Non-Blocking Conditions for EGS Networks

Italo Busi and Achille Pattavina

***Abstract*—A class of multistage interconnection networks, known as extended generalized shuffle (EGS) network, is here considered; it is built with arrays of elementary switching elements, interconnected by shuffle patterns, and external splitters and combiners. New results are given here that reduce the splitter fanout required to make the network strictly non-blocking.**

***Index Terms*—Interconnection networks, Extended Generalized Shuffle, non-blocking conditions.**

I. Introduction

An *interconnection network* is the set of devices needed to make available input-output connections within a node of a communication network. A *multistage* interconnection network includes a cascade of switching stages, each composed by a stack of switching matrices. The basic property of a multistage interconnection network is its capability of making available upon request one of its internal paths in order to connect an input and an output port by proper operation of the internal resources. For the case of *full interstage connectivity*, in which each matrix has access to any matrices of the following stage, Clos provided the condition to design a minimum-cost three-stage network that is always non-blocking for any input-output new connection request [1].

Banyan networks were later proposed as multistage arrangements of very simple matrices, called switching elements (SEs), with typical size 2×2. *Partial interstage connectivity* is accomplished by these networks, which provide a single internal path for any input-output pair. Non-blocking conditions can be provided by making available more internal paths through horizontal stage extension, vertical replication of the multistage network, or a combination of the two solutions [2].

Extended generalized shuffle (EGS) networks represent a very particular multistage arrangement of SEs in which all stages are mutually interconnected through a regular interstage connection pattern called "EGS interstage pattern" [3]. EGS networks have been considered attracting in free-space optical implementations due to their regular interstage pattern [4]. Other interesting applications areas of EGS patterns have been the minimization of interstage link crossovers in case of guided-wave implementations [5] and the realization of Clos networks by using MEMS devices [6].

Here we consider EGS networks in which all the SEs have the same size, i.e. 2×2. An $N \times N$ EGS network with $N = 2^n$ includes a stage of N splitters $1 \times F$, s stages of $N \cdot F/2$ SEs 2×2 and a final stage with N combiners $F \times 1$. The number s of internal switching stages ranges from 1 to $2n - 1$, where $n = \log_2 N$. Interstage pattern i connects switching stages i and $i+1$; pattern 0 connects splitters to switching stage 1. An EGS network 8×8 with $F = 4$ and $s = 4$ is shown in Fig. 1. It includes 8 splitters of size 1×4, 4 switching stages with $F \cdot N/2 = 16$ SEs of size 2×2 and 8 combiners of size 4×1. All interstage patterns are of EGS type; in fact, starting from first output of upstream SEs/splitters connected to first input of downstream SEs/combiners, consecutive upstream outputs are connected to consecutive downstream elements selecting their first available input.

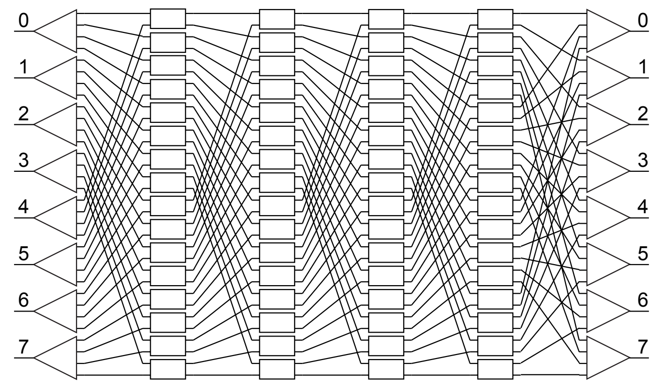

Fig. 1. 8×8 EGS network with $F = 4$ and $s = 4$.

Strict-sense non-blocking (SNB) conditions on the splitter fanout F are known [3] that guarantee at least one idle internal path for any arbitrary idle input-output couple, independent of the current network occupancy by other input-output connections. In the case of $s \leq n$ these conditions are necessary and sufficient, while for $s > n$ they are only sufficient. We show here how these conditions lead to overdimensioning a non-blocking network for networks larger than a given size.

II. Sufficient Non-Blocking Conditions

The sufficient conditions to make the EGS networks strict-sense non-blocking are found as corollary of a more general theorem [3]. In order to prove how overdimensioning occurs, we prove these conditions for our case of EGS network based on 2×2 SEs, following the same approach used in Ref. [3].

