Partial Hydrodynamic Lubrication With Large Fractional Contact Areas

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An alternative average Reynolds equation for use under conditions of large fractional contact area is proposed. The flow factors for this form of the equation are calculated for a variety of longitudinal surfaces and the results are shown to be relatively insensitive to the initial height distribution. Pressure and shear flow factors for the Christensen height distribution and a variety of Peklenik surface pattern parameters are also derived from the work of Patir and Cheng, Lo and Tripp. These are represented by semi-empirical equations over the full range of contact conditions. The implications of the results, with respect to the lubrication of metal forming processes, is discussed.

Introduction

In the mixed lubrication regime, which is common in metal forming processes, part of the interface load is carried by asperity contact and part by the pressurized lubricant in surface valleys. Thus, in order to model the mixed lubrication regime with liquid lubricants, it is important to be able to relate the pressures generated in the valleys to the surface topography, geometry and kinematics and the lubricant rheological properties. The roughness gives rise to a random component in the lubricant pressure. This demands an averaged form of the basic Reynolds equation to smooth out the stochastic component in the pressure. In the last two decades, many papers have been published which attempt to predict the average fluid pressure between two lubricated rough surfaces to create an average Reynolds equation. The flow factor method introduced by Patir and Cheng (1978), probably represents the most useful approach.

For a steady, one-dimensional problem, the form proposed by Patir and Cheng reduces to

$$\frac{d}{dx}\left(\phi_x' \frac{\bar{h}^3}{12\mu}\frac{dp_b}{dx}\right) = \frac{U_1 + U_2}{2}\frac{dh_t}{dx} + \frac{U_1 - U_2}{2} R_q \frac{d\phi_s}{dx} \quad (1)$$

where x is the distance along the film, p_b is the average hydrodynamic pressure, ϕ_x' and ϕ_s are the pressure and shear flow factors, $\bar{h}$ is the nominal surface separation (separation of mean planes of undeformed surface), h_t is the mean film thickness (volume between surfaces divided by area) and R_q is the RMS composite roughness. This formulation becomes difficult to handle for small values of $\bar{h}/R_q$ and is meaningless for zero or negative values. Since these will occur with values of fractional contact area A greater than about 50 percent, Patir and Cheng's formulation is unsuitable for conditions of high fractional contact area which occur in many bulk metal-forming processes.

Lo (1994) has suggested a way to avoid some of these problems by modifying Patir and Cheng's equation to

$$\frac{d}{dx}\left(\phi_x'' \frac{(\bar{h} + 2R_q)^3}{12\mu}\frac{dp_b}{dx}\right) = \frac{U_1 + U_2}{2}\frac{dh_t}{dx} + \frac{U_1 - U_2}{2} R_q \frac{d\phi_s}{dx} \quad (2)$$

where ϕ_x'' is an alternative pressure flow factor.

Lo also calculated the flow factors appropriate to his formulation all the way from the thin film case to the percolation threshold. The latter corresponds to the surface separation at which pressure gradients no longer influence lubricant flow because the lubricant is completely trapped in surface pockets. He used a combination of porous media and percolation theory applied to surfaces with various lays as characterized by the surface pattern parameter (Peklenik, 1968).

The use of $(\bar{h} + 2R_q)$ rather than say $(\bar{h} + 3R_q)$, while effective for the cases Lo concentrated on, is entirely arbitrary. This arbitrary feature is avoided in the formulation

$$\frac{d}{dx}\left(\phi_x \frac{h_t^3}{12\mu}\frac{dp_b}{dx}\right) = \frac{U_1 + U_2}{2}\frac{dh_t}{dx} + \frac{U_1 - U_2}{2} S_q \frac{d\phi_s}{dx} \quad (3)$$

where ϕ_x is yet another alternative pressure flow factor and S_q is the composite surface roughness of the undeformed surfaces based on a measurements over an area of the surfaces (rather than from a profile). For uncorrelated surfaces S_q may be defined by

$$S_q = \sqrt{S_{q1}^2 + S_{q2}^2} \quad (4)$$

where S_{q1} and S_{q2} are the roughnesses of the individual surfaces.

