Some Remarks on Transonic Potential Flow Theory

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The validity of the commonly used transonic potential equation for flows with shock waves is examined. It is concluded that in such cases the potential formulation is inconsistent with the basis assumptions of the theory because of the nonconservation of momentum across a shock. The relationship of this momentum source to wave drag is also discussed. Another topic examined is the rationalization of means to make solutions of the transonic potential equation agree better with solutions of the Euler equations.

1 Introduction

At present, the main means of predicting transonic flow characteristics is by numerically solving either the full potential equation [1, 2] or its approximate form, the transonic small disturbance equation [3, 4]. To justify the use of a potential equation to describe transonic flows with shock waves it is usual to assume that entropy changes through a weak shock are negligible and hence, from Crocco's theorem [5] the flow can be considered irrotational. However, the derivation of Crocco's results requires that mass, momentum, and energy be conserved, and since in the present transonic potential, computer-codes axial momentum is not conserved if there are shock waves in the flow, it is obvious that there is an inconsistency in the model. This momentum error is often used to define a wave drag of the airfoil. The present study is concerned with examining the origin and effect of the inconsistency of potential flow theory when shock waves are present in the flow and also the relationship of the momentum error to wave drag.

In Section 2, a perturbation analysis of the potential theory through a normal shock is conducted and it is shown that the momentum error produced by the potential formulation leads to a "wave drag" proportional to the shock strength, whereas the correct result is the cube of the shock strength. Also, a possible theoretical basis for modifying potential theory to give more realistic shock jumps is described. In Section 3, the effect of not conserving momentum on the irrotationality assumption is examined and it is concluded that the potential formulation is only valid for a free-stream Mach number close to unity. An analysis is given which derives a consistent potential theory but it is concluded that this theory would give worse results than the conventional theory.

2 Comments on Isentropic Shock Waves

In this section the behavior of the flow through a one-dimensional isentropic shock wave is examined.

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The isentropic density relation is

$$\frac{\rho}{\rho_\infty} = \left[1 + \left(\frac{\gamma-1}{2}\right) M_\infty^2 (1-q^2)\right]^{\frac{1}{\gamma-1}} \qquad (1)$$

where $q = U/U_\infty$. The pressure relation is

$$\frac{p}{p_\infty} = \left[1 + \left(\frac{\gamma-1}{2}\right) M_\infty^2 (1-q^2)\right]^{\frac{\gamma}{\gamma-1}} \qquad (2)$$

The pressure, density, and velocity are expanded as a series in the perturbation velocity u. Let

$$q = 1 + u \qquad (3)$$

and expand the relations of equations (1) and (2) in powers of u. Hence

$$\rho = \rho_\infty \left\{1 - M_\infty^2 u - M_\infty^2 [1 + (\gamma-2) M_\infty^2] \frac{u^2}{2}\right\} \qquad (4)$$

$$p = p_\infty \left\{1 - \gamma M_\infty^2 u - \frac{\gamma u^2}{2} M_\infty^2 \beta^2\right\} \qquad (5)$$

where

$$\beta^2 = 1 - M_\infty^2 \qquad (6)$$

Through a shock wave, mass, momentum, and energy should be conserved. If this is not true then there are errors E_1, E_2, and E_3 in the conservation laws of mass, momentum, and energy, respectively. Hence

$$E_1 = (\rho_1 U_1 - \rho_2 U_2)/\rho_\infty U_\infty = \left\{\beta^2 [u]_-^+ - \frac{k}{2} [u^2]_-^+\right\} \qquad (7)$$

$$E_2 = (p_1 + \rho_1 U_1^2 - p_2 - \rho_2 U_2^2)/(p_\infty + \rho_\infty U_\infty^2)$$
$$= \gamma M_\infty^2 \left\{\beta^2 [u]_-^+ - \left(\frac{k-\beta^2}{2}\right) [u^2]_-^+\right\} / (1 + \gamma M_\infty^2) \qquad (8)$$

$$E_3 = 0 \qquad (9)$$

where subscript 1 denotes a value upstream of the shock, the subscript 2 denotes a value downstream of the shock, and

$$k = M_\infty^2 [3 + (\gamma-2) M_\infty^2] \qquad (10)$$

The notation $[\ \ ]_-^+$ defines the jump across the shock. The energy equation is satisfied because the isentropic density and pressure relations of equations (1) and (2) are derived by assuming that energy is conserved.

