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EFFECTS OF NEGATIVE BIAXIAL LOADINGS AND NOTCH ON FAILURE ASSESSMENT DIAGRAMS

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ABSTRACT

The Failure Assessment Diagram (FAD) is a simplified and robust flaw assessment methodology which simultaneously connects two dominant failure criteria: Linear Elastic Fracture mechanics (LEFM) on one end and Plastic collapse on other end. This interaction is the realm of Elastic Plastic Fracture Mechanics (EPFM.) It is popularly known as the R6 approach which graphically characterizes the impact of plasticity on crack driving force. In the recent years, there has been continuous interest in using Failure Assessment Diagrams (FAD) to assess the failure of cracked structures subjected to biaxial loadings. Biaxiality is defined as the ratio of stress applied parallel and normal to the crack. Some aircraft components operate under negative biaxial ratios up to -0.5.

In this paper, a detailed study on FAD was conducted using FEA computed J-integral methods to investigate the effect of biaxial loading using different FAD approaches for geometries with notches. Geometries with a crack that emanates at a fillet region were simulated with various biaxial loading ratios from -0.5 to +0.5 using 2014-T6 material. FAD curves were numerically generated for cracks at notched regions subjected to various biaxial loadings using J-integral values from finite element analyses and validated its practical application. Comparison studies were made between uniaxial and biaxial loading cases with FAD curves created using standard approaches for four different crack sizes.

Under small scale yielding, this study clearly shows that FAD curves are not influenced by negative biaxial loading at low load (up to 40% of yield strength). It was clearly confirmed that the majority of previously developed analytical FAD curves do not effectively account for notch and plasticity effects due to negative biaxilaity. Based on this study, tension normal to the crack and compression parallel to the crack is the worst combination and it has a very pronounced effect on FAD curve shapes. The standard analytical FAD curves are non-conservative compared to the approach recommended here, particularly under the worst case condition. The proposed method is expected to predict lower failure loads relative to currently accepted analytical methods.

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INTRODUCTION

There are several structural integrity assessment methodologies available in predicting a critical state of cracked structural components. The Failure Assessment Diagram (FAD) is one of the popular two criteria failure approaches which represent the range of material behavior from brittle fracture to plastic collapse. In the recent past, there have been increasing interest on FAD [1-3] subjected to biaxial loading but most are based on uniaxial or positive biaxial loading conditions. Biaxiality is defined as the ratio of stress applied parallel and normal to the crack. A large amount of information is available in the open literature but the majority are based on uniaxial stress state conditions [4-5]. There is no adequate information in the open literature about the influence of negative biaxiality on FAD. Lacking validation, the application of existing FAD approaches are more questionable for negative biaxial loading. Some aircraft components operate under negative biaxial loading (up to -0.5).

This study attempts to confirm FAD suitability by numerically quantifying the impact of negative biaxial loading on FAD at fillet regions. Hong [4] proposed modified FAD curve in 1994 which accounts for the stress concentration effect but applicability of this approach for negative biaxial loading is again questionable. The finite element based FAD curves have been generated for various biaxial ratios ranging from -0.5 to +0.5 and validated its applicability with material based FAD curve developed by Ainsworth [3] and other existing analytical curves.

2. BACKGROUND OF FAILURE ASSESSMENT DIAGRAM

The Failure Assessment Diagram (FAD) represents a transition or interaction between two distinctly separate mechanisms of failure as shown in Fig.1. Simple analyses assume that failure will occur when the applied load reaches the lower of either a load to cause failure calculated using LEFM or a collapsed load [1-3], ignoring any interaction between brittle fracture and plastic collapse. FAD procedure requires the calculation of L_r and K_r parameters [6] which depend on load, geometry and material properties

$$L_r = \frac{P}{P_L} = \frac{\sigma_{ref}}{\sigma_y}, \quad \sigma_{ref} = L_r \sigma_y \tag{1}$$

$$K_r = \sqrt{\frac{J_e}{J}} \tag{2}$$

where J is a crack driving force parameter, P is applied load, P_L is the limit load and σ_y is yield strength. J_e is crack driving force from linear analysis value which can be evaluated from finite element analysis or from the SIF hand book solutions.

