

Vibrational Enhancement of Nuclear Level Density within Response Function Method

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Abstract. The main results are summarized on the development of the response function method for a description of the vibrational state effect on nuclear level density. The enhancement factor is calculated and compared within different approaches. The results of the RF approach are in the better agreement with the ones within the method of attenuated phonon occupation numbers. They also agree reasonably with results of a finite temperature extension of the interacting boson model.

INTRODUCTION

Level-density ρ is one of the main quantities to define the characteristics of nuclear decay. The collective states have a rather strong effect on level density, specifically at low excitation energies [1-7]. Calculation of a ratio of level densities with (ρ) and without (ρ_0) allowing for vibrational states [enhancement (variation) factor $K = \rho / \rho_0$] is the simplest method to estimate the effect of the collective states on level densities. The level densities can be calculated within the framework of a statistical approach with the usual partition function [1] or its modification [8]. It was found [9] that factor K is practically independent of the partition function used. Therefore we use a standard approach as the simplest way to calculate the enhancement factor of level density. In this contribution the effect of the vibrational state on nuclear level density is studied on the basis of the response function (RF) approach [10,11]. The method allows taking into account the damping of vibrational states in a rather accurate way.

RESPONSE FUNCTION METHOD

The expression of level density ρ in $K = \rho / \rho_0$ is used in the form within the framework of the standard saddle-point method [1]:

$$\rho(U, A) = (4\pi^2 D)^{-1/2} \exp S(\alpha_0, \beta_0), \quad (1)$$

where $S(\alpha_0, \beta_0) = \beta_0 E + \ln Z(\alpha_0, \beta_0) - \alpha_0 A = \beta_0 E - \beta_0 \Omega(\alpha_0, \beta_0) - \alpha_0 A$ is the entropy of the nucleus with mass number A at excitation energy $U = E - E_{g.s.}$ with $E_{g.s.}$ for the ground state energy; $Z(\alpha, \beta) = \text{Tr}[\exp(-\beta \tilde{H})]$ the partition function; $\tilde{H} = \hat{H} - \mu \hat{A}$; $\mu = \alpha / \beta$; and $\Omega(\alpha, \beta) = -\ln Z(\alpha, \beta) / \beta$ is the thermodynamic potential. The symbol “Tr” denotes the trace procedure over all variables of all particles; \hat{H} is the Hamiltonian; \hat{A} is the operator of particle number. The function D in (1) is the determinant of the second partial derivatives of partition function logarithms with respect to parameters α and β . The saddle-point parameters α_0, β_0 define temperature ($T = 1 / \beta_0$) and chemical potential ($\mu = \alpha_0 / \beta_0$). They are solutions of the system of thermodynamic-state equations: $A = \partial \ln Z / \partial \alpha$, $E = -\partial \ln Z / \partial \beta$. We follow the random phase approximation (RPA) and consider vibrational states of multipolarity L as collective states formed by the two-body coherent interaction $V_{k,res}(i, j)$ of separable form:
$$V_{k,res}(i, j) = k \sum_{\mu} q_{L\mu}^*(\vec{r}_i) q_{L\mu}(\vec{r}_j), \quad q_{L\mu}(\vec{r}) = r^L Y_{L\mu}(\hat{r}),$$
 where $k \equiv k(L)$ is a coupling constant for interaction

with the one-body operator $q_{L\mu}(\vec{r})$ for the form-factor. The thermodynamic potential can be presented in the following form $\Omega = \Omega_0 + \Delta\Omega$, where $\Omega_0 = -(1/\beta)\ln Z_0$ is a thermodynamic potential of the independent particle model and $\Delta\Omega = -(1/\beta)\ln \Delta Z$ is an addition to the thermodynamic potential Ω_0 due to the presence of the residual interaction $\hat{V}_{k,res}$. In order to obtain $\Delta\Omega$ for fixed L ($\Delta\Omega \equiv \Delta\Omega_L$) we use the Bloch transformation with the result

$$\Delta\Omega_L = \int_0^k dk' \frac{\langle \hat{V}_{k',res} \rangle_{k'}}{k'} = \frac{2L+1}{2} \int_0^k \langle \hat{Q}_{L0}^+ \hat{Q}_{L0} \rangle_{k'} dk',$$

$$\hat{Q}_{L\mu} \equiv \sum_j q_{L\mu}(\vec{r}_j). \text{ Here, the bracket } \langle \dots \rangle_{k'} \text{ denotes}$$

the average of the canonical ensemble with the density matrix $\hat{\rho} = \exp(-\beta\tilde{H}_{k'})/\text{Tr}(\exp(-\beta\tilde{H}_{k'}))$, where $\tilde{H}_{k'} = \hat{H}_{k'} - \mu\hat{A}$; $\hat{H}_{k'} = \hat{H}_0 + \hat{V}_{k',res}$.