Since $[u]^{+}_{-} = u_1 - u_2$ and $[u^2]^{+}_{-} = u_1^2 - u_2^2$ it can be seen that

$$E_1 = \sigma\left\{\beta^2 - \frac{k}{2}\,\hat{u}\right\} \tag{11}$$

$$E_2 = \gamma M_\infty^2\,\sigma\left\{\beta^2 - \left(\frac{k-\beta^2}{2}\right)\hat{u}\right\}/(1+\gamma M_\infty^2) \tag{12}$$

where

$$\left.\begin{aligned}\sigma &= u_1 - u_2\\ \hat{u} &= u_1 + u_2\end{aligned}\right\} \tag{13}$$

Hence if the shock strength σ is zero then there are no errors in the solution. If the transonic small disturbance equation is formulated as

$$(\beta^2 - \hat{k}u)u_x + v_y = 0 \tag{14}$$

where $\hat{k}$ is a function of the free-stream Mach number, then for normal shock waves the jump relation is

$$\hat{u} = 2\beta^2/\hat{k} \tag{15}$$

Hence if $\hat{k} = k$ then the error in mass conservation is zero and there is a momentum error

$$E_2 = \frac{\gamma M_\infty^2}{(1+\gamma M_\infty^2)}\,\frac{\sigma\beta^4}{k} \tag{16}$$

This is equivalent to an upstream force on the shock wave. If, as is usual in transonic flow calculations, free-stream conditions are enforced at the downstream boundary then a contour integral of momentum around the flow indicates a total conservation of momentum. Hence the momentum error across the shock must be balanced by a pressure force on the airfoil. This is sometimes referred to (erroneously) as the wave drag. It is directly due to an inconsistency of the isentropic equations through a shock wave. This conclusion was obtained by Steger and Baldwin [6]. If the transonic parameter $\hat{k}$ in equation (14) is chosen to be $(k-\beta^2)$ then momentum is conserved but there is a mass error

$$E_1 = -\beta^4\sigma/(k-\beta^2) \tag{17}$$

Note that since transonic small disturbance theory assumes $\beta^4 = 0$ the conservation equations are satisfied to the order of approximation of the theory. However, the foregoing results are also applicable to the full potential equation for which no formal limit on β^2 is required.

It can be inferred from the preceding analysis that since transonic small disturbance theory has traditionally only one flexible parameter, $\hat{k}$, it is impossible to remove both the mass and momentum errors across a shock. However, it may be advantageous to choose the transonic parameter k such that a linear combination of the errors is minimized. Thus, if an error E is defined as

$$E = w_1E_1 + w_2E_2 \tag{18}$$

where w_1 and w_2 may be functions of u_1, then E can be minimized for a given u_1. Thus

$$E = \sigma\left\{\beta^2\left[w_1 + \frac{\gamma M_\infty^2}{1+\gamma M_\infty^2}w_2\right] - \left[kw_1 + \frac{\gamma M_\infty^2}{1+\gamma M_\infty^2}w_2(k-\beta^2)\right]\hat{u}\right\} \tag{19}$$

If $w_1 = 1$, $w_2 = 0$, this reduces to the conventional mass conserving result. If

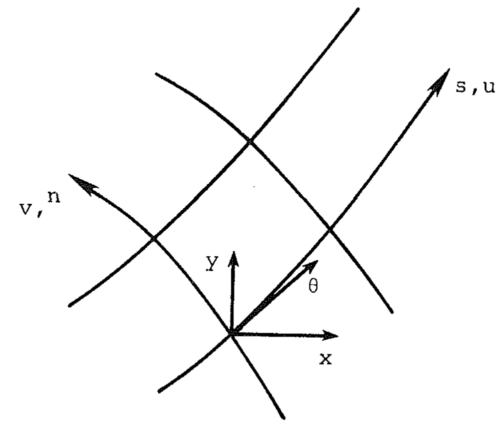

Fig. 1 Sketch of a streamline coordinate system

$$w_1 = \beta^2 - \left(\frac{k-\beta^2}{2}\right)\hat{u}_E; \quad w_2 = -\left(\beta^2 - \frac{k}{2}\,\hat{u}_E\right)$$

then $E = 0$ if k is chosen such that $u = \hat{u}_E$ where $\hat{u}_E$ is the value of $u_1 + u_2$ given by the Euler equations. To a first approximation

$$\hat{u}_E = u_1\left\{2 - \frac{M_\infty^2(\gamma+1)}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)^2}\right\} - \frac{M_\infty^2(\gamma+1)}{2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)} \tag{20}$$

Such a modified small disturbance equation is used by Nixon [7].