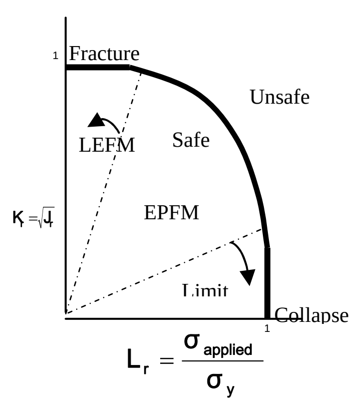

Fig.1 Different levels in Failure Assessment Diagram

2.1 Option-1. Material & Geometry Independent

Option-1 curve is independent of material, geometry and type of loading and crack size. Only yield strength and plane strain fracture toughness would be enough to use this approach. The equation of Option-1 is [3],

$$f_1(L_r) = \left(1 - 0.1L_r^{\,2}\right)\left[0.3 + 0.7\exp\left(-0.65L_r^{\,6}\right)\right] \tag{3}$$

2.2 Option-2. Material Dependent/Geometry Independent

There are two popular approaches [2, 13] to predict elastic-plastic part of J value. The first approach is GE/EPRI approach based on deformation plasticity theory. This approach requires extensive finite element analysis. The second approach is a reference stress based approach developed by Ainsworth [3]. Option-2 is the reference stress or material based FAD approach [2], where elastic-plastic J is evaluated from elastic component of J and stress-strain curve of the material. It is also independent of geometry.

$$\frac{J}{J_e} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2}\left(\frac{\sigma_{ref}}{\sigma_y}\right)^2 \frac{\sigma_{ref}}{E\varepsilon_{ref}}, \tag{4}$$

where $\sigma_{ref} = \frac{P}{P_L}\sigma_y$

σ_{ref} is reference stress and ε_{ref} denotes the true strain corresponding to the stress σ_0 determined from true stress-strain curve. The same equation can be rearranged as prescribed in R6 method.

$$f_2(L_r) = \left(\frac{E\varepsilon_{ref}}{\sigma_{ref}} + \frac{L_r^{\,3}\sigma_y}{2E\varepsilon_{ref}}\right)^{-1/2} \tag{5}$$

Option-2 curve is only dependent on material and independent of loading and structures. The true stress-strain curve is required to use this approach.

2.3 Option-3. Material & Geometry Dependent

Option-3 is a more advanced, energy based approach [2] and is directly evaluated from elastic and elastic-plastic fine element analysis with J-integral estimation scheme using deformation plasticity theory. It is dependent upon load, geometry, crack size and material properties.

$$f_3(L_r) = \sqrt{\left(\frac{J_e}{J}\right)} \tag{6}$$

2.4 Failure Assessment Diagrams at fillet region

The FAD has three standard options prescribed in R6 method developed by British Central Electricity Generating Board (CEGB). The stress concentration effects due to structural discontinuities are not accounted in the standard Option-1 and Option-2 approaches. Hong has proposed a modified Option-1 and Option-2 curves in 1994 [4] which account for the stress concentration factors. In order to use FAD curves at fillet region, Hong proposed the following modifiers which have to be subtracted from original FAD curves.

$$\beta = 1.582\beta_1[\exp\left(-\left(1-1.25L_r\right)^2\right)-0.368], \quad L_r \leq 0.8 \qquad (7)$$

$$= \beta_1, \qquad L_r > 0.8$$

where

$$\beta_1 = 0.0416\left(\frac{R}{a}\right)^{\left(0.735-0.0907\ln\left(\frac{R}{a}\right)\right)} \qquad (8)$$

and R/a is the ratio of the local radius of fillet to the flaw depth. In this paper, a detailed finite element analysis has been performed for various biaxial load combinations for geometries with crack that occurs at fillets regions. Attempts were made to assess the applicability of above equations for geometries subjected to various biaxial loadings.

3. MATERIAL PROPERTIES

The aluminum material (AL2014-T6) was chosen for the material input to the analysis [9]. The material along with true stress-strain curve is characterized by Ramberg-Osgood power law hardening model as shown in Fig.2.

$$\frac{\varepsilon}{\varepsilon_O} = \frac{\sigma}{\sigma_O} + \alpha\left(\frac{\sigma}{\sigma_O}\right)^n \qquad (9)$$

Where σ_0 is taken as reference stress or the 0.2% yield strength. $\varepsilon_0$is σ_0/E, where E is the Young's modulus of the material. α is the yield offset and n is the material hardening exponent.