We next obtain the following expression for the addition to the thermodynamic potential $\Delta\Omega$,

$$\Delta\Omega_L = -\frac{2L+1}{2\pi} \int_0^k dk' \int_{-\infty}^{+\infty} \frac{\hbar}{1-e^{-\beta\hbar\omega}} \cdot \text{Im}\chi_{k',L}(\omega) d\omega, \quad (2)$$

using the Green's function methods [12] and the relationship between the retarded Green's function and the response function.

This quantity should be equal to zero in the absence of the coherent interaction. In order to provide this condition we modify Eq. (2) and subtract from its integrand a contribution corresponding to response function $\Pi_L(\omega) \equiv \chi_{k=0,L}(\omega)$ of the independent particle model without forming collective states:

$$\Delta\Omega_L = \Delta\tilde{\Omega}_L = -\frac{2L+1}{2\pi} \int_0^k dk' \int_{-\infty}^{+\infty} \frac{\hbar}{1-e^{-\beta\hbar\omega}} \times$$

$$\text{Im}(\chi_{k',L}(\omega) - \Pi_L(\omega)) d\omega. \quad (3)$$

This expression corresponds to the contribution to the thermodynamic potential from vibrational states with fixed multipolarity L . A full addition to the thermodynamic potential due to the presence of the vibrational states is the sum of $\Delta\Omega_L$: $\Delta\Omega = \sum_L \Delta\Omega_L$.

Note that the level densities ρ_0 and ρ should be calculated at the same excitation energy and due to this we will use in the calculation of K the modified

expression $\delta Z = \exp(-(\Delta\Omega(T) - \Delta\Omega(T=0))/T)$ for the change of the partition function instead of ΔZ . This leads to the relationship $K = \delta Z(T)$ if the changes of the determinant D and nuclear temperature T in the presence of vibrational states are neglected.

COMPARISON OF DIFFERENT EXPRESSIONS FOR THE ENHANCEMENT FACTOR

There are simple methods for calculations of the vibrational enhancement factor K with phenomenological allowing for damping vibrational states [3-7,18]. They are based on different empirical extensions of an analytical expression for the vibrational enhancement factor as a quantity that is proportional to the boson partition function. At first the effect of the vibrational-state damping on the enhancement factor was considered in [3] ($K = K_{CE}$). The complex energies of vibrational states in the boson partition function are used in K_{CE} . Attenuated occupation numbers of boson states are used in the expression for K considered in [6,7] ($K = K_{DN}$). The reduction of vibrational enhancement with temperature in the form of a Fermi function is adopted in modular system EMPIRE-II of codes for nuclear-reaction calculations [18] ($K = K_{EM}$).

First we investigate the accuracy of approximation ΔZ by the partition function of the boson state $\Delta Z^{(B)}$. As mentioned before this kind of approximation was used for construction of the factors K_{CE} and K_{DN} . We use a RPA form of RF: $\chi_{k,L}(\omega) = \Pi_L(\omega)/(1-k\Pi_L(\omega))$, $N=Z$. A noncollective response function $\Pi_L(\omega)$ is taken in one resonance approximation $\Pi_L(\omega) = B((\omega - \tilde{\omega}_0)^{-1} - (\omega + \tilde{\omega}_0^*)^{-1})$, where B is the normalization constant; $\tilde{\omega}_0 = \omega_0 - i\eta_0$ with ω_0 (η_0) for the real (imaginary) part that is taken as not dependent on both the temperature and frequency. Note that the complex frequencies $\tilde{\omega}_k = \omega_k - i\eta_k$, $\tilde{\omega}_0 = \omega_0 - i\eta_0$ are roots of the equations $(\chi_{k,L})^{-1} = 0$, $(\Pi_L)^{-1} = 0$. We obtain the following expression for an addition to the thermodynamical potential after contour integration over ω in a complex plane

