

Transmission of Information and Interaction in the Mutual Motion of Two Physical Bodies MSR (Motion Shapes Reality)

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Abstract When two bodies are in mutual motion, it should not be considered that one of them is stationary while the other moves or vice versa, but that both bodies move in relation to the center of mass (which is motionless, conditionally) and that they move at speeds dependent on the relationships of their masses, which is the consequence of the law of conservation of momentum. The time of a light signal travelling between two bodies A and B in mutual motion at the velocity of v_0 depends on the relationship between the masses of these bodies m_A and m_B , so light signal travel time from the body A to the body B differs from the light signal travel time from the body B to the body A. In accordance with this, the following notions are defined: the relationship of the time difference (interval) between two successively emitted light signals from one body and the time difference (interval) of receiving these two signals by the other body, as well as the intensity and relationship between the relative velocities v of the two bodies measured from one body and from the other body. In addition, the expressions are derived for the Doppler shift in the function of velocity v_0 of the mutual motion of two bodies A and B and the relationship between the masses of these bodies m_A and m_B . The results of this study prove that the formulae of the special theory of relativity (STR) have not been duly derived (since they disregard the masses of the bodies in mutual motion) and that they do not offer correct results.

Keywords: physics, the special theory of relativity

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1. Introduction

The information about a phenomenon is obtained on the basis of recording the information which has reached either our senses or the devices used for registering that phenomenon. This information is transmitted at the final velocity dependent on the information transmitter and the medium through which the information is transmitted.

Also, the interactions between individual bodies (particles) are transmitted at the final velocity which depends on the information transmitter and the medium through which the interaction is transmitted.

When it comes to the bodies which are mutually motionless, the information and interactions are received at the same rate as they are emitted since successive signals travel the same distance. However, when it comes to the bodies which are in mutual motion, the rate of receiving and emitting information (interaction) is not identical because each forthcoming information (interaction) travels a different distance length.

Thus, considering the bodies in mutual motion, it is necessary to find the dependence of the sent and received information (interactions) in the function of the speed of transmitting this information (interactions) and the speed of the movement of the bodies.

The special theory of relativity (hereinafter referred to as STR), proposed by Albert Einstein at the beginning of the 20th century, is generally accepted nowadays.

STR regards the mutual motion of two inertial systems WITH NO MASS assuming that it is irrelevant whether one system is motionless and the other is moving or the second system is stationary while the first system is moving [1]. Thus, bodies are regarded as mathematical points with no mass, and not as physical bodies with mass, which does not correspond to reality.

The subject of this paper is proposing a new theory named MSR (Motion Shapes Reality) on the transmission of light signals between two bodies in mutual motion, based on the law of conservation of momentum.

When talking about the motion of two bodies, it is necessary to bear in mind that each body has its mass!

So, while two bodies are moving, one of them should not be considered to be still while the other is moving and vice versa, but both bodies should be considered to be moving in relation to the center of mass (which is motionless, conditionally) and that they move at velocities dependent on the relationships of their masses, which is the consequence of the law of conservation of momentum [2], which is a consequence of Newton's third law [3].

2. Transmitting Information and Interactions during Motion

The information about a phenomenon is obtained on the basis of recording the information which has reached either our senses or the devices used for registering that phenomenon. This information is transmitted at the final velocity dependent on the information transmitter and the medium through which the information is transmitted.

Also, the interactions between individual bodies (or particles) are transmitted at the final velocity which depends on the information transmitter and the medium through which the interaction is transmitted.

When it comes to the bodies which are mutually motionless, the information and interaction are received at the same rate as they are emitted since successive signals travel the same distance. However, when it comes to the bodies which are in mutual motion, the rate of receiving and emitting information (interaction) is not identical because each upcoming information (interaction) covers a different distance length.

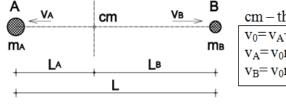
Thus, in relation to the bodies in mutual motion it is necessary to find the dependence of the sent and received information (interactions) in the function of the speed of transmitting this information (interactions) and the speed of the bodies.

2.1. The Motion of Two Bodies in the Same Line

When talking about the motion of two bodies, it is necessary to bear in mind that each body has its mass.

Figure 2.1 illustrates the motion of two bodies A and B having the masses m_A and m_B .

According to the law of conservation of momentum, it follows: $m_A v_A = m_B v_B$



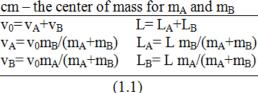


Figure 2.1.

Thus, when it comes to the motion of two bodies with masses m_A and m_B where one of them moves at the velocity of v_0 in relation to the other, one of them should not be considered to be still while the other is moving and vice versa, but both bodies should be considered to be moving in relation to the center of mass (which is motionless, conditionally) and that they move at the velocities of v_A and v_B which are dependent on the relationship of the masses m_A and m_B .