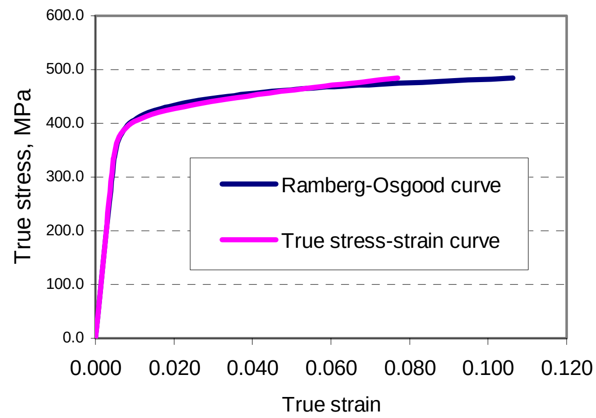

Fig.2 True stress strain curve for AL2014-T6 material

The values of materials properties and constants were as follows.

σ_O = 390MPa, E=72398 MPa,

α =0.372, n=18.

4. GEOMETRY AND FINITE ELEMENT MODEL

4.1 Geometry

The flat plate model [5] with hole is considered for the current study as shown in Fig.3. In order to assess the effect of stress concentration and negative biaxiality, the crack size at the fillet region were systematically varied (R/a ratio) with respect to fillet radius. The analyses were performed by varying R/a ratio from 1 to 5. For each R/a ratio, three different biaxial loadings (B=-0.5, 0, +0.5) were considered.

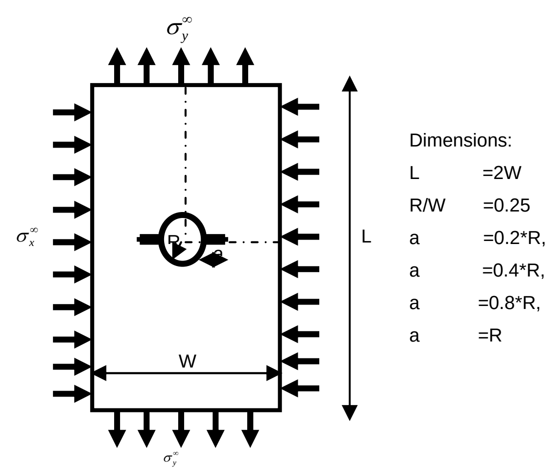

Fig.3 Flat plate model with hole under biaxial loading

4.2 Finite Element Model and biaxial loadings.

In order to capture the steep strain gradient at the crack tip, a focused crack tip mesh using eight noded isoparametric element was considered and a very fine mesh were introduced at the crack front as shown in Fig.4. To prevent deformation locking at higher loading conditions, the reduced integration technique was chosen for elastic-plastic analysis. Considering the symmetry of geometry and loadings, only one-fourth of the full geometry was generated.

The finite element analyses were performed using ABAQUS for four different crack sizes at the fillet region. The crack driving force (J-integral) was calculated under plane strain conditions by systematically varying the magnitude of load applied normal and parallel to the crack. To generate the FE based FAD curves (Option-3), two different FE approaches were considered. The first approach is a linear analysis, where only the isotropic material properties were considered. In the second approach, the deformation plasticity theory based on the Ramberg-Osgood equation was considered to calculate the nonlinear portion of the J-integral value.

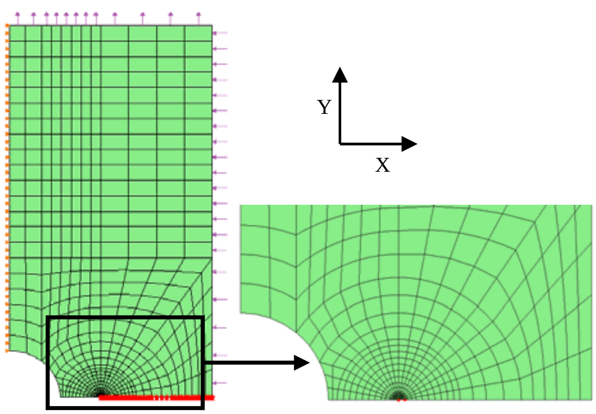

Fig.4 FE model with biaxial loading and focused mesh at the crack front.

5. EFEECT OF BIAXIALITY

The biaxial ratio is nothing but a load ratio between load parallel and normal to the crack front.

$$B = \frac{\sigma_x^\alpha}{\sigma_y^\alpha} \quad (10)$$

In the case of uniaxial loading, B is assumed as zero (B=0). In the current work three different biaxial ratios are considered ranging from -0.5 to +0.5.