$$\Delta\Omega_L = -\frac{2L+1}{\beta} \ln(\Delta Z^{(B)} \Delta Z^{(M)}), \quad (4)$$

where $\Delta Z^{(B)} = \frac{\exp(\beta\hbar\omega_0/2) |1 - \exp(-\beta(\hbar\omega_0 + i\gamma_0))|}{\exp(\beta\hbar\omega_k/2) |1 - \exp(-\beta(\hbar\omega_k + i\gamma_k))|}$ is a relative partition function for one degree of freedom for ensemble of Bose particles with complex energies; $\gamma_\alpha = \hbar\eta_\alpha$;

$$\Delta Z^{(M)} = \left| \prod_{n=1}^{\infty} \frac{(1 - k\Pi_L(-\omega_n))}{(1 - k\Pi_L(\omega_n))} \right|^{1/2}$$

is a term from Matsubara poles $\omega_n = 2\pi in / \beta\hbar$. In the case $\gamma_0 \rightarrow +0$ the quantity $\Delta\tilde{\Omega}_L$ is equal to the relative thermodynamical potential of the Bose particles.

The expression for $\Delta\tilde{\Omega}_L$ is obtained in the following form

$$\Delta\tilde{\Omega}_L = -\frac{2L+1}{\beta} \ln(\Delta Z^{(B)} \Delta\tilde{Z}^{(M)} \Delta\tilde{Z}^{(0)}), \quad (5)$$

where

$$\Delta\tilde{Z}^{(M)} = \prod_{n=1}^{\infty} \left| \frac{\exp(k\Pi_L(-\omega_n))(1 - k\Pi_L(-\omega_n))}{\exp(k\Pi_L(\omega_n))(1 - k\Pi_L(\omega_n))} \right|^{1/2}$$

is a term from Matsubara poles;

$$\Delta\tilde{Z}^{(0)} = \exp\left(\beta\hbar \frac{\omega_k^2 - \omega_0^2}{4\omega_0} \operatorname{Re}\left(\operatorname{cth}\left(\beta(\hbar\omega_0 + i\gamma_0)/2\right)\right)\right)$$

is the term from the poles of the noncollective response function $\chi_{0,L}$. In the case $\gamma_0 \rightarrow +0$ the expression for $\Delta\tilde{\Omega}_L$ coincides with the RPA one that was obtained in [20] within Green function method with imaginary time.

The calculations $\delta\tilde{Z}$, δZ were performed by Eqs.(4)-(5) with allowance for the quadrupole vibrational state with $\hbar\omega_k = 30A^{-2/3}$ MeV and $\hbar\omega_0 = 40A^{-1/3}$ MeV; $\gamma_\alpha = \hbar/\tau(\omega_\alpha)$, where relaxation time $\tau(\omega_\alpha)$ was taken according to the kinetic approach [15]. It was found that the addition $\delta\tilde{Z}$ can be approximated by boson partition function ratio $\delta Z^{(B)}$ with an accuracy of about 30%. The approximation of δZ by $\delta Z^{(B)}$ is worse than that for $\delta\tilde{Z}$ (see Fig. 1). As illustrated in Fig.2, the calculations of δZ , $\delta\tilde{Z}$ according to the one resonance RF show strong dependence on the expression for relaxation time $\tau(\omega_k)$ of the vibrational quadrupole state [15-17].

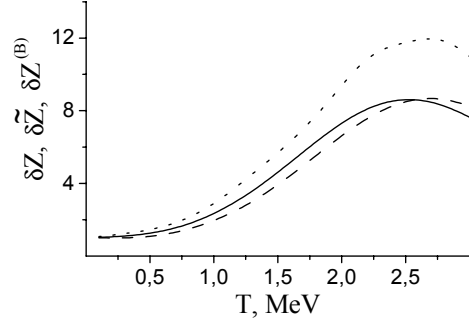


FIGURE 1. The dependence in ^{56}Fe of δZ and $\delta\tilde{Z}$ on the temperature: — δZ ; --- $\delta Z^{(B)}$; $\delta\tilde{Z}$.