The existence of the momentum deficit through the shock given by equation (16) is often assumed to be the drag. The drag coefficient, C_D, is given by the relation

$$C_D = \frac{(p_\infty + \rho_\infty U_\infty^2)E_2}{\frac{1}{2}\rho_\infty U_\infty^2} = 2\beta^4\sigma \tag{21}$$

whereas the formal limit of entropy producing drag as given by Murman and Cole [8] is, in the present notation

$$C_D = \frac{(\gamma+1)}{6M_\infty^2}\,\sigma^3 \tag{22}$$

and which is third order in σ in comparison to the linear dependence on σ of equation (21). Note that to get the complete drag, these drag relations must be integrated along the shock wave.

3 Comments on Momentum Deficit and Irrotationality

In the following analysis it is assumed that mass is conserved, since this allows a simple definition of the stream function coordinate system. It will be assumed that there are possible sources or sinks in momentum and energy. In Fig. 1, s is the streamline direction and n is a coordinate normal to the streamlines. The velocity u is in the stream-wise direction and by definition there is zero flow velocity across the stream tube. The conservation equations for mass, streamwise, and normal momentum and energy are as follows.

$$\frac{\partial\rho U}{\partial s} = 0 \quad \text{(conservation of mass)} \tag{23}$$

$$\rho U \frac{\partial U}{\partial s} = -\frac{\partial p}{\partial s} + \frac{\partial \epsilon_1}{\partial s} \quad \text{(conservation of streamwise momentum)} \tag{24}$$

$$\rho U^2 \frac{\partial \theta}{\partial s} = -\frac{\partial p}{\partial n} + \frac{\partial \epsilon_2}{\partial n} \quad \text{(conservation of normal momentum)} \tag{25}$$

$$\frac{\partial (h + U^2/2)}{\partial s} = \frac{\partial \epsilon_3}{\partial s} \quad \text{(conservation of energy)} \tag{26}$$

where $\partial \epsilon_1/\partial s$, $\partial \epsilon_2/\partial n$, and $\partial \epsilon_3/\partial s$ are the effects due to sources in streamwise momentum, normal momentum, and energy, respectively.

The entropy gradients in the streamwise and normal directions are defined by

$$T\frac{\partial S}{\partial s} = \frac{\partial h}{\partial s} - \frac{1}{\rho}\frac{\partial p}{\partial s} \tag{27}$$

$$T\frac{\partial S}{\partial n} = \frac{\partial h}{\partial n} - \frac{1}{\rho}\frac{\partial p}{\partial n} \tag{28}$$

Integration of the energy equation, equation (26), gives

$$h + U^2/2 = h_0(n) + \epsilon_3(n) \tag{29}$$

where $h_0(n)$ is the reservoir condition. If the fluid is considered a calorically and thermally perfect gas, then

$$h = C_p T = \frac{\gamma}{\gamma - 1} \cdot \frac{p}{\rho} \tag{30}$$

where C_p is the specific heat at constant pressure. Using equations (25) and (29), equations (28) can be written as

$$T\frac{\partial S}{\partial n} = \frac{\partial h_0}{\partial n} - \left(U\frac{\partial U}{\partial n} - U^2\frac{\partial \theta}{\partial s}\right) - \frac{1}{\rho}\frac{\partial \epsilon_2}{\partial n} + \frac{\partial \epsilon_3}{\partial n} \tag{31}$$

The quantity $(\partial U/\partial n - U\partial\theta/\partial s)$ is the vorticity, ζ, of the flow and hence equation (31) can be written as

$$T\frac{\partial S}{\partial n} = \frac{\partial h_0}{\partial n} - U\zeta - \left(\frac{1}{\rho}\frac{\partial \epsilon_2}{\partial n} - \frac{\partial \epsilon_3}{\partial n}\right) \tag{32}$$

The entropy gradient in the streamwise direction can be written, using equation (24), as

$$T\frac{\partial S}{\partial s} = \frac{\partial \epsilon_3}{\partial s} - \frac{1}{\rho}\frac{\partial \epsilon_1}{\partial s} \tag{33}$$

Thus there is a streamwise entropy production due to the errors ϵ_1, ϵ_3. It is assumed that any shock waves in the flow are sufficiently weak that the entropy production due to physical phenomena is negligible.