Wilson and Chang (1994) introduced a similar formulation to treat the mixed lubrication of rolling under conditions which lead to over 95 percent contact. Under such circumstances, Lo's formulation is difficult to use because $(\bar{h} + 2R_q)$ becomes negative. In solving some inverse problems it is also very convenient to use the same film thickness variable h_t in both the Poiseuille (LHS) and Couette (RHS) terms of the equation. The use of the areal based S_q roughness rather than the profile based R_q roughness is also preferable in treating non-isotropic surfaces (surfaces with a pronounced lay) because profiles taken parallel to the lay direction fail to reflect the real roughness. Thus, it is believed that the formulation given as Eq. (3) is best for use under conditions of large fractional contact area.

The pressure flow factor ϕ_x is a measure of the influence of

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roughness on the pressure driven (Poiseuille) flow. In the present formulation it is defined by

$$\phi_x = \frac{12\mu Q_p}{h_t^3 \dfrac{dp_b}{dx}} \tag{5}$$

where Q_p is the average flow per unit width under the influence of a pressure gradient dp_b/dx in a situation with no surface motion. A value of ϕ_x greater than unity reflects the channeling effect of valleys running in the flow direction, while a value of ϕ_x less than unity reflects the blocking effect of ridges running across the flow direction. Thus ϕ_x depends not only on the form of the surfaces and the degree of contact, but also on the orientation of non-isotropic surface lays relative to the coordinate direction. In the present paper, only the simplest case of a one-dimensional lubrication problem with the lay direction aligned with the coordinate direction is considered. For two-dimensional lubrication problems, it is also necessary to define pressure flow factors in orthogonal coordinate directions. If the lay direction does not line up with one of the coordinate directions, cross flow terms may result as described by Tripp (1983).

The definitions of the flow factors ϕ_x' and ϕ_x'' in Patir and Cheng's and Lo's formulations are of similar form to Eq. (5) except that h_t is replaced by $\bar{h}$ (Patir and Cheng) or $(\bar{h} + 2R_q)$ (Lo).

The shear flow factor ϕ_s is a measure of the influence of roughness on the flow carried by surface motion (Couette flow). In all three formulations it is defined by

$$\phi_s = \frac{2S_q Q_c}{(U_1 - U_2)} \tag{6}$$

where Q_c is the average flow per unit width under circumstances where there is no general pressure gradient, the surface velocities are equal and opposite ($U_1 = -U_2$), and all the roughness is concentrated on one surface. Values of ϕ_s other than zero reflect the ability of a rough surface to carry lubricant in its valleys.

In describing the flow factors it is convenient to use the nondimensional nominal surface separation H and nondimensional mean film thickness H_t defined by

$$H = \bar{h}/S_q \tag{7}$$

and

$$H_t = h_t/S_q \tag{8}$$

respectively.

Given the form of Eq. (3), one might expect that it would be best to express the flow factors as functions of H_t. However, the best choice of independent variable actually depends on the lubrication problem to be tackled and may even change from one part of the interface to another. For example, in analyses of metal-forming processes such as rolling, the use of H is preferable in the inlet zone while the use of H_t is better in the work zone. This is because, in the inlet zone, the *nominal surface separation* is controlled by the convergence of the essentially rigid surfaces, while, in the work zone, the *mean film thickness* is controlled by continuity of lubricant flow. Thus, in general, it does not really matter whether H or H_t is used provided that a relationship between them can be constructed from the surface height distributions. These can also be used to calculate the relationship between the fractional contact area A and either H or H_t, which is important in the treatment of mixed lubrication.

While Wilson and Chang introduced the formulation of Eq. (3), they only provided the pressure flow factor for the simple case of longitudinal saw-tooth surfaces. In the present paper, pressure flow factors for longitudinal surfaces with different height distributions will be developed and compared. Pressure and shear flow factors for more realistic (not purely longitudinal) surfaces with different surface pattern parameters and approximately Gaussian height distributions will also be developed. It will be assumed that a rough workpiece surface is being flattened by a smooth tooling surface. This case may readily be extended to the case where the tooling surface is essentially unidirectional with a lay direction aligned with the sliding (x) direction. This is a common situation in practical metal rolling operations.