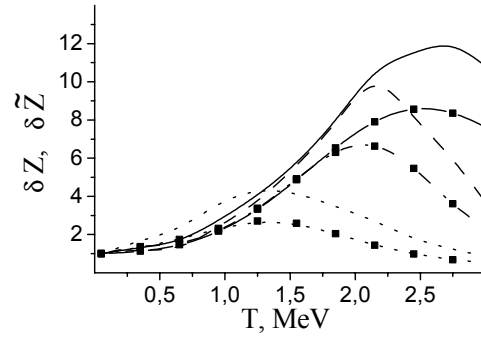


FIGURE 2. The dependence in ^{56}Fe of δZ and $\delta\tilde{Z}$ (■) on the temperature with allowance for the 2^+ state: — relaxation time according to the kinetic approach; --- and different expressions for relaxation time within the doorway state mechanism [16].

Figures 3 and 4 show the results of the vibrational enhancement factor calculation in ^{56}Fe and ^{146}Sm due to presence of the quadrupole 2^+ states within the RF method and different phenomenological approaches. The expressions of the Fermi-gas model are used for calculation of the mean-field components of the thermodynamic potential, the temperature, and level density ρ_0 . A variation in the temperature and determinant D due to the presence of the collective states is also taken into account in the calculation of K . We adopt a semiclassical approach based on the Landau-Vlasov kinetic equation to calculate RF $\chi_{k,L}(\omega)$ [14]. Non-coherent residual interaction is taken into account for the description of the collective state damping. This interaction is included within the relaxation time method with retardation effects during two-body collisions [15-17]. It allows considering the damping of vibrational states in a rather simple and accurate way. The calculations within framework of the RF approach were performed with different expressions for relaxation times $\tau(\omega) = \hbar/\gamma(\omega)$ of the collective states [15-17]. We present in Figs.3 and 4

the results with a collective relaxation time according to the kinetic approach. The coupling constant k of the coherent interaction was found from the fitting of a peak energy of the strength function (the imaginary part of the RF) to the experimental energies $\hbar\omega_{2^+}$ in cold nuclei.

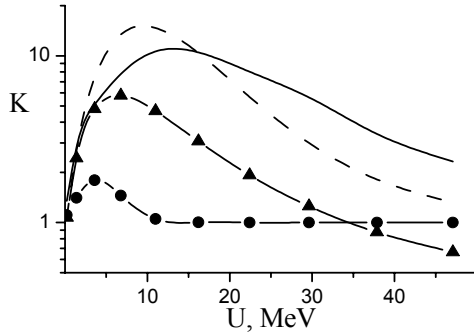


FIGURE 3. The dependence of the enhancement factor on excitation energy for ^{56}Fe : — the RF method; --- K_{DN} ; \blacktriangle — K_{CE} ; \bullet — K_{EM} .

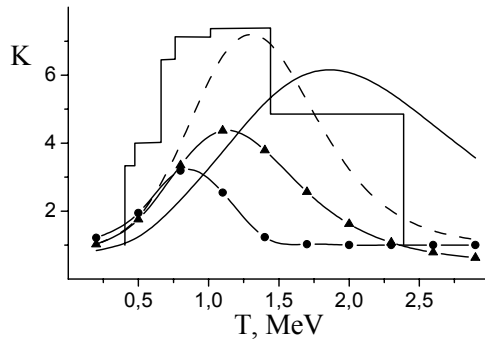


FIGURE 4. The dependence of the vibrational enhancement factor on the temperature for ^{146}Sm ; denotations are the same as in Fig. 2; histogram - temperature-dependent IBM model [19].

It can be seen that calculations within the RF approach agree better with the ones within the method of the attenuated phonon occupation numbers K_{DN} . They are in rather close agreement with the results of a finite temperature extension of the interacting boson model [19]. The calculations within the RF method demonstrate rather strong dependence of the level density on the relaxation time shape. This can give an additional possibility for investigation of the collective-state relaxation in heated nuclei.

The results show that expression for K with the attenuated phonon occupation numbers ($K = K_{DN}$) is the best simple method of the enhancement factor calculations.

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