Consider now the case where both normal momentum and the energy are conserved with no source terms. In this case

$$\left.\begin{aligned} \frac{\partial \epsilon_2}{\partial n} &= 0 \\ \frac{\partial \epsilon_3}{\partial n} &= 0 \end{aligned}\right\} \tag{34}$$

and hence if the reservoir condition h_0 is such that

$$\frac{\partial h_0}{\partial n} = 0 \tag{35}$$

(this is usually the case for transonic gas flows) and if the entropy production gradient normal to the streamlines is zero, i.e.,

$$\frac{\partial S}{\partial n} = 0 \tag{36}$$

Then equation (32) gives the irrotational condition

$$\zeta = 0 \tag{37}$$

If ϵ_1, ϵ_3, are expanded as series in terms of a perturbation velocity u, $[U = U_\infty(1+u)]$, such that

$$\epsilon_1 = \epsilon_1^{(1)}u + \epsilon_2^{(2)}u^2 \ldots$$
$$\epsilon_3 = \epsilon_3^{(1)}u + \epsilon_3^{(2)}u^2 \ldots$$

and if

$$T = T_\infty(1 + \alpha_T^{(1)}u + \alpha_T^{(2)}u^2 \ldots)$$
$$p = p_\infty(1 + \alpha_p^{(1)}u + \alpha_p^{(2)}u^2 \ldots)$$

then

$$\frac{\partial S}{\partial s} = \frac{1}{T_\infty}(\epsilon_3^{(1)} + 2\epsilon_2^{(2)}u) \times (1 - \alpha_T^{(1)}u - (\alpha_T^{(2)} - \alpha_T^{(1)2})u^2)\frac{\partial u}{\partial s} - \frac{R}{p_\infty}(\epsilon_1^{(1)} + 2\epsilon_1^{(2)}u) \times (1 - \alpha_p^{(1)}u - (\alpha_p^{(2)} - \alpha_p^{(1)2})u^2)\frac{\partial u}{\partial s} \tag{38}$$

where the subscript ∞ denotes free-stream conditions and R is the gas constant. Equation (38) can be integrated to give

$$\Delta S = \frac{1}{T_\infty}\left(\epsilon_3^{(1)}[u]_-^+ + \epsilon_3^{(2)}[u^2]_-^+\right) - \frac{1}{T_\infty}\alpha_T^{(1)}[u^2]_-^+\epsilon_3^{(1)} - \frac{R}{p_\infty}\left(\epsilon_1^{(1)}[u]_-^+ + \epsilon_1^{(2)}[u^2]_-^+\right) + \frac{R}{p_\infty}\alpha_p^{(1)}[u^2]_-^+\epsilon_1^{(1)} + 0(u)^3 \tag{39}$$

where Δ denotes a difference from some reference condition.

Finally, it should be noted that by using equations (27) and (30),

$$\frac{p}{\rho^\gamma} = Ke^{\frac{(\gamma-1)S}{R}} \tag{40}$$

where K is a constant. If the free-stream conditions are the reference conditions for equation (40) then

$$K = \frac{p_\infty}{\rho_\infty^\gamma} \tag{41}$$

In an inviscid irrotational continuous flow it can be shown [5] that the conservation of mass and energy, together with the isentropic relations for p, ρ ensures conservation of momentum. However, if there is a discontinuity normal to the streamlines in the flow, then it is shown earlier that this set of equations does not conserve momentum through the discontinuity. In many transonic calculations this momentum deficit is erroneously referred to as wave drag. Since the isentropic approximation to transonic flow requires the basic assumption that mass, momentum, and energy be conserved, there is an obvious inconsistency in the overall theory. This momentum error only occurs at a shock wave and from equation (39) this error shows up as an entropy production term. However, it is possible that a self-consistent potential theory can be derived and this possibility is examined in the subsequent analysis.

Consider now the case of a transonic flow that has sufficiently weak shock waves that no entropy production from purely thermodynamic means is significant. Assume also the shock wave is normal to the streamlines, thus ensuring conservation of normal momentum. Finally, assume that total enthalpy is conserved throughout the flow; this is consistent with the isentropic model since the necessary density/velocity relation is found by assuming conservation of total enthalpy.