Longitudinal Surfaces

Initially, the flow factors for surfaces with purely longitudinal lay directions will be derived. Such surfaces consist of ridges and valleys aligned along the direction of surface motion and flow. They cannot prevent flow unless they are in complete contact, so that there is no percolation limit as defined by Tripp (1983). For all combinations of longitudinal surfaces, roughness has no influence on the Couette flow term expressed using h_t, so that

$$\phi_s = 0 \tag{9}$$

If the surfaces are out of contact, the pressure flow factor ϕ_x is given simply by

$$\phi_x = 1 + 3H_t^{-2} = 1 + 3H^{-2} \tag{10}$$

Nomenclature

A = fractional contact area
E = expectancy operator
$H = \bar{h}/S_q$, nondimensional nominal surface separation
$H' = h'/3S_q$, nondimensional distance from mean plane
$H_t = h_t/S_q$, nondimensional mean film thickness
H_{tc} = value of H_t corresponding to percolation threshold
P = probability density
Q_c, Q_p = average Couette and Poiseuille flows per unit width
R_q = profile based RMS roughness
$S_q = \sqrt{S_{q1}^2 + S_{q2}^2}$, area based composite RMS roughness
S_{q1}, S_{q2} = roughnesses of the individual surfaces
U_1, U_2 = surface velocities (in x direction)
$Z = \bar{h}/3S_q$, surface separation variable
$\bar{h}$ = nominal surface separation (between mean planes of undeformed surfaces)
h' = distance from mean plane
h_t = mean film thickness (volume between surfaces divided by total area)
p_b = average pressure in surface valleys
x = distance along film (in sliding direction)
γ = Peklenik surface pattern parameter
μ = lubricant viscosity
$\phi_s = 2S_qQ_c/(U_1 - U_2)$, shear flow factor
$\phi_x = (12\mu Q_p/h_t^3(dp_b/dx))$, pressure flow factor
ϕ_x' = pressure flow factor in Patir and Cheng's (1978) formulation
ϕ_x'' = pressure flow factor in Lo's (1994) formulation

for any nonskewed surface height distribution. However, for surfaces in contact, ϕ_x depends on the surface height distribution. In the present work, three cases were considered as shown in Fig. 1 (with exaggerated surface slope). In characterizing the contact process it has been assumed that the process of asperity flattening is equivalent to removing the "overlapping" material.

The first case is the saw-tooth surface shown in Fig. 1. For this case we can draw on the results of Wilson and Chang (1994) to write

$$A = 1 - 0.7598\sqrt{H_t} \quad (11)$$

$$\phi_x = 3.464H_t^{-1} \quad (12)$$

and

$$H_t = \frac{(H + \sqrt{3})^2}{4\sqrt{3}} \quad (13)$$

For the sinusoidal surface shown in Fig. 1, the methods described by Wilson and Chang may be used to obtain

$$A = 0.5 - 0.3183 \sin^{-1}(0.7071H) \quad (14)$$

$$\phi_x H_t^3 = 1.5H + 0.5H^3 + \sqrt{1 - 0.5H^2}\,(0.6003 + 0.8253H^2) + (\sin^{-1}(0.7071H))(0.9549H + 0.3183H^3) \quad (15)$$

and

$$H_t = 0.5H + 0.4502\sqrt{1 - 0.5H^2} + 0.3183H \sin^{-1}(0.7071H) \quad (16)$$

Note that the flow factor in Eq. (15) appears naturally in the term $\phi_x H_t^3$ which is a measure of the lubricant flow induced by a fixed pressure gradient. It is also the form which appears in the most common nondimensional form of the average Reynolds equation, so that it has not been rearranged. This quantity will be referred to as the nondimensional pressure flow.

