as iterative method. $f = 33.69 Hz$

An example of an iteration where the Newton-Raphson solver is used is shown in figure 3.6. Again the figure shows the signals after the force amplitude of the fundamental frequency has been adjusted. Four harmonics are controlled, one iteration requires approximately 20 measurements of eight seconds each.

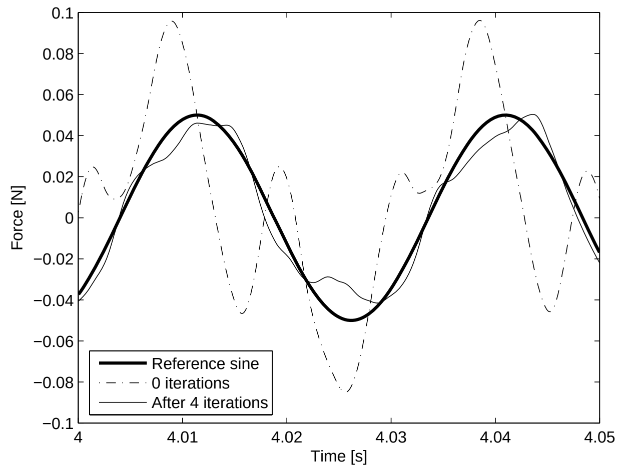

Figure 3.6. The Newton-Raphson solver used as iterative method. $f = 33.81Hz$

Unfortunately the test system chosen was too lightly damped, therefore it wasn’t possible to complete the entire sweep over the resonance. There is a frequency span of 0.28 Hz in which the algorithm has been unable to find a solution.

A FRF of the points successfully controlled and measured using the force control algorithm is shown and compared to the FRFs of the previous sine sweep measurements in figure 3.7.

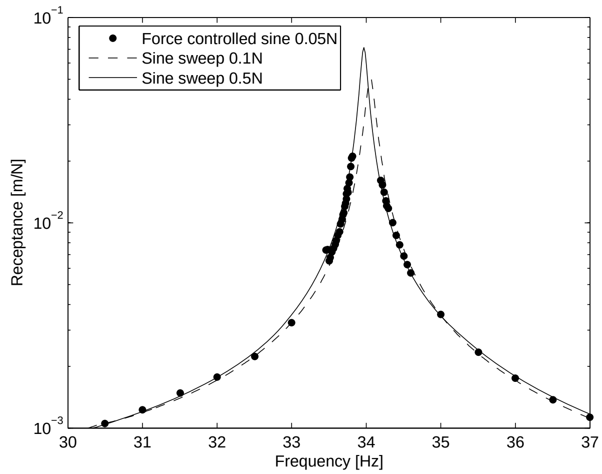

Figure 3.7. The FRF of the successfully controlled points compared with normal sine sweep measurements.

As can be seen in figure 3.7, the measurements obtained with the force controlled sine follows the transfer function from the 0.5 N sine sweep. This is expected since it’s a linear system and a distorted force signal shouldn’t affect the frequency response function. The general idea was to create a distorted force signal and use the force control algorithm to get rid of the distortion. This has been successfully completed, see figure 3.5 and 3.6.

4. CONCLUSIONS AND FUTURE WORK

When using stepped-sine excitation in a conventional measurement setup, the actual force signal will be harmonically distorted, due to the nonlinearities. A forced control algorithm has therefore been developed [4] where the actual voltage signal is designed to compensate for the nonlinearities in the force signal. This work presents the first step to verify that the control algorithm can be used in practical applications.

As shown in chapter 3, the measured force signal will be significantly distorted at resonances on a lightly damped linear structure. This is due to the nonlinearities produced in the shaker at large-amplitude vibrations. By using the control algorithm it has been shown that it is possible to excite with a non-distorted force signal closer to the resonance than with conventional methods. The frequency response function obtained from these measurements, indicates that the true damping in the structure is much smaller compared to what a measurement with random noise excitation would indicate.

Due to the small damping, there still is a small frequency band of 0.28 Hz where the control algorithm has been unable to find a solution. This problem is not expected in measurements at more damped structures. Except for the small frequency band mentioned, the control system has successfully reduced the distortion of the force signal. Both the Newton-Raphson procedure and the modified version which uses Broyden’s method have been tested with promising results.

In future measurements the control algorithm will be applied to structures with significant nonlinearities.

5. REFERENCES

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