2.1.1. Light Signals between Two Bodies in Motion

Let us consider the travel time of a light signal with the speed of c_0 between two bodies A and B in mutual motion at the speed of v_0 , according to the Figure 2.1. It has been accepted that the plus sign is used for the velocity when the bodies move away from each other. So, if the bodies approach each other, the sign in front of the velocity symbol should be changed in the following expressions.

a) The light signal from the body A towards the body B

A light signal emitted from the body A at the speed of c_0 reaches the body B during the time t_A ; the body B needs the same time (t_A) to move away for the additional distance of $\Delta L_B = t_A v_B$, moving at the speed of v_B in relation to the center of mass cm, so that the total distance covered by the light signal amounts to:

$$L'_{B} = t_{A}c_{0} = L + \Delta L_{B} = L + t_{A}v_{B}$$

Solving for t_A , the travel time of the light signal from A to B is obtained:

$$t_{A} = L/(c_{0} - v_{B})$$
 (2.1.1.a)

b) The light signal from body B towards body A

When a light signal emitted from the body B at the speed of c_0 reaches the body A during the time t_B , the body A needs the same time to move away for the additional

distance of $\Delta L_A = t_B v_A$, moving at the speed of v_A in relation to the center of mass cm, so that the total distance covered by the light signal amounts to:

$$L'_A = t_B c_0 = L + \Delta L_A = L + t_B v_A$$

Solving for t_B , the travel time of the light signal from B to A is obtained:

$$t_{\rm B} = L/(c_0 - v_{\rm A})$$
 (2.1.1.b)

c) The relationship of signals from A to B and from B and A

Comparing the travel times of t_A and t_B of light signals from the body A to the body B and from the body B to the body A, it is perceived that they are different, i.e. $t_A \neq t_B$ (except for the bodies of identical masses $m_A=m_B$ when both $v_A=v_B$ and $L_A=L_B$):

$$t_A / t_B = (c_0 - v_A) / (c_0 - v_B).$$
 (2.1.1.c)

2.1.2. Time Intervals of Two Bodies In Motion

Let us consider the time difference (interval) between two successively emitted light signals from the body A and the time difference (interval) of receiving these signals by the body B, according to Figure 2.1.

In order to simplify the text, all 'A' marks will be replaced by 'E' (emitter), and all 'B' marks will be replaced by 'R' (receiver).

As seen in section 2.1.1.a, the travel time of the first light signal from E to R is:

$$t_1 = L/(c_0 - v_R).$$

Let us define the travel time from the body E to the body R of the second light signal emitted from the body E after particular time Δt_E in relation to the first signal.

During the time Δt_E the body E covers the additional distance of $\Delta L_E = \Delta t_E v_E$ and the body R covers the additional distance of $\Delta L_{R1} = \Delta t_E v_R$. While the light signal travels from the body E to the body R during the time t_2 , the body R covers the additional distance of $\Delta L_{R2} = t_2 v_R$; thus, the total distance covered by the second light signal from the body E to the body R amounts to:

$$\begin{split} L'_2 &= t_2 c_0 = L + \Delta L_{R1} + \Delta L_{R2} + \Delta L_E \\ &= L + \Delta t_E v_R + t_2 v_R + \Delta t_E v_E \end{split}$$

Solving it for t₂, the following is obtained:

$$\mathbf{t}_2 = \left(\mathbf{L} + \Delta \mathbf{t}_{\mathbf{E}} \mathbf{v}_0\right) / \left(\mathbf{c}_0 - \mathbf{v}_{\mathbf{R}}\right).$$

The difference in the duration of the travelling of the first and second signal from E to R amounts to:

$$t_2 - t_1 = (L + \Delta t_E v_0) / (c - v_R) - L / (c_0 - v_R)$$

= $\Delta t_E v_0 / (c_0 - v_R)$

thus, the time interval of the signal reception is:

$$\begin{aligned} \Delta \mathbf{t}_{\mathbf{R}} &= \Delta \mathbf{t}_{\mathbf{E}} + \left(\mathbf{t}_{2} - \mathbf{t}_{1}\right) = \Delta \mathbf{t}_{\mathbf{E}} \left[1 + \mathbf{v}_{0} / \left(\mathbf{c}_{0} - \mathbf{v}_{\mathbf{R}}\right)\right] \\ &= \Delta \mathbf{t}_{\mathbf{E}} \left(\mathbf{c}_{0} + \mathbf{v}_{\mathbf{E}}\right) / \left(\mathbf{c} - \mathbf{v}_{\mathbf{R}}\right) \\ \Delta \mathbf{t}_{\mathbf{R}} &= \gamma \Delta \mathbf{t}_{\mathbf{E}} \qquad \gamma = \left(\mathbf{c}_{0} + \mathbf{v}_{\mathbf{E}}\right) / \left(\mathbf{c}_{0} - \mathbf{v}_{\mathbf{R}}\right) \left(2.1.2.1\right) \end{aligned}$$

If light signals are simultaneously emitted from the end points of a line segment of L_E length on the body E towards the body R, they will reach the body R at the same time and the same length $L_R = L_E$ will be recorded on the body R. Thus, there is no contraction of the length.

