Consistent Tri-directional Reasoning with Human Concepts

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1 Introduction

In many situations, a process-operator expresses his knowledge about a process in terms of human concepts, e.g. "A low temperature causes a high pressure", but is not able to give the mathematical relation underlying the process. When giving decision-support to a human operator, the *explanation* that accompanies the given hypothesis is as important as the hypothesis itself and should, therefore, be expressed in human concepts. In this article, we present a reasoning system that is able to *reason with human concepts*, by using fuzzy sets to represent these concepts. Moreover, the presented reasoning system is able to reason forward (*deduction*) and backward (*abduction*) as well as learn from examples (*induction*). Most other reasoning theories only consider one reasoning form, however, in our view, the three reasoning forms must be considered simultaneously [1]. As is illustrated in Figure 1, each threesome of cause, effect and corresponding rule-strength makes up three *inseperably* connected relationships, where an instantion of two out of three membership values completely sets the third. Each reasoning form (and corresponding reasoning operator) is associated with one such relationship, and therefore should be (chosen) consistent with eachother. Another aspect that is overlooked by other reasoning theories is that in assigning a *rule-strength* between two fuzzy sets, the influence of the other sets is implicitly neglected. In order to decouple individual rules from their inherent multi-rule environment, we have introduced the concept of sub-effects [1]. Figure 2 shows a schematic overview of the reasoning system.

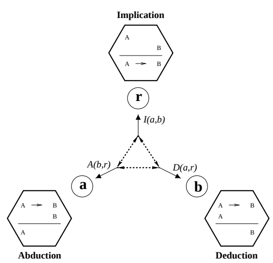

Figure 1: The three relationships

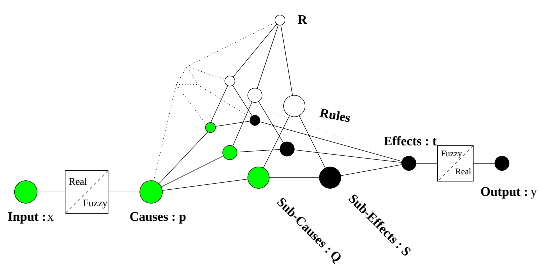

Figure 2: The Reasoning System

2 Fuzzyfication

Fuzzyfication is defined as a function $\mathcal{F}$ which performs a *unique* mapping from a subset of the real domain into the fuzzy domain:

$$\mathcal{F}_x : X \rightarrow [0,1]^{N_X} \; : \; \mathcal{F}_x(x) = \mathbf{p} = \{\, p_i = \mu_{X_i} \mid i = 1, \ldots N_X \,\} \qquad N_X \in \mathbb{N}\backslash\{0\} \tag{1}$$

, where N_X equals the number of fuzzy sets (or human concepts, e.g. "a low temperature") on X. Similarly to $\mathbf{p}$, $\mathbf{t}$ represents the membership values of N_Y fuzzy sets on Y. In order to predict real values, defuzzyfication must be the inverse operation of fuzzyfication, hence finding that real value that resembles most the given unique membership set [2].

$$\mathcal{F}_x^{-1} : [0,1]^{N_X} \rightarrow X \; : \; \mathcal{F}_x^{-1}(\mathbf{p}) = arg\min_x ||\, \mathcal{F}(x) - \mathbf{p}\,|| \qquad N_X \in \mathbb{N}\backslash\{0\} \tag{2}$$

3 Reasoning

By introducing the concept of a sub-effect $Y_j \triangleleft X_i$, representing the individual contribution of cause X_i to the total effect Y_j, this sub-effect is completely determined by cause X_i and the corresponding rule strength between X_i and Y_j. The relation $X_i \rightarrow Y_j \triangleleft X_i$ is defined by the deduction operator $\mathcal{D}$ that is chosen depending on the application domain. Finally, the resulting effect Y_j is completely determined by all the corresponding sub-effects

Table 1: Overview of operators

	Relationship	Generic	Choosen
Deduction	$X_i \rightarrow Y_j \lhd X_i$	$s_{i,j} = \mathcal{D}(p_i, r_{i,j})$	$s_{i,j} = p_i \cdot r_{i,j}$
Aggregation	$Y_j = \mathcal{AGG}_i(Y_j \lhd X_i)$	$t_j = \mathcal{AGG}_i(s_{i,j})$	$t_j = \sum_i s_{i,j}$

that can be combined using the aggregation operator $\mathcal{AGG}$ that is also chosen depending on the application domain. In this article we have chosen multiplication for the deduction and summation for the aggregation operator (see also table 1). In [1] we have defined a basic set of properties for each operator and give several functions that comply with these properties. Once the deduction operator $\mathcal{D}$ and the aggregation operator $\mathcal{AGG}$ are chosen, the relationships in the model become known and the three reasoning forms can be determined.

