# Chromagenic Filter Design

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## ABSTRACT

A *chromagenic camera* captures a pair of RGB images of a scene. Both images are captured as in a conventional digital imaging device but one of the pair is optically pre-filtered using a so-called *chromagenic filter*. It has been shown that the information in such a pair of images makes it easier to solve certain problems in colour vision. For example it can help to solve the illuminant estimation problem, provided that the chromagenic filter used when capturing the image pair is chosen carefully. In this paper we investigate two schemes for deriving the "optimal" filter for a chromagenic device in the context of the illuminant estimation problem and we show that the choice of filter does indeed have a significant effect on algorithm performance.

## 1. INTRODUCTION

Recently, an algorithm for estimating the scene illuminant (a key step in solving the colour constancy problem) using a pair of chromagenic images has been proposed[1]. A pair of images (which differ only in that one of the pair is optically pre-filtered) is captured, and the relationships between corresponding pixels in each image are used to obtain information about the scene illuminant. As might be expected, the choice of filter used when capturing the image pair is an important factor in the success or failure of the algorithm. In this paper we investigate two schemes for deriving the "optimal" chromagenic filter in the context of the illuminant estimation problem. In a first approach we define certain properties that we believe it is desirable a chromagenic filter should have and we show that it is possible to derive a closed form procedure to determine a filter that satisfies these properties. We will show that the filter derived in this way works well in the context of illuminant estimation, but it is nevertheless sub-optimal in the sense that it is not guaranteed to minimise the error in this task. In our second approach we address this shortcoming by determining the filter that explicitly minimises the illuminant estimation error. Since the algorithm for illuminant estimation is relatively complex, it is not possible in this case to derive a closed form optimisation, and so instead we sample the set of all possible filters to discover the optimal one. We begin the paper by providing a brief overview of the *Chromagenic Algorithm* for illuminant estimation and the role that the chromagenic filter plays in this estimation procedure (§2). We present our two approaches to filter design in §3 and evaluate their performance in §4.

## 2. CHROMAGENIC ILLUMINANT ESTIMATION

An estimate of a scene illuminant can be obtained using a pair of chromagenic images in two steps: a pre-processing step, applied once for a given chromagenic camera, and a second operation applied to a given pair of images whose scene illuminant it is wished to estimate.

**Pre-processing**, We choose a set of $m$ plausible scene illuminants $E_i(\lambda)$. In addition we select a set of $n$ surface reflectances $S_j(\lambda)$ representative of the surfaces which occur in the world. Now, for the $j$th illuminant we define a $3\times n$ matrix $Q_i$ whose $j$th column contains $(R_{ij}, G_{ij}, B_{ij})$ the sensor response to the $j$th surface imaged under the $i$th illuminant. Similarly we define $Q_i^F$, also a $3\times n$ matrix whose $i$th column contains $(R_{ij}^F, G_{ij}^F, B_{ij}^F)$ the sensor response to the $j$th surface imaged under the $i$th illuminant and filtered by a chromagenic filter $F(\lambda)$. Then for each plausible illuminant we define a $3\times 3$ transform matrix $T^i=Q_i^F Q_i^+$ (where + denotes pseudo-inverse) which maps unfiltered sensor responses to filtered sensor responses.

**Operation.** Given a chromagenic image pair, each consisting of $p$ pixels, let $Q$ and $Q^F$ denote the $3 \times p$ matrices of unfiltered and filtered sensor responses respectively. For each plausible scene illuminant we calculate a fitting error $e_i = \left\| T^i Q - Q^F \right\|$ under the assumption that $E_i(\lambda)$ is the scene illuminant. We then hypothesise that the scene illuminant is the illuminant corresponding to the transform $T^i$ which best describes the relationship between filtered and unfiltered RGBs. That is, we choose the illuminant with minimum fitting error so that our estimate of the scene illuminant is $E_{est}(\lambda)$ where $est = \min_i(e_i) \quad i = 1,2,...m$.

## 2. CHROMAGENIC FILTER DESIGN

From the summary of the illuminant estimation algorithm above, it is clear that the choice of chromagenic filter plays a key role in the ability of the algorithm to successfully identify the scene illuminant. To take an extreme case, if a filter $F(\lambda)$ is chosen such that the transform $T^i$ for all illuminants is identical, then the algorithm has no discrimination power at all. Ideally then we require a filter that leads to the transforms $T^i$ for each illuminant being as different as possible. We can frame this requirement in the form of a mathematical optimisation by posing the problem as that of finding the filter that maximises the transform variance

$$\sigma_F^2 = \frac{1}{m} \sum_{i=1}^{m} \left( \underline{t}_F^i - \underline{\mu}_F \right)^2 \tag{1}$$