Theorem 1: An EGS network $N \times N$ with $s > n$ switching stages ($n = \log_2 N$) is SNB if the splitter fanout is

$$F_r \geq \begin{cases} \frac{3}{2} 2^{\frac{2n-s}{2}} + s - n - 1 & s \text{ } even \\ 2^{\frac{2n-s+1}{2}} + s - n - 1 & s \text{ } odd \end{cases} \quad (1)$$

Proof: The proof consists in considering the tagged connection 0-0 to represent a generic connection from an idle inlet to an idle outlet. In order to find the the worst traffic load pattern that blocks the maximum number of paths for this connection, the corresponding *channel graph* is used which

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I. Busi is with Alcatel-Lucent, Italy (e-mail: italo.busi@alcatel-lucent.com).

A. Pattavina is with the Dept. of Electronics and Information, Politecnico di Milano, Italy (e-mail: pattavina@elet.polimi.it).

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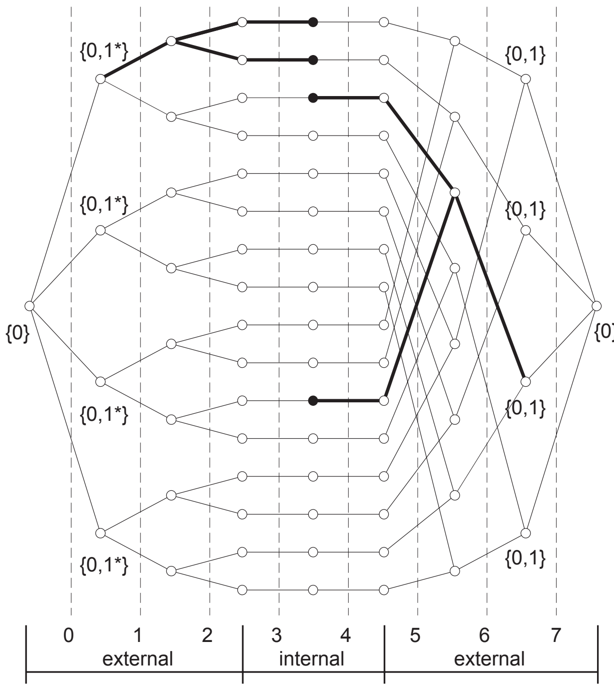

Fig. 2. Channel graph of the EGS network 32×32 with $s = 7$ and $F = 4$.

represents the set of all the paths between a generic inlet and a generic outlet. Fig. 2 shows the channel graph of the EGS network 32×32 with $F = 4$ and $s = 7$ referred to the inlet-outlet couple 0-0; hence it includes 7 stages of nodes preceded and followed by a single node which represents the splitter and the combiner where the selected inlet and outlet terminate, respectively. The figure also shows within curly brackets the network inlet(s) or outlet(s) associated to the graph nodes with the convention that x^* denotes the inlet/outlet whose address is the bit reversal of the address x; for example 1*=100 if $N = 8$. Therefore the nodes labelled $\{0, 1^*\}$ represent the SEs of first switching stage reached by network inputs 0 and 16, if $N = 32$. Apparently the channel graph is the same for every inlet-outlet couple, but we will refer to this specific channel graph so as to identify other inlet-outlet connections interfering with the paths available for the inlet-outlet pair 0-0.

The concept of *shell* in a channel graph has been defined in Ref. [7] as a set of interstage links that belongs to two specific interstage patterns: in this network we define shell 0 to be formed by the interstages 0 and s, shell 1 to be formed by interstages 1 and $s-1$ and in general shell k is formed by interstages k and $s-k$ (Fig. 2 includes four shells). Shells from 0 to $s-n$ are called *external shells*, while shells from $s-n+1$ to $\lfloor (s-1)/2 \rfloor$ are called *internal shells*. We note that a connection that blocks the tagged connection 0-0 in shell k blocks 2^{s-n-k} paths, if the shell is external, only 1 path, if the shell is internal. Fig. 2 shows that connections 1*-x and y-1 (i.e. 16-x and y-1, if N=32) block 2 paths each, with $x \neq 0, 1$ and $y \neq 0, 1^*$; blocked paths are represented by black circles.

