Some Aspects regarding the Basic Index Number Theory*

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Abstract
The index number problem can be framed as the problem of decomposing the value of a well-defined set of transactions in a period of time into an aggregate price term times an aggregate quantity term. It turns out that this approach to the index number problem does not lead to any useful solutions. The problem of decomposing a value ratio pertaining to two periods of time into a component that measures the overall change in prices between the two periods (this is the price index) times a term that measures the overall change in quantities between the two periods (this is the quantity index) is considered.
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The Mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are as many kinds of average as there are purposes; and we may almost say in the matter of prices as many purposes as writers. Hence much vain controversy between persons who are literally at cross purposes. (Edgeworth (1888)).

If we further distinguish physical commodities by their geographical location or by the season or time of day that they are produced or consumed, then there are billions of commodities that are traded within each year in any advanced economy. For many purposes, it is necessary to summarize this vast amount of price and quantity information into a much smaller set of numbers. The question addressed is: how exactly should the microeconomic information involving possibly millions of prices and quantities be aggregated into a smaller number of price and quantity variables? This is the basic problem of index numbers.

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It is possible to pose the index number problem in the context of microeconomic theory; i.e., given that we wish to implement some economic model based on producer or consumer theory, what is the ‘‘best’’ method for constructing a set of aggregates for the model? When constructing aggregate prices or quantities, however, other points of view (that do not rely on economics) are possible. Some of these alternative points of view are considered.

The simplest price index is a fixed basket type index; i.e., fixed amounts of the n quantities in the value aggregate are chosen and then the values of this fixed basket of quantities at the prices of period 0 and at the prices of period 1 are calculated. The fixed basket price index is simply the ratio of these two values where the prices vary but the quantities are held fixed. Two natural choices for the fixed basket are the quantities transacted in the base period, period 0, or the quantities transacted in the current period, period 1. These two choices lead to the Laspeyres (1871) and Paasche (1874) price indices, respectively.

Unfortunately, the Paasche and Laspeyres measures of aggregate price change can differ, sometimes substantially. Thus, taking an average of these two indices to come up with a single measure of price change is considered. It is argued that the ‘‘best’’ average to take is the geometric mean, which is Irving Fisher’s (1922) ideal price index. Instead of averaging the Paasche and Laspeyres measures of price change, taking an average of the two baskets is considered. This fixed basket approach to index number theory leads to a price index advocated by Correa Moylan Walsh (1901; 1921). Other fixed basket approaches are, however, also possible. Instead of choosing the basket of period 0 or 1 (or an average of these two baskets), it is possible to choose a basket that pertains to an entirely different period, say period b. In fact, it is typical statistical agency practice to pick a basket that pertains to an entire year (or even two years) of transactions in a year prior to period 0, which is usually a month. Indices of this type, where the weight reference period differs from the price reference period, were originally proposed by Joseph Lowe (1823).

Another approach to the determination of the functional form or the formula for the price index is considered. This approach is attributable to the French economist Divisia (1926) and is based on the assumption that price and quantity data are available as continuous functions of time. The theory of differentiation is used in order to decompose the rate of change of a continuous time value aggregate into two components that reflect aggregate price and quantity change. Although the approach of Divisia offers some insights, it does not offer much guidance to statistical agencies in terms of leading to a definite choice of index number formula.

The advantages and disadvantages of using a fixed base period in the bilateral index number comparison are considered versus always comparing the current period with the previous period, which is called the chain system. In the chain system, a link is an index number comparison of one period with the previous period. These links are multiplied together in order to make comparisons over many periods.

The decomposition of value aggregates into price and quantity components

- The decomposition of value aggregates and the product test

A price index is a measure or function which summarizes the change in the prices of many commodities from one situation 0 (a time period or place) to another situation 1. More specifically, for most practical purposes, a price index can be regarded as a weighted mean of the change in the relative prices of the commodities under consideration in the two situations. To determine a price index, it is necessary to know:

- which commodities or items to include in the index;
- how to determine the item prices;
- which transactions that involve these items to include in the index;
- how to determine the weights and from which sources these weights should be drawn;
- what formula or type of mean should be used to average the selected item relative prices.