where $\underline{t}_F^i$ is the 3×3 transform $T^i$ for the $i$th illuminant and a given filter $F$, represented as a 9×1 vector and $\underline{\mu}_F$ is the mean of all transforms for filter $F$. Now, let us define a $9m \times 1$ vector $\underline{v}_F$ such that its elements are the vectors $\underline{t}_F^i - \underline{\mu}_F$ concatenated together. Then the inter-transform variance for the filter $F$ is given by $\sigma_F^2 = \underline{v}_F^t \underline{v}$. We would like to find the filter $F$ that maximises this variance. To do so we must make the role of the filter explicit. To do this, let us assume that our filter can be represented as a linear combination of a finite number $l$ of basis vectors. That is $F(\lambda) = c_1 \underline{b}_1 + c_2 \underline{b}_2 + ... + c_l \underline{b}_l = B\underline{c}_F$ where $B$ is the matrix whose columns are basis vectors $\underline{b}_k$ and $\underline{c}_F$ is the $l \times 1$ vector of weights required to define the filter $F$. Considering each of the columns of $B$ as a filter, we can express the inter-transform variance for $\underline{b}_k$ as $\sigma_{\underline{b}_k}^2 = \underline{v}_{\underline{b}_k}^t \underline{v}_{\underline{b}_k}$ and it follows that the variance for the filter $F$ is

$$\sigma_F^2 = \left( c_1 \underline{v}_{\underline{b}_1} + c_2 \underline{v}_{\underline{b}_2} + ... + c_l \underline{v}_{\underline{b}_l} \right)^t \left( c_1 \underline{v}_{\underline{b}_1} + c_2 \underline{v}_{\underline{b}_2} + ... + c_l \underline{v}_{\underline{b}_l} \right) = \underline{c}^t V^t V \underline{c} \tag{2}$$

where $V$ is a $9m \times 1$ matrix whose $k$th column is $\underline{v}_{\underline{b}_k}$. The role of the filter is now explicit: it depends on the vector $\underline{c}$. So to maximise the inter-transform variance we must find $\underline{c}$ to maximise $\underline{c}^t V^t V \underline{c}$. As it stands there are two problems with this procedure. First is the fact that this expression can be trivially maximised by choosing $\underline{c}$ to be infinite. Secondly, while maximising the inter-transform variance is desirable, we would also like to ensure that the transforms for each illuminant affords a good mapping from filtered to unfiltered sensor responses. That is, we would like the fitting errors $e_i = \left\| T^i Q_i^F - Q_i^F \right\|$ to be small.

We can address both issues in the following way. Let us first define a vector $g_k$ whose elements are the fitting errors between filtered and unfiltered sensor responses, for all illuminant transforms and all surfaces, for the $k$th basis filter. If we further define a matrix $G$ such that the columns of $G$ are the vectors $\underline{g}_k$ for all basis filters, then the total fitting error for the filter $F$ as defined in (4) is $\underline{c}^t G^t G \underline{c}$. We would like to find $\underline{c}$ such that $\underline{c}^t G^t G \underline{c}$ is minimised while the variance $\underline{c}^t V^t V \underline{c}$ is maximised. We can achieve this by maximising the objective function

$$\underline{c}^t V^t V \underline{c} + \beta\left(1 - \underline{c}^t G^t G \underline{c}\right) \quad (3)$$

where β is a Lagrange multiplier and the addition of the constraint on the fitting error ensures that infinite $\underline{c}$ no longer maximises the objective function. We can determine the vector $\underline{c}$ that maximises (3) by solving the standard eigenvector problem

$$\left(G^t G\right)^{-1} V^t V \underline{c} = \beta \underline{c}. \quad (4)$$

**Sampling.** While the optimisation procedure outlined above ensures that our filter has certain desirable properties, it does not explicitly ensure minimisation of the error in the problem we are trying to solve, i.e. illuminant estimation. Unfortunately we are unable to derive a closed-form optimisation to derive the "best" (in the sense of lowest illuminant estimation error) chromagenic filter in this context. Instead, we propose to search the whole space of all possible filters and to evaluate each filter in terms of the resulting estimation error.