The third surface to be considered is the Christensen surface illustrated in Fig. 1. Christensen (1970) originally developed the height distribution for this case to approximate the normal or Gaussian distribution of height in random surface models. In Fig. 1 a periodic surface with the same height distribution has been drawn to allow direct comparison with the periodic sawtooth and sinusoidal surface already described. For the Christensen distribution the probability density P of the deviation h' from the mean line is

$$P = (35 - 105H'^2 + 105H'^4 - 35H'^6)/96S_q \quad (17)$$

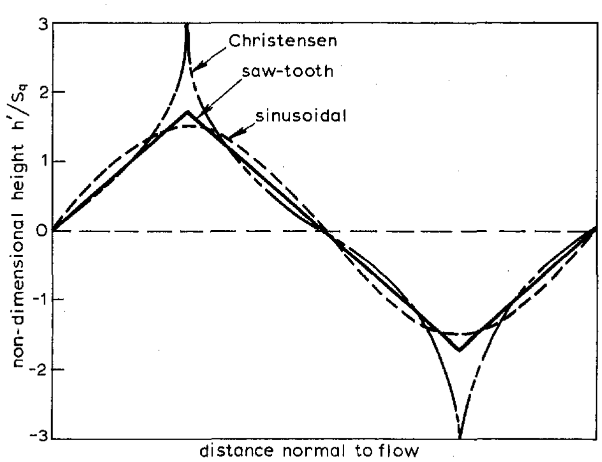

Fig. 1 Longitudinal surface profiles

where

$$H' = h'/3S_q \quad (18)$$

The fractional contact area A was given by Christensen as

$$A = (16 - 35Z + 35Z^3 - 21Z^5 + 5Z^7)/32 \quad (19)$$

and the relationship between H_t and H as

$$H_t = 3(35 + Z(128 + Z(140 + Z^2(-70 + Z^2(28 - 5Z^2)))))/256 \quad (20)$$

where

$$Z = \bar{h}/3S_q = H/3 \quad (21)$$

The flow factor ϕ_x may be calculated from

$$\phi_x H_t^3 = E(\bar{h} + h')^3/S_q^3 = 81\int_{-H/3}^{1} P(Z + H')^3 dH' \quad (22)$$

where E is the expectancy operator. Using Eqs. (17) and (18) to evaluate the integral yields

$$\phi_x H_t^3 = 0.7383 + 1.5H + 1.2305H^2 + 0.5H^3 + 0.0911H^4 - 0.002H^6 + 4.8225 \times 10^{-5}H^8 - 5.9537 \times 10^{-7}H^{10} \quad (23)$$

Figure 2 compares the pressure flow factors for the three longitudinal surface models with surface contact (Eqs. (12), (15), and (23)) together with the relationship for the noncontact case (Eq. (10)). The results are all plotted as nondimensional pressure flows, $\phi_x H_t^3$, as discussed above, against the nondimensional mean film thickness H_t.

As expected, all the nondimensional pressure flows show a monotonic increase with H_t. The results for the contact analyses lie quite close together and somewhat below the no-contact solution. This is apparently because of the blocking effect of the contact patches. At sufficiently high values of H_t, all the curves run together, as expected.

The results for the Christensen surface are slightly above those for the saw-tooth surface, while those for the sinusoidal case lie below both of these. In the case of longitudinal lays, flow is dominated by the form of the valleys. Thus the difference between the three curves reflects the lower flow resistance of the "V" shaped valleys of the Christensen and sawtooth surfaces compared with the "U" shaped valleys of the sinusoidal surface when all have equal RMS roughness.

With the present formulation the flow factor is apparently relatively insensitive to the height distribution. In fact, the simple expression for the sawtooth case (Eq. (12)) is an excellent approximation for both the Christensen and sinusoidal surfaces under conditions of contact. One point to remember is that, even though the flow factors for different surfaces are similar, the values of the fractional contact area A can be quite different and this may lead to differences in the load support due to asperity contact and in frictional behavior in the mixed regime.