In the remaining analysis the error ϵ_3 is set to zero, implying conservation of total enthalpy and the pressure/density relation, equations (40) and (41) are written as

$$(p/p_\infty)/(\rho/\rho_\infty)^\gamma = 1+\delta \qquad (42)$$

where δ is of order $[u]^{\pm}_{-}$, the shock strength, from equations (39) and (40). If it is assumed the shock waves are weak, then powers of δ greater than unity can be neglected. Substitution of equation (42) into the energy equation, equation (29) gives

$$\frac{\gamma}{(\gamma-1)} \cdot \frac{p_\infty}{\rho_\infty}\left(\frac{\rho}{\rho_\infty}\right)^{\gamma-1}(1+\delta) = \frac{\gamma}{\gamma-1}\,\frac{p_\infty}{\rho_\infty} + (U_\infty^2/2)\left(1-\frac{U^2}{U_\infty^2}\right) \qquad (43)$$

In terms of the perturbation velocity, this is

$$\frac{\rho}{\rho_\infty} = \left\{1-\frac{\gamma-1}{2}M_\infty^2(2u+u^2)\right\}^{\frac{1}{\gamma-1}} / (1+\delta)^{\frac{1}{\gamma-1}} \qquad (44)$$

An expansion to second order in u gives

$$\frac{\rho}{\rho_\infty} = 1 - M_\infty^2\,u - \frac{u^2}{2}[1+M_\infty^2(\gamma-2)]M_\infty^2 - \frac{\delta}{\gamma-1} \qquad (45)$$

The pressure relation is found by taking

$$\frac{p}{P_\infty} = (1+\delta)(\rho/\rho_\infty)^\gamma$$

$$\frac{p}{P_\infty} = 1-\gamma M_\infty^2\,u - \frac{u^2M_\infty^2\beta^2}{2} - \frac{\delta}{\gamma-1} \qquad (46)$$

where $\beta^2 = 1-M_\infty^2$.

Across a normal shock wave the errors in the conservation of mass, momentum, and energy are as follows: The error terms ϵ_1, δ only contribute to the values on the downstream side of the shock:

$$E_{\text{mass}} = \rho_1 U_1 - \rho_2 U_2 = \rho_\infty U\Big(\beta^2[u]^{\pm}_{-} - \frac{k}{2}[u^2]^{\pm}_{-}\Big) + \frac{\rho_\infty U_\infty \delta}{\gamma-1}. \qquad (47)$$

$$E_{\text{momentum}} = p_1 + p_1U_1^2 - \rho^2 - \rho_2 U_2^2 =$$

$$\rho_\infty U_\infty^2\Big(\beta^2[u]^{\pm}_{-} - \frac{(k-\beta^2)}{2}[u^2]^{\pm}_{-}\Big) + \frac{\rho_\infty U_\infty^2\delta}{\gamma-1}\left(1+\frac{1}{\gamma M_\infty^2}\right) \qquad (48)$$

$$E_{\text{energy}} = 0 \qquad (49)$$

In the preceding equations, $k=[3+(\gamma-2)M_\infty^2]M_\infty^2$ and $[\ \ ]^{\pm}_{-}$ denotes a jump across the shock wave. Note that the result of equation (49) confirms the consistency of putting the energy error equal to zero.

Now assume that the solution algorithm conserves mass. Thus the error in equation (47) is zero and then the solution has shock jumps given by

$$u_2 = \frac{1}{k}\left(\beta^2 \pm \left[(\beta^2-ku_1)^2 - \frac{2k\delta}{\gamma-1}\right]^{1/2}\right) \qquad (50)$$

From equations (39)–(42)

$$\delta = \frac{-(\gamma-1)}{p_\infty}\{(\epsilon_1^{(1)}[u]^{\pm}_{-} + \epsilon_1^{(2)}[u^2]^{\pm}_{-}) - \alpha_p^{(1)}\epsilon_1^{(1)}[u^2]^{\pm}_{-}\} \qquad (51)$$

and since, by definition,

$$E_{\text{momentum}} = -(\epsilon_1^{(1)}[u]^{\pm}_{-} + \epsilon_1^{(2)}[u^2]^{\pm}_{-}) \qquad (52)$$

equation (48) may be written as

$$-\{\epsilon_1^{(1)}[u]^{\pm}_{-} + \epsilon_1^{(2)}[u^2]^{\pm}_{-}\} = \rho_\infty U_\infty^2\Big[\beta^2[u]^{\pm}_{-} - \frac{k[u^2]^{\pm}_{-}}{2} + \frac{\delta}{\gamma+1}\Big] + \rho_\infty U_\infty^2\frac{\beta^2}{2}[u^2]^{\pm}_{-} - \Big[\epsilon_1^{(1)}[u]^{\pm}_{-} + \epsilon_1^{(2)}[u^2]^{\pm}_{-} - \alpha_p^{(1)}\epsilon_1^{(1)}[u^2]^{\pm}_{-}\Big] \qquad (53)$$