- *Deduction*, i.e. determine $\hat{\mathbf{t}}^k$, given $\mathbf{p}^k$ and $\mathbf{R}$, is performed by a simple matrix multiplication:

$$\hat{\mathbf{t}}^k = \mathcal{DED}[\, \mathbf{p}^k \,;\, \mathbf{R}\,] = \mathbf{p}^k \cdot \mathbf{R} \tag{3}$$

  , where superscript k indicates the k-th observation and $\mathbf{R}$ the complete rulebase.

- *Induction*, i.e. determine $\mathbf{R}$, given K observations of $\mathbf{p}^k$ and $\mathbf{t}^k$, follows by minimizing the distance between the estimated $\mathbf{t}^k$, being $\hat{\mathbf{t}}^k$, and the observed $\mathbf{t}^k$ for all samples. This is equivalent to minimizing the following error-function:

$$E = \frac{1}{K} \cdot \sum_k \|\, \mathcal{F}(\hat{y}^k) - \mathcal{F}(y^k)\, \|^2 = \frac{1}{K} \cdot \sum_k \|\, \hat{\mathbf{t}}^k - \mathbf{t}^k \,\|^2 = \frac{1}{K} \cdot \sum_k \|\, \mathbf{p}^k \cdot \mathbf{R} - \mathbf{t} \,\|^2 \tag{4}$$

  In order to determine the rulebase that minimizes E, we employ the Mean Square Error estimation:

$$\mathbf{R} = (\mathbf{P}^T \cdot \mathbf{P})^{-1} \cdot \mathbf{P}^T \cdot \mathbf{T} \tag{5}$$

  , where the k-th row of $\mathbf{P}$ and $\mathbf{T}$ corresponds to one observation set of $\mathbf{p}^k$ and $\mathbf{t}^k$ respectively.

- *Abduction*, i.e. determine $\hat{\mathbf{p}}^k$, given $\mathbf{t}^k$ and $\mathbf{R}$, is performed by finding all sets of causes that result in the given set of effects if deduction was applied on these causes:

$$\hat{\mathbf{p}}^k = \mathcal{ABD}[\, \mathbf{t}^k \,;\, \mathbf{R}\,] = \{\, \mathbf{p} \mid \mathbf{p} \cdot \mathbf{R} = \mathbf{t}^k \,\} \tag{6}$$

These reasoning forms can be used to predict the real valued output as well as the input of a complex process, after being learned from examples and is able to give explanations by providing the most significant rules.

- *Predict real-valued output* $\hat{y}^k$, given the input x^k and known rulebase $\mathbf{R}$:

$$\hat{y}^k = \mathcal{F}_y^{-1}\big(\, \mathcal{DED}[\, \mathcal{F}_x(x^k) \,;\, \mathbf{R}\,]\,\big) = \mathcal{F}_y^{-1}\big(\, \mathcal{F}_x(x^k) \cdot \mathbf{R}\,\big) \tag{7}$$

- *Predict real-valued input* $\hat{x}^k$, given the output y^k and known rulebase $\mathbf{R}$:

$$\hat{x}^k = \mathcal{F}_x^{-1}\big(\, \mathcal{ABD}[\, \mathcal{F}_y(y^k) \,;\, \mathbf{R}\,]\,\big) = \mathcal{F}_x^{-1}\big(\, \{\, \mathbf{p} \mid \mathbf{p} \cdot \mathbf{R} = \mathcal{F}_y(y^k)\}\,\big) \tag{8}$$

In this article we have presented a consistent tri-directional reasoning system and shown how it can be applied to model a complex process.

References

[1] E.P. van Someren. Extension of the fuzzy logic algebra with a class of reasoning operators. Graduation Thesis, Information and Communication Theory Group, TU Delft, October 1990.

[2] E.P. van Someren and M.J.T. Reinders. Intelligent molecular diagnostic system. In *Proceedings of the fifth Annual Conference of the Advanced School for Computing and Imaging ASCI'99*. Heijen, The Netherlands, 15-17 June 1999. (*Submitted*).