Nonlongitudinal Lays

The pressure flow factors ϕ_x and shear flow factors ϕ_s in Eq. (3) have also been estimated for Christensen surfaces with other than purely longitudinal surface lays by using a combination of Patir and Cheng's and Lo's results for flow factors and the results of Tripp (1983) for the percolation threshold. This involved converting Patir and Cheng's and Lo's results to the current formulation and extending them to match the limiting conditions imposed by Tripp's results. This process involved some judgment because the three sources do not agree exactly. However, the differences are relatively small and it is believed that the final result can be

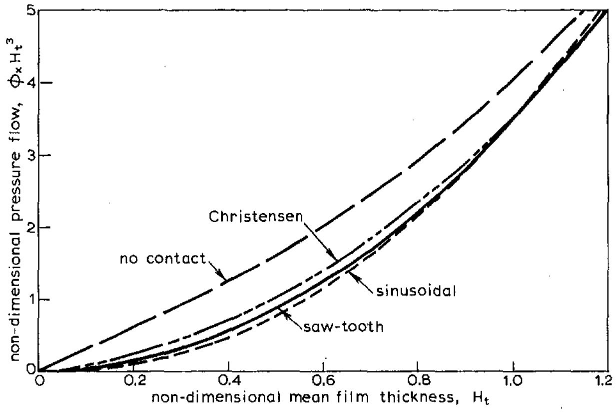

Fig. 2 Pressure flow factors for longitudinal surfaces

used with some confidence. The results are shown in Figs. 3 and 4.

Figure 3 shows the flow factors plotted in terms of the nondimensional pressure flow $\phi_x H_t^3$ against H_t for different values of the Peklenik surface pattern parameter γ. The curves all increase monotonically with H_t. At small values of H_t, the curves become tangent to the H_t axis at a value of H_t corresponding to the percolation threshold.

The nondimensional pressure flow increases with the Peklenik parameter, as expected. A value of γ of infinity corresponds to the purely longitudinal surface previously considered. As γ is reduced the rough surface will have an increasing content of transverse features which will block the pressure driven flow component. No curve is shown for the purely transverse case ($\gamma = 0$). This case is very sensitive to the tail of the height distribution. Thus, for a Christensen height distribution, the percolation threshold corresponds to a value of H_t of 3. For larger values of H_t, no contact occurs. On the other hand, for an exact Gaussian distribution there is always a finite probability that some transverse ridges will be in contact, and pressure driven flow will be blocked by these, for all values of H_t, no matter how large.

Figure 4 shows the shear flow factors ϕ_s plotted against H_t for different values of the Peklenik surface pattern parameter γ. At large values of H_t, the shear flow factors decrease with increasing H_t, and increasing γ, in the manner described by Patir and Cheng. However, at small values of H_t, below the percolation threshold, the lubricant will be trapped and carried along in the pockets in the rough surface. In this condition,