The tagged connection cannot be blocked in shell 0, because it includes the splitter and combiner stages. The first shell can be blocked by the inlet 1^* and the outlet 1. The connection originating from inlet 1, as well the one terminating onto outlet 1, blocks 2^{s-n-1} paths. By considering the two sets of blocked paths as disjoint, the maximum number of paths blocked in the first shell is $n_{b1} = 2^{s-n}$. The k-th shell can be blocked by 2^{k-1} inlets and 2^{k-1} outlets, and hence in the worst case there are 2^k blocking connections. If the shell is external, every connection blocks 2^{s-n-k} paths, and considering these 2^k sets of blocked paths as disjoint, the maximum number of blocked paths in the k-th external shell is

$$n_{bk} = 2^{s-n} \qquad 1 \leq k \leq s-n \tag{2}$$

If this shell is internal every connection blocks a single path; hence the maximum number of blocked paths in the k-th internal shell is

$$n_{bk} = 2^k \qquad s-n+1 \leq k \leq \lfloor (s-1)/2 \rfloor \tag{3}$$

If s is odd, there are $(s+1)/2$ shells and considering all the sets of blocked paths as disjoint, the maximum number of blocked paths given by Eqs. 2-3 is

$$n_b = \sum_{k=1}^{(s-1)/2} n_{bk} = 2^{s-n} \left(2^{\frac{2n-s+1}{2}} + s - n - 2 \right) \tag{4}$$

If s is even, there are $s/2$ shells and one central interstage. Unlike all the interstages in the external and internal shells, every link in the central interstage "sees" the same number of network inlets and outlets. Therefore its contribution to the number of blocked paths must not be counted twice as with the shells. There are $2^{s/2-1}$ inlets and outlets that can block this interstage, so the number of paths blocked in the central stage is $2^{s/2-1}$. The maximum number of blocked paths is then given by Eqs. 2-3, i.e.

$$n_b = \sum_{k=1}^{s/2-1} n_{bk} + 2^{\frac{s}{2}-1} = 2^{s-n} \left(\frac{3}{2} 2^{\frac{2n-s}{2}} + s - n - 2 \right) \tag{5}$$

The EGS network is non-blocking if the number of paths available for the tagged connection 0-0 is greater than the maximum number of blocked paths. In an EGS network the number of paths available for every connection is $2^{s-n}F$ and hence from the condition

$$2^{s-n} F_r \geq n_b + 1$$

Eq. 1 follows immediately. ■

The above theorem gives only sufficient conditions for the internal fanout F of an EGS network built with 2×2 SEs to be non-blocking. This is due to the assumption of disjointness of the blocked paths seen from the input side and from the output side. For example, in an 8×8 EGS network with $s = 4$ and $F = 2$ it is easy to show that the connections 4-4, 1-1, 2-2 and 6-3 properly routed determine a blocking state for the connection 0-0; hence the condition $F_r = 3$ derived through Eq. 1 represents in this case a condition both necessary and sufficient to make this network non-blocking. Thus a network blocking state can be found by examining a specific network configuration; nevertheless no algorithm is known to find all the cases in which the conditions (1) are both necessary and sufficient. Ref. [3] reports a refinement of the sufficient conditions given in Eq. 1 making non-blocking a network built with 2 SEs, that is our case. There it is claimed that such refinement refers to networks with $n \geq 8$ that is with size larger than or equal to 256×256. In the following section we show and prove that such refinement actually applies to a larger set of network size, that is starting from size 32×32.

TABLE I
SPLITTER FANOUT F MAKING AN EGS NETWORK NON-BLOCKING.