Rhodes and Randon [7] have reviewed the impact of local stress effect at the crack tip using von Mises yielding criterion under plane stress condition. For example, under negative biaxial ratio, the earlier yielding occurs for the same load as compared with positive biaxial ratio. Negative biaxiality is a worst load combination. The crack tip constraint will be reduced by compressive load applied parallel to the crack front.

$$\sigma_{xx}^2 - \sigma_{xx}\sigma_{yy} + \sigma_{yy}^2 = \sigma_{yt}^2 \quad (11)$$

$$\frac{\sigma_{xx}^2}{\sigma_{yy}^2} - \frac{\sigma_{xx}\sigma_{yy}}{\sigma_{yy}^2} + \frac{\sigma_{yy}^2}{\sigma_{yy}^2} = \frac{\sigma_{yt}^2}{\sigma_{yy}^2} \quad (12)$$

$$\frac{\sigma_{yy}}{\sigma_{yt}} = \frac{1}{\sqrt{1 - \frac{\sigma_{xx}}{\sigma_{yy}} + \left(\frac{\sigma_{xx}}{\sigma_{yy}}\right)^2}} \quad (13)$$

where σ_{xx} and σ_{yy} and are components of applied stress and σ_{yt} is the yield stress in uniaxial tension. Using equation [12], it can be seen in Fig. 5 that yielding increases for negative biaxial loadings.

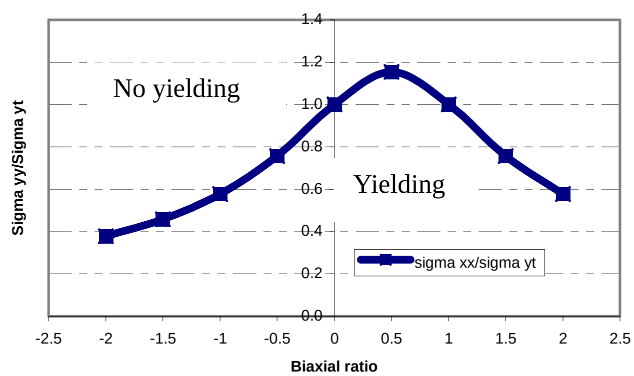

Fig.5 Effect of biaxiality on yielding.

In linear analysis, J-integral shows no dependence whatsoever on the magnitude of load parallel to crack [10-11] but Socie [12] has highlighted that there is a significant effect for a crack emanating from hole under biaxial loading and this effect will disappear once crack size reaches equal to fillet radius. Y.C Lam [14] has confirmed that crack behavior is greatly influenced by compressive load parallel to the crack.

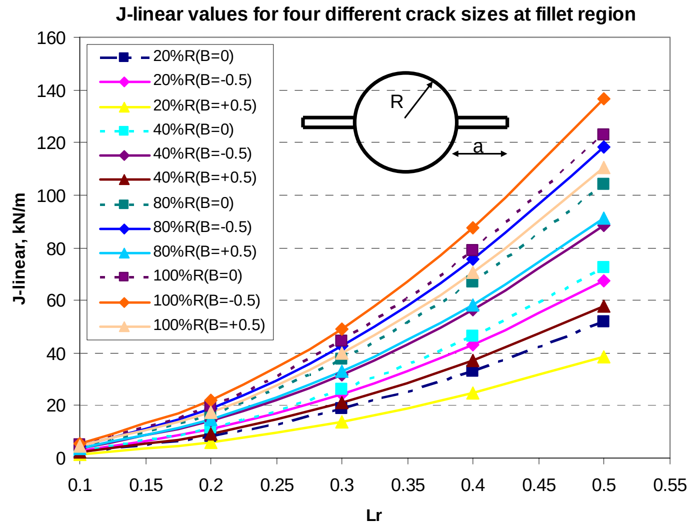

Fig.6 Crack driving force values from linear analysis for four different cracks ratios (a/R = 20%, 40%, 80%, 100%)

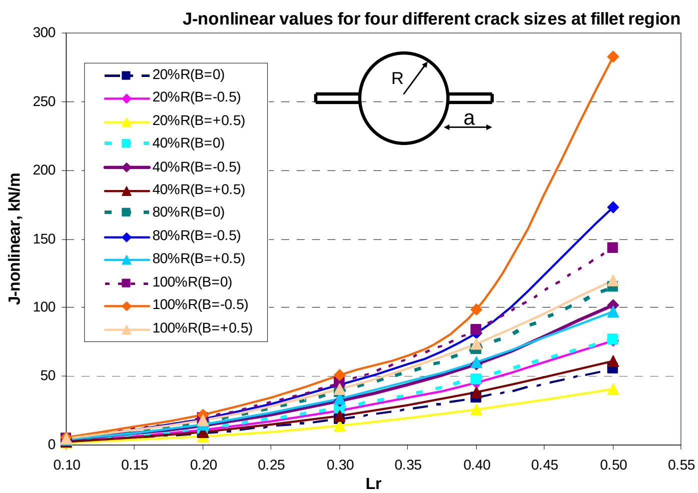

Fig.7 Crack driving force values from nonlinear analysis for four different crack ratios (a/R = 20%, 40%, 80%, 100%)