$$\phi_s = H_t \tag{24}$$

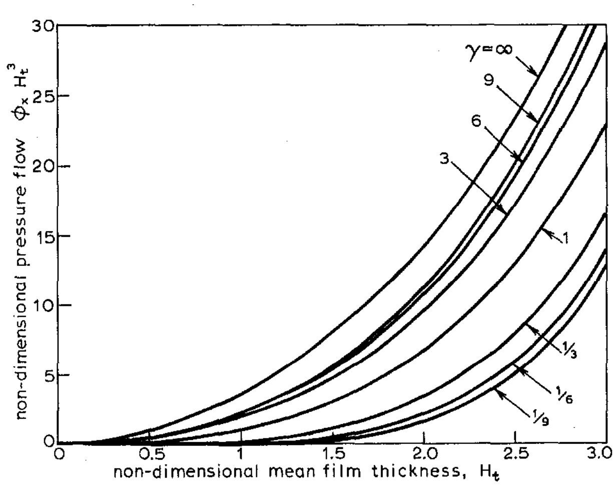

Fig. 3 Pressure flow factors for Christensen surfaces

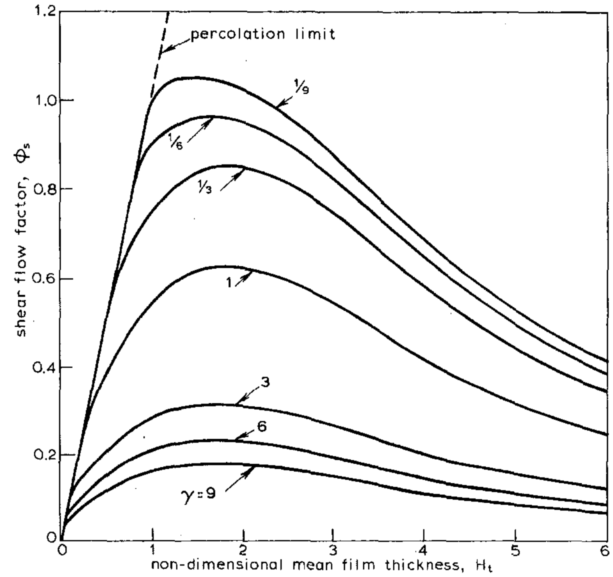

Fig. 4 Shear flow factors for Christensen surfaces

Thus, as H_t is decreased, ϕ_S at first increases, then goes through a maximum and finally joins the percolation limit line given by Eq. (24) at the point corresponding to the percolation threshold. Again, there is a problem with the purely transverse case ($\gamma = 0$), for the reasons already discussed.

Both the pressure and shear flow factors ϕ_x and ϕ_s, respectively, have been fitted by semiempirical equations over a large range of values of the Peklenik surface pattern parameter γ and the nondimensional mean film thickness H_t. The equations are not suitable for very small values of γ (less than about 0.1) for the reasons discussed above. For values of H_t greater than 3, where there is no contact with the Christensen distribution, the equations already given by Patir and Cheng may be used.

For the pressure factor ϕ_x an appropriate form for the semiempirical equation for $H_t < 3$ is

$$\phi_x H_t^3 = a_2(H_t - H_{tc})^2 + a_3(H_t - H_{tc})^3 \tag{25}$$

where H_{tc} is the value of H_t corresponding to the percolation threshold and a_2 and a_3 are functions of γ. These can be fitted well enough for most practical purposes by the semiempirical equations

$$H_{tc} = 3(1 - (0.47476/\gamma + 1)^{-0.25007}) \tag{26}$$

$$a_2 = 0.051375 \ln^3(9\gamma) - 0.0071901 \ln^4(9\gamma) \tag{27}$$

and

$$a_3 = 1.0019 - 0.17927 \ln \gamma + 0.047583 \ln^2 \gamma - 0.016417 \ln^3 \gamma \tag{28}$$

For the shear factor ϕ_s an appropriate form for the semiempirical equation for $H_t < 5$ is

$$\phi_s = b_0 + b_1 H_t + b_2 H_t^2 + b_3 H_t^3 + b_4 H_t^4 + b_5 H_t^5 \tag{29}$$

where the coefficients b_0, b_1, b_2, b_3, b_4 and b_5, are functions of γ given by

$$b_0 = 0.12667\gamma^{-0.6508} \tag{30}$$

$$b_1 = \exp(-0.38768 - 0.44160 \ln \gamma - 0.12679 \ln^2 \gamma + 0.042414 \ln^3 \gamma) \tag{31}$$

$$b_2 = -\exp(-1.1748 - 0.39916 \ln \gamma - 0.11041 \ln^2 \gamma + 0.031775 \ln^3 \gamma) \tag{32}$$

$$b_3 = \exp(-2.8843 - 0.36712 \ln \gamma - 0.10676 \ln^2 \gamma + 0.028039 \ln^3 \gamma) \quad (33)$$

$$b_4 = -0.004706 + 0.0014493 \ln \gamma + 0.00033124 \ln^2 \gamma - 0.00017147 \ln^3 \gamma \quad (34)$$

and

$$b_5 = 0.00014734 - 4.255 \times 10^{-5} \ln \gamma - 1.057 \times 10^{-5} \ln^2 \gamma + 5.0292 \times 10^{-6} \ln^3 \gamma \quad (35)$$