Since from conservation of mass the first term in square brackets in equation (50) is zero, equation (53) becomes

$$-\Big[\alpha_p^{(1)}\epsilon_1^{(1)} - \rho_\infty U_\infty^2\frac{\beta^2}{2}\Big][u^2]^{\pm}_{-} = 0 \qquad (54)$$

Hence for a consistent formulation either the flow is continuous,

$$u_1 = u_2 \qquad (55)$$

or the flow has a shock wave with the jump relation

$$u_1 = -u_2 \qquad (56)$$

or, in terms of the shock strength $\sigma = u_1 - u_2$

$$\sigma = 2u_1 \qquad (57)$$

To the same order of accuracy the conventional "isentropic" jump relation is

$$\sigma = 2u_1 - \frac{2\beta^2}{k} \qquad (58)$$

and hence for $\beta^2 \neq 0$ the consistent theory gives a stronger shock than the inconsistent conventional theory. Since the conventional shock is already too strong it is probable that a consistent theory is too inaccurate for practical calculations.

From equations (50) and (56) the term δ is given by

$$\delta = -2\beta^2u_1(\gamma-1) \qquad (59)$$

for a flow with a discontinuity. The error ϵ_1 is given by a combination of equations (59) and (51); thus

$$\epsilon_1^{(1)}[u]^{\pm}_{-} + \epsilon_1^{(2)}[u^2]^{\pm}_{-} - \alpha_p^{(1)}[u^2]^{\pm}_{-}\epsilon_1^{(1)} = 2p_\infty\beta^2u_1 \qquad (60)$$

The preceding discussion can be summarized as follows.

(*a*) The conventional potential theory is inconsistent because axial momentum is not conserved.

(*b*) A consistent, irrotational, one-dimensional theory can be derived if the shock wave is normal to the free stream.

(*c*) It is probable that the results of using this consistent theory are more inaccurate than results of the inconsistent conventional theory.

In view of these conclusions, it is suggested that the conventional theory can enhanced by the addition of variables such as modifying the potential equation either by analytic means [9] or nonconservative differencing [10]. Since both conventional and modified theories are inconsistent, it would seem that a modified theory is as valid as the conventional theory.

For the irrotational assumption to hold the quantity $|\delta| << 1$ and hence from equation (59)

$$\beta^2 u_1 << 1 \tag{61}$$

Now to a first approximation

$$\beta^2 - ku_1 = 1 - M_1^2$$

where M_1 is the Mach number just upstream of the shock. Hence equation (61) can be written as

$$\frac{\beta^2 (M_1^2 - M_\infty^2)}{k} << 1 \tag{62}$$

It can be concluded that apart from its treatment of the tangency boundary conditions, the full potential equation is formally no more accurate than the small disturbance equation since both require $\beta^2 u_1 << 1$.

In equations (11) and (12) it can be seen that if mass is conserved then momentum is conserved only if

$$[u^2]_-^+ = 0$$

which, in the case of conventional potential theory, is *not* possible since $[u^2]_-^+$ is solely determined by the mass conservation equation. In the consistent theory, although the same requirement of $[u^2]_-^+$ is needed, there is an additional parameter in the mass conservation equation, the "δ" term that allows this requirement to be satisfied. It should be noted that even for very weak shock waves, the consistent theory *does not approach the conventional theory; this only occurs* for continuous flow.

Finally, it should be noted that since $\delta < 0$ for equation (59) the entropy due to the momentum change through the shock wave decreases, which contravenes the second law of thermodynamics. Thus the consistent potential theory is not physically plausible.

Concluding Remarks

Several aspects of the transonic potential theory have been examined and it is concluded that there are several inconsistencies in the theory. It is also suggested that there are some commonly held misunderstandings in the interpretation of the results of potential theory calculations.

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