In this study, using a flat plate model with hole, it was confirmed that the crack driving force (J-integral) from linear analysis is significantly altered at the fillet region subjected to biaxial loading, particularly under negative biaxial loading as compared with positive biaxial loadings as shown in Fig.6. It was also confirmed that crack driving force from nonlinear analysis has very significant effect from negative biaxial loading if externally applied exceeds 40% of yield strength as shown in Fig.7

6. DISCUSSION ON EFFECT OF BIAXILITY AND NOTCH ON FAD NOMENCLATURE

To study the effect of various biaxial loadings for a crack at the fillet region, finite element analyses were performed with four different crack sizes at fillet region and FAD curves were generated using crack driving forces for various biaxial loading combinations.

6.1 Effect of stress concentration on FAD under uniaxial loading

Failure assessment diagrams were created from finite element analysis results using crack driving force (J-integral) values subjected to uniaxial loading conditions and compared with Option-1 and 2 approaches. From the study, it was confirmed that FAD's from FE results are matching well with analytical results if L_r ratio is less than 0.6 and has major variations at higher loading conditions (L_r>0.7) and deviations increase as crack size increases as shown in figures 8-11

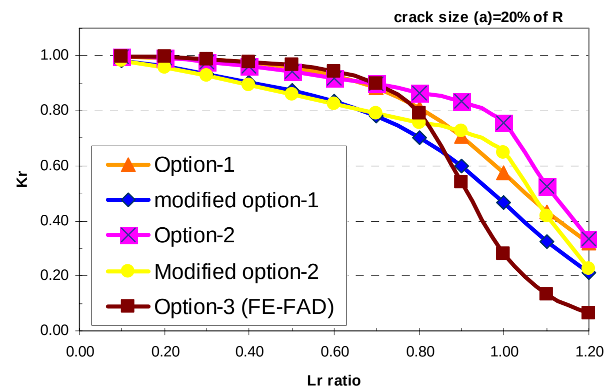

Fig.8 Uniaxial loading – a/R=20%

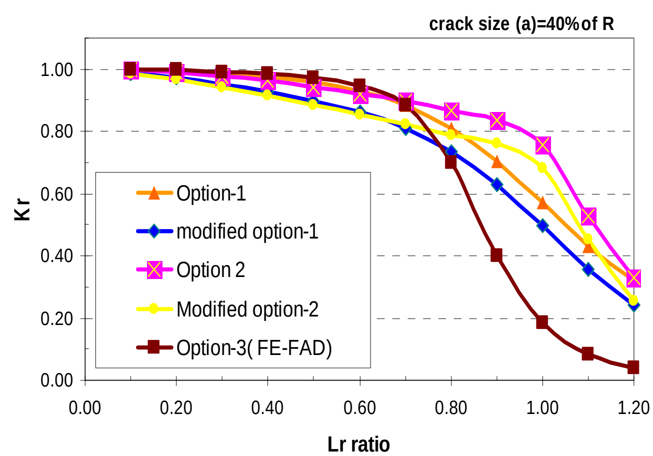

Fig.9 Uniaxial loading - a/R=40%

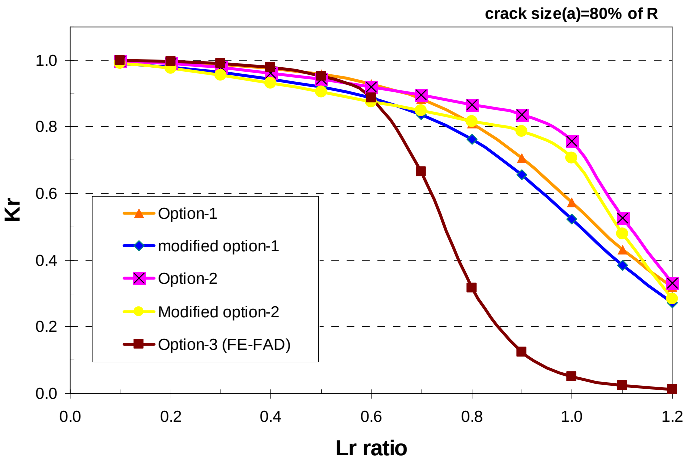

Fig.10 Uniaxial loading - a/R=80%

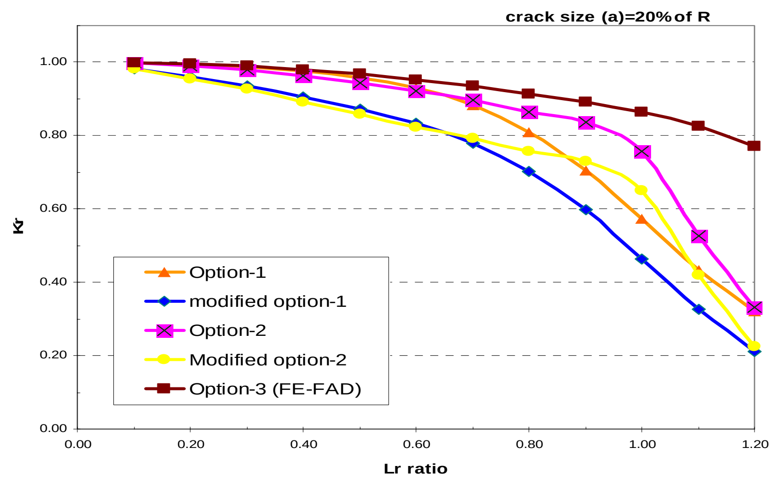

Fig.12 Positive biaxial loading - a/R=20%

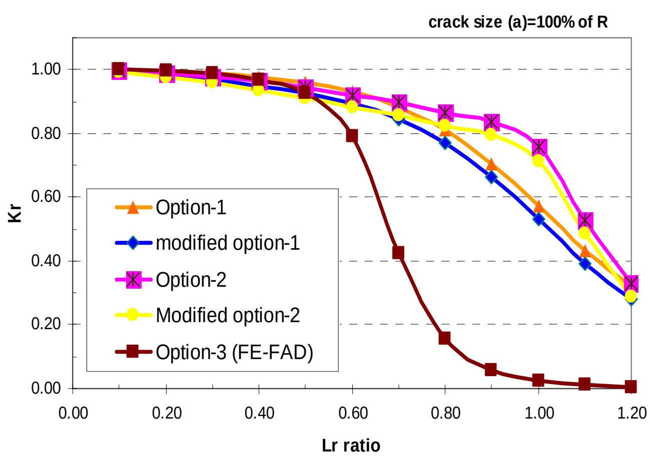

Fig.11. Uniaxial loading - a/R=100%

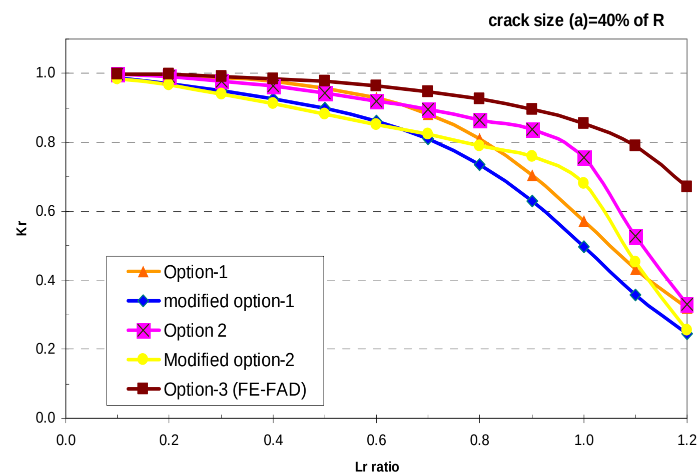

Fig.13 Positive biaxial loading - a a/R=40%

6.2 Effect of stress concentration on FAD under positive biaxial loading

In the case of positive biaxial load cases at stress concentration region, the FADs have shown no major changes on its shapes and it matches well with analytical FAD approaches. For small crack sizes, up to a/R = 0.40, the FEA based FAD suggests that the analytical approaches are always conservative. Hong's modified FAD is giving satisfactory results for positive biaxial loadings. The main reason is that the stress magnitude at fillet region is two times higher than the nominal value for positive loading which is lower than the uniaxial loading condition. Therefore Hong's proposed equation is more conservative for biaxial loading as shown in Fig. 12-15. It is also matching with Tan's study [8] on FAD subjected to positive biaxial loading.