	N=8	16	32	64	128	256	512	1024
s=4	3							
5	3	4						
6		4	6					
7		4	5	8				
8			5	7	12			
9			$5 \to 4$	6	9			
10				6	8	13	24	
11				$6 \to 5$	7	10	17	32
12					$7 \to 6$	9	14	25
13					$7 \to 6$	$8 \to 7$	11	18
14						$8 \to 7$	10	15
15						$8 \to 7$	$9 \to 8$	12
16							$9 \to 8$	$11 \to 10$
17							$9 \to 8$	$10 \to 9$
18								$10 \to 9$
19								$10 \to 9$

III. REDUCTION OF THE SPLITTER FANOUT

Eqs. 4-5 give the maximum number of blocked paths in an EGS network. According to the expression of F_r given in Eq. 1, n_b can also be expressed as

$$n_b = 2^{s-n}\left(F_r - 1\right) \tag{6}$$

that is as the number of paths in the EGS network with $F = F_r - 1$. The sufficient condition for F_r has been found by assuming that all the path sets made unavailable by the blocking connections are disjoint. So if we find two non-disjoint sets of blocked paths in the network with $F = F_r - 1$, the maximum number of blocked paths given by Eq. 6 decreases by one. Thus in these cases a more convenient sufficient condition $F_r - 1$ holds, instead of F_r.

The channel graph of an EGS network with $F = 4$ and $s = 7$ is shown in Fig. 2. According to the proof of theorem 1 the blocking connection originated from the inlet 1* blocks 2^{s-n-1} consecutive paths in the channel graph. We can assume these paths to be the first 2^{s-n-1} ones, without loss of generality. We can also note that there are F nodes in the first stage between the interstages 0 and 1 and also in the last stage between the interstages $s - 1$ and s. Furthermore there are $2F$ nodes in the stage 2 and $s - 1$ of the channel graph. There are also $2F$ edges between stages $s - 1$ and s, and hence there is a one-to-one correspondence between these edges and the nodes in stage $s - 1$.

By observing the right side of the channel graph, we can notice that the first blocked node in the central stage, which represents the first blocked path, is connected with the first node in stage $s - 1$. Hence if we route the connection terminating onto outlet 1 through this link, the two sets of blocked paths have at least this element in common. In the same way the second node in the central stage, associated to the second path, is connected to the second node in stage $s - 1$. Hence in order to have disjoint sets of blocked paths, the connection terminating onto the outlet 1 does not have to be connected through this node.

If $2^{s-n-1} < 2F$, as in the channel graph shown in Fig. 2, there is at least a node in stage $s - 1$ that is not connected with blocked nodes in the central stage, and hence by routing the connection terminating onto the outlet 1 through this link, we obtain two disjoint sets of blocked paths.

On the other hand if $2^{s-n-1} \geq 2F$, the first $2F$ blocked nodes in the central stage are connected to all the $2F$ nodes in stage $s - 1$, and hence the set of paths blocked by the connection terminating onto the outlet 1 has always at least one element in common with the set of the paths blocked by the connection originating from the inlet 1*. In this case there are at least two non-disjoint sets of blocked paths, and hence the maximum number of blocked paths becomes $n_b \leq 2^{s-n}\left(F_r - 1\right) - 1$. Thus it is not possible to block the EGS network with $F = F_r - 1$, since the number of its paths is $2^{s-n}(F_r - 1)$; so F_r is only a sufficient condition and $F_r - 1$ is a more restrictive sufficient condition.

We have thus proven that if

$$2^{s-n-1} \geq 2F_r - 2 \tag{7}$$

it is not possible to block an EGS network with $F = F_r - 1$. Hence as long as this condition holds, the fanout value F_r given by Eq. (1) that guarantees a non-blocking network must be reduced by one.

Table I shows for an EGS network with $n = 3, \ldots, 10$ and $n < s \leq 2n - 1$ the splitter fanouts F_r that make the network strictly non-blocking. The fanout values reported with an arrow represent configurations in which the splitter fanout reduction occurs, based on Eq. 7. Among the fanout values that are reduced we reconfirm those claimed in Ref. [3].

We remind that a replicated banyan network (RBN) $N \times N$ with K planes is strict-sense non-blocking (SNB) [8] if and only if $K = F_r$ banyan planes are equipped, each with m additional stages, with F_r given by Eq. (1) and $s - n = m$. Splitters and combiners have therefore the same size in EGS and RBN networks and hence non-blocking RBN and EGS networks require the same number of SEs. It is worth noting that the necessary SNB condition of RBNs relies on the series-parallel type of the channel graph and therefore path overcounting does not occur. We have provided here a new sufficient SNB condition for EGS networks, so that a non-blocking EGS network with size $N \geq 32$ requires less SEs than a RBN of the same size, as long as Eq. 7 is satisfied.

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