All the above questions regarding the definition of a price index, except the last, can be answered by appealing to the definition of the value aggregate to which the price index refers. A value aggregate V for a given collection of items and transactions is computed as:

$$V = \sum_{i=1}^{n} p_i q_i \qquad (1)$$

where p_i represents the price of the i^{th} item in national currency units, q_i represents the corresponding quantity transacted in the time period under consideration and the subscript i identifies the i^{th} elementary item in the group of n items that make up the chosen value aggregate V.

Included in this definition of a value aggregate is the specification of the group of included commodities (which items to include) and of the economic agents engaging in transactions involving those commodities (which transactions to include), as well as principles of the valuation and time of recording that motivate the behaviour of the economic agents undertaking the transactions (determination of prices). The included elementary items, their valuation (the pi), the eligibility of the transactions and the item weights (the qi) are all within the domain of definition of the value aggregate.

The value aggregate V refers to a certain set of transactions pertaining to a single (unspecified) time period. Now the same value aggregate for two places or time periods, periods 0 and 1, is considered.

For the sake of convenience, period 0 is called the base period and period 1 is called the current period and it is assumed that observations on the base period price and quantity vectors, $p^0 = [p^0_1, \ldots, p0n]$ and $q^0 = [q^0_1, \ldots, q^0_n]$ respectively, have been collected.4

The value aggregates in the base and current periods are defined in the obvious way as:

$$V^0 \equiv \sum_{i=1}^{n} p_i^0 q_i^0; \; V^1 \equiv \sum_{i=1}^{n} p_i^1 q_i^1 \qquad (2)$$

In the previous paragraph, a price index was defined as a function or measure which summarizes the change in the prices of the n commodities in the value aggregate from situation 0 to situation 1. In this paragraph, a price index $P(p^0, p^1, q^0, q^1)$ along with the corresponding quantity index (or volume index) $Q(p^0, p^1, q^0, q^1)$ is defined to be two functions of the 4 variables p^0, p^1, q^0, q^1 (these variables describe the prices and quantities pertaining to the value aggregate for periods 0 and 1) where these two functions satisfy the following equation:

$$V^1/V^0 = P(p^0, p^1, q^0, q^1)\ Q(p^0, p^1, q^0, q^1) \tag{3}$$

If there is only one item in the value aggregate, then the price index P should collapse down to the single price ratio, $p^1{}_1/p^0{}_1$, and the quantity index Q should collapse down to the single quantity ratio, $q^1{}_1/q^0{}_1$. In the case of many items, the price index P is to be interpreted as some sort of weighted average of the individual price ratios, $p^1{}_1p^0{}_1$, . . . , $p^1{}_n/p^0{}_n$.

Thus the first approach to index number theory can be regarded as the problem of decomposing the change in a value aggregate, V1/V0, into the product of a part that is attributable to price change, $P(p^0, p^1, q^0, q^1)$, and a part that is attributable to quantity change, $Q(p^0, p^1, q^0, q^1)$. This approach to the determination of the price index is the approach that is taken in the national accounts, where a price index is used to deflate a value ratio in order to obtain an estimate of quantity change. Thus, in this approach to index number theory, the primary use for the price index is as a deflator.

Note that once the functional form for the price index $P(p^0, p^1, q^0, q^1)$ is known, then the corresponding quantity or volume index $Q(p^0, p^1, q^0, q^1)$ is completely determined by P; i.e., rearranging the previous equation:

$$Q(p^0, p^1, q^0, q^1) = \left(V^1/V^0\right)/P(p^0, p^1, q^0, q^1) \tag{4}$$

Conversely, if the functional form for the quantity index $Q(p^0, p^1, q^0, q^1)$ is known, then the corresponding price index $P(p^0, p^1, q^0, q^1)$ is completely determined by Q.

Thus using this deflation approach to index number theory, separate theories for the determination of the price and quantity indices are not required: if either P or Q is determined, then the other function is implicitly
determined by the product test equation.

In the next section, two concrete choices for the price index $P(p^0, p^1, q^0, q^1)$ are considered and the corresponding quantity indices $Q(p^0, p^1, q^0, q^1)$ that result from using equation are also calculated. These are the two choices used most frequently by national income accountants.

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