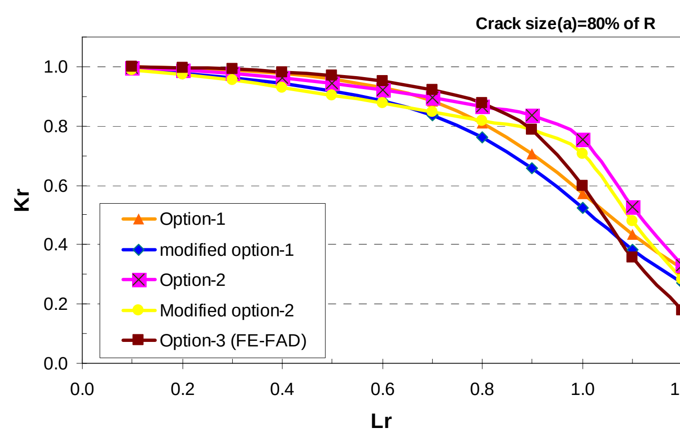

Fig.14 Positive biaxial loading - a/R=80%

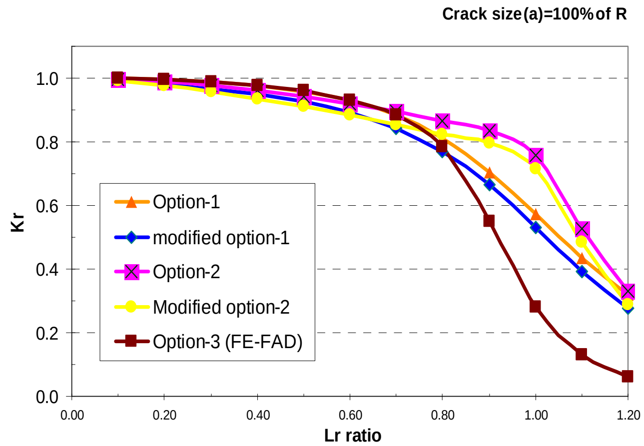

Fig.15 Positive biaxial loading - a/R=100%

6.3 Effect of stress concentration on FAD under negative biaxial loading

The elastic (Je) and elastic-plastic (J_{ep}) values were obtained from finite element analysis by systematically varying the magnitude of externally applied load and crack sizes at the fillet region. The FADs were constructed for flat plate model subjected to negative biaxial loadings and compared with existing analytical approaches. It can be seen from Fig. 16-19 that that FADs from FE results are matching well with analytical curves when L_r ratio is less than or equal to 0.4, i.e. if externally applied load is less than 40% of yield strength but it has very significant effect at higher load conditions. The primary reason is excess plastic deformation due negative biaxial loading which existing analytical FADs are not able to account. It is clearly confirmed that existing analytical FADs are not directly extended to negative biaxial loading conditions.