We begin by once more defining a filter as a linear combination of basis filters, as above. Suppose for the moment that we use a 3-d basis ($l$=3). Let us note first, that a filter $F$ and its corresponding normalised filter

$$F_n(\lambda) = \frac{1}{\left\|c_1 \underline{b}_1 + c_2 \underline{b}_2 + c_3 \underline{b}_3\right\|^2}\left(c_1 \underline{b}_1 + c_2 \underline{b}_2 + c_3 \underline{b}_3\right) \quad (5)$$

are essentially equivalent, since they differ only by the overall intensity of light they transmit. We note further, that if we assume an orthonormal basis then

$$\left\|c_1 \underline{b}_1 + c_2 \underline{b}_2 + c_3 \underline{b}_3\right\|^2 = \left\|c_1 + c_2 + c_3\right\|^2. \quad (6)$$

So that a filter defined by $\underline{c} = \left(c_1, c_2, c_3\right)^t$ will have equivalent performance to the filter defined by $\underline{c} / \left\|\underline{c}\right\|^2$. Now, since any point $\underline{c} / \left\|\underline{c}\right\|^2$ lies on the surface of a sphere with unit radius it follows that to search all filters we need only investigate all points on the surface of the unit sphere. Since exploring all such points is infeasible, we restrict ourselves to a finite set of sample points uniformly distributed about the surface of the sphere. There are a number of proposals (e.g.[2]) as to how best to obtain such a set of sample points. However, the most straightforward approach is to simply generate points *(x,y,z)* where $x$, $y$ and $z$ are each chosen from a uniform distribution on the interval [-1, 1]. We discard points *(x,y,z)*, which fall outside the unit sphere (and any occurrences of the point *(0,0,0)*) and scale the remaining points to lie on the unit sphere. In this way we can obtain as many points as we choose, randomly positioned on the surface of the sphere. This idea can easily be extended to the case in which we use an $l$-dimensional basis, where $l$>3 in which case we sample the surface of an $l$-dimensional hypersphere. For each sample point we generate the corresponding filter, and measure its goodness in terms of the accuracy of illuminant estimation it affords. Further details as to how we define illuminant estimation accuracy are given in the next section where we discuss filter performance.

## 4. FILTER PERFORMANCE

To test the relative performance of the methods of chromagenic filter design introduced above, we conducted a set of illuminant estimation experiments. We follow the experimental design suggested by Barnard *et al*[3] in their evaluation of a number of leading illuminant estimation algorithms. We test the performance of the chromagenic algorithm on 6000 synthetic images, each containing between 2 and 64 surfaces. To use the chromagenic algorithm we must first derive transforms for each of a set of plausible illuminants: we used a subset of the lights under which the test images are rendered. The data used is described in detail in[3]. We compare the chromagenic algorithm's estimate of the scene light to the actual (known) scene illuminant using a standard[3] *angular error* measure of estimation performance: the angle (in degrees) between the RGB of the illuminant and the estimate of this RGB provided by the algorithm.

We evaluate performance for the methods of filter design introduced above. We investigated two different choices of bases for the filters: an orthornormal cosine basis and a basis derived from a Principal Components Analysis (PCA) of the transmittance functions of a set of 53 Wratten filters. We also used each of these Wratten filters themselves in the algorithm to determine the level of performance that can be achieved using existing filters.

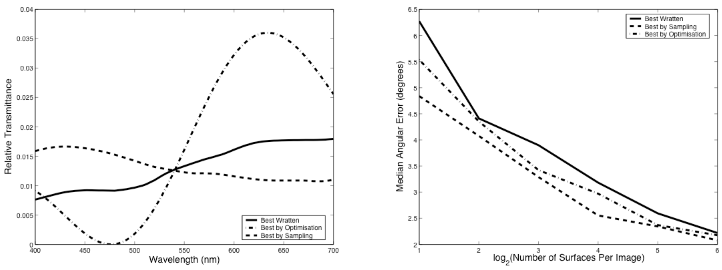


**Figure 1:** Tested filters (left-hand plot) and performance statistics (right-hand plot). See text for details.

The left-hand plot in Figure 1 shows the best performing filter for three different cases: the filter derived using a closed form optimisation (dashed-dotted line), the filter found using the sampling approach (dashed line), and the best performing Wratten filter (solid line). In both cases we found a filter basis derived from a PCA analysis of the Wratten filters to give better performance than a cosine basis. In the closed-form optimisation a 6-d basis gave the best results, whilst in the sampling approach we used a 5-d basis. The right-hand plot shows median angular error performance of the chromagenic algorithm using each of these three filters, as a function of the number of surfaces per image. The three filters are quite different, and as might be expected, so too is their illuminant estimation performance. We note first that using a standard Wratten filter provides very good illuminant estimation performance, and importantly, the performance is significantly better (in a strict statistical sense[1] than all previous illuminant estimation algorithms. Secondly, taking a principled approach to filter design leads to a further improvement in algorithm performance: Figure 1 shows that both designed filters outperform the standard Wratten filter, particularly for scenes with few surfaces. We tested the statistical significance (using a non-parametric Sign Test[4]) of this performance difference and found that at the 99% confidence level both designed filters give performance which is significantly better than that of the Wratten filter. In conclusion then, a principled approach to filter design does lead to a significant improvement in the performance of the chromagenic algorithm. In future work we intend to study the interaction between the chromagenic filter and the camera sensors, as well as to consider other factors, which influence the design of the filter.

## References


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