Discussion

The results given above provide a way of modeling the hydrodynamic component of lubrication in the mixed regime under conditions which lead to large fractional contact areas. They are expected to be of particular value in analyses of bulk metal forming processes where the high interface pressures and reduction in workpiece asperity hardness due to bulk plastic strain led to fractional contact areas approaching unity in the absence of hydrodynamic action (Wilson and Sheu, 1988). They are also expected to be useful in modeling the lubrication of machine elements under severe conditions where wear and plastic asperity flattening may lead to large fractional contact areas.

The results can also help to provide insight into the behavior of the lubricant in metal-forming processes under conditions of high fractional contact area. As the value of H_t is reduced, and the percolation threshold is approached, the value of the pressure flow factor ϕ_x tends to zero, and the influence of the pressure gradient on the lubricant flow becomes less and less important. Thus, the Poiseuille terms on the left of Eq. (3) become negligible compared with the Couette terms on the right. This will result in a condition which is very similar to that occurring in the work zone of a metal-forming operation in the thick film lubrication regime, where it is possible to neglect the influence of the pressure gradient on lubricant flow (Wilson and Chang, 1994). Under such conditions the pressures in the lubricant are no longer decided by hydrodynamics but by workpiece plasticity. Also, since the Poiseuille terms are no longer important, lubricant flow or transport will be controlled by the Couette terms and the motion of the surfaces and their roughnesses become the deciding factors in lubricant transport and mean film thickness.

A particularly interesting feature of the new formulation is that it extends smoothly through the percolation threshold to the condition where the lubricant is completely trapped in surface pockets. In this condition, the Poiseuille term becomes identically zero so that lubricant viscosity or speed no longer affect the process. This leads to a truly "hydrostatic" lubrication condition which has been observed by Lo (1994) in low-speed rolling experiments with isotropic strip surfaces. However, complete trapping is not possible with purely longitudinal surfaces and Wilson and Chang's theoretical work shows that pressure gradients continue to influence lubricant flow with longitudinal surfaces to very low speeds. Thus, it seems reasonable to expect that surface lay effects will be very important under low speed mixed lubrication conditions, which is contrary to initial expectations.

Conclusion

A new formulation for treating the lubrication of rough surfaces under conditions of high fractional contact area has been developed. This avoids problems with earlier formulations by using the mean lubricant film thickness h_t as the sole measure of surface separation. The approach allows the treatment of cases where the fractional contact area approaches unity as well as where the lubricant is completely trapped in surface pockets.

For purely longitudinal surfaces, the flow factors based on the new formulation are relatively insensitive to the surface height distribution. The simple analysis for the saw-tooth case developed by Wilson and Chang (1994) is an excellent approximation for both the sinusoidal and Christensen surface under conditions of contact. For surfaces with transverse components of roughness, the results are consistent with the work of Patir and Cheng (1978), Lo (1994) and Tripp (1983). Semiempirical equations are provided to facilitate the use of the formulation in future lubrication analyses.

The results suggest that surface topography may have interesting influences on the lubrication of metal-forming processes in the low speed mixed lubrication regime. In particular, the Peklenik surface pattern parameter may decide whether a state of hydrostatic or hydrodynamic lubrication occurs under low speed conditions. In the hydrostatic case, lubricant is trapped in surface pockets and lubrication is independent of speed or lubricant viscosity, while in the hydrodynamic case the lubricant can flow through surface channels under the influence of local pressure gradients.

Work is in progress to incorporate the present formulation into models of rolling, ironing, and sheet-forming processes. The predictions of these models will be compared with experimental measurements of film thickness, fractional contact area, and friction. Further theoretical work is also planned to incorporate thermal and non-Newtonian effects into the basic formulation as well as to calculate flow factors for surface models which better approximate the complex topographies found in practical forming operations.

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