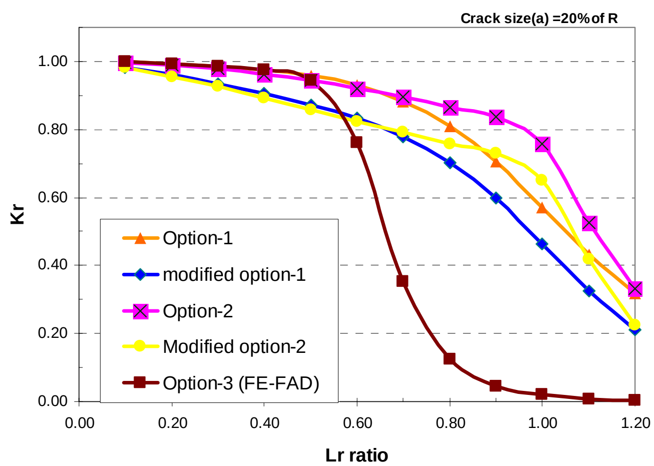

Fig.16 Negative biaxial loading - a/R=20%

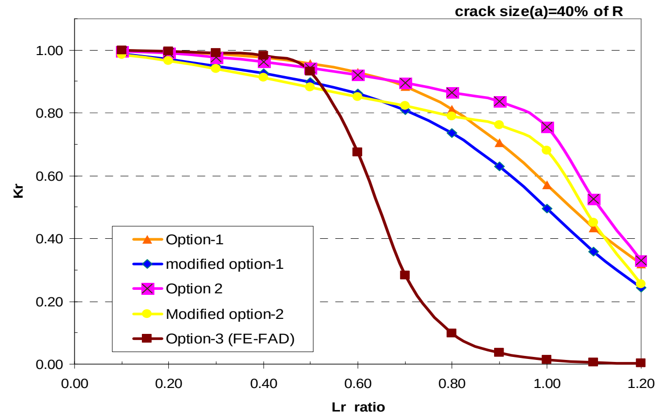

Fig.17 Negative biaxial loading - a/R=40%

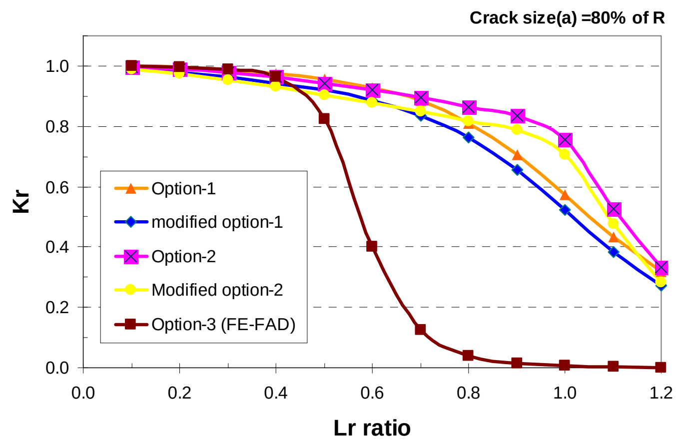

Fig.18 Negative biaxial loading - a/R=80%

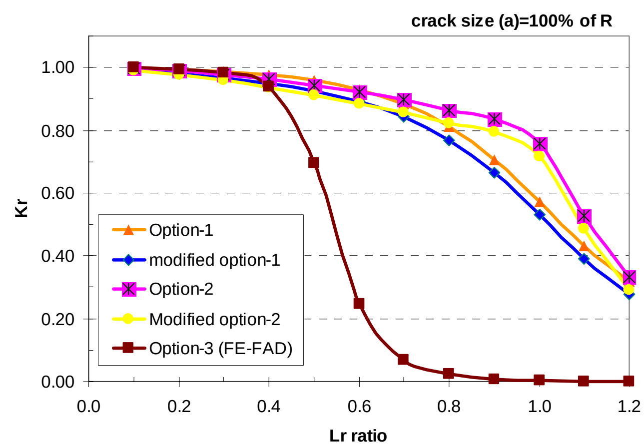

Fig.19 Negative biaxial loading - a/R=100%

It is also confirmed that Hong's modified FADs are not effectively capturing the excess plasticity effect at stress concentration region. The reason is that the stress magnitude at the fillet region is four times higher than the nominal stress value under negative loading conditions whereas for positive biaxial loading, the stress magnitude at stress concentration region is two times higher than the nominal value. Therefore Hong's proposed equation is not a conservative method for negative biaxial loading.

7. CONCLUSION

Failure assessment diagrams have been generated from finite element analysis for four different crack sizes at the fillet region subjected to uniaxial, positive biaxial and negative biaxial loadings. To assess the effect of excess yielding due to negative biaxial loading, the validation studies were carried out with existing analytical FADS.

1. From the study, it was clearly confirmed that failure assessment diagrams are very sensitive to negative biaxial loadings at geometry discontinuity regions. Existing analytical based FAD curves are not appropriate choices for negative biaxial loading conditions.

2. Hong's equations and proposed approach for modified FAD curves at the fillet regions agree well for positive biaxial loadings whereas for uniaxial loading case, it is matching for L_r ratio up to 0.6.

3. Hong's equations do not agree for negative biaxial loading if L_r ratio is greater than 0.4. This is due to excess plastic deformation effects and reduced constraint at the crack tip from negative biaxial loadings.

4. The FADs from uniaxial load cases can be extended to positive biaxial loadings as curve moves outward from uniaxial loading but analytical FADs are not appropriate for use in negative biaxial loading situations as the failure envelope moves inward which leads to significantly lower predicted failure loads as compared with uniaxial and positive biaxial loading cases.

Further validations are required on FAD subjected to negative biaxial loadings. Additional studies need to confirm its limit of applicability using different geometries before making explicit conclusions.

8. NOMENCLATURE

W = Width of the specimen, m
L = Length of the specimen, m
R= = Fillet radius, m
a = Crack size, m
σ_{yy} =Stress normal to crack front, MPa
σ_{yy}^{α} = Far field external load normal to crack front, MPa
σ_{xx} =Stress parallel to crack front, MPa
σ_{xx}^{α} = Far field external stress parallel to crack front, MPa
σ_{yt} = Yield strength, MPa
σ_{ref} = Reference stress, MPa
L_r = Ratio between applied load and yield strength
K_r = Ratio between stress intensity factor and fracture toughness

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