However, the body A's crossing the distance $\Delta L_E = v_E \Delta t_E$ at the speed of v_E during the time Δt_E will be read out on the body R as the crossing of the distance (ΔL_E) during the time Δt_R , so the velocity of the body E in relation to the center of mass will be read out on the body R:

Since the body R moves at the velocity v_R in relation to the center of mass, the total velocity of movement of the body E in relation to the body R will be recorded on the body R:

$$v = v_{R} + v'_{E} = v_{R} + v_{E} / \gamma$$

$$v = v_{0} + v_{E} (1 - 1 / \gamma)$$
(2.1.2.1b)

In the case when m_R is insignificantly small in relation to m_{E_a} an approximate expression is obtained:

$$\Delta t_{R} = \gamma \Delta t_{E}$$

$$\gamma = c_{0} / (c_{0} - v_{0}) \qquad (2.1.2.2)$$

$$v = v_{0}$$

In the case when m_E is insignificantly small in relation to m_{R_c} an approximate expression is obtained:

$$\Delta t_{\mathbf{R}} = \gamma \Delta t_{\mathbf{E}}$$

$$\gamma = (c_0 + v_0) / c_0 \qquad (2.1.2.3)$$

$$v = v_0 c_0 / (c_0 + v_0)$$

The previous formulae are derived for the situation of the two bodies moving away from each other. As already mentioned in the introductory part of chapter 2.1.1, if the bodies approach each other, the sign before the velocity symbol should be changed in the previous expressions. Thus, the expressions are as follows:

$$\Delta t_{R} = \gamma \Delta t_{E}$$

$$\gamma = (c_{0} - v_{E}) / (c_{0} + v_{P}) \qquad (2.1.2.4)$$

$$v = v_{0}c_{0} / (c_{0} - v_{E})$$

In the case when m_R is insignificantly small in relation to m_E an approximate expression is obtained:

$$\Delta t_{R} = \gamma \Delta t_{E}$$

$$\gamma = c_{0} / (c_{0} + v_{0}) \qquad (2.1.2.5)$$

$$v = v_{0}$$

In the case when m_E is insignificantly small in relation to m_R an approximate expression is obtained:

$$\Delta t_{R} = \gamma \Delta t_{E}$$

$$\gamma = (c_{0} - v_{0}) / c_{0} \qquad (2.1.2.6)$$

$$v = v_{0}c_{0} / (c_{0} - v_{0})$$

2.1.3. The Doppler Shift

The Doppler shift z is defined in the expression [4]:

$$z = \Delta f / f_{R} = (f_{E} - f_{R}) / f_{R} = f_{E} / f_{R} - 1$$

where:

 $\begin{array}{ll} f_{\rm E} & {\rm emitted \ frequency} & f_{\rm E} = 1/\Delta t_{\rm E} \\ f_{\rm R} & {\rm observed \ frequency} & f_{\rm R} = 1/\Delta t_{\rm R} \\ \Delta f & {\rm the \ difference \ between \ the \ emitted \ and \ observed \\ frequency} & \Delta f = f_{\rm R} - f_{\rm E} \end{array}$

$$f_{\rm E} / f_{\rm R} = (1/\Delta t_{\rm E}) / (1/\Delta t_{\rm R}) = \Delta t_{\rm R} / \Delta t_{\rm E}$$
$$z = \Delta t_{\rm R} / \Delta t_{\rm E} - 1 = \gamma - 1.$$
(2.1.3)

a) The Doppler shift when E and R are moving away from each other

Applying the formulae (2.1.2.1), we have:

$$z = \left[1 + v_0 / (c_0 - v_R)\right] - 1 \quad z = v_0 / (c_0 - v_R)$$
$$z = \frac{v_0 (m_E + m_R)}{c_0 (m_E + m_R) - vm_E} \quad v = \frac{zc_0 (m_E + m_R)}{(z+1)m_E + m_R} \quad (2.1.3.1)$$

In the case when m_R is insignificantly small in relation to m_E an approximate expression is obtained:

$$z \approx v_0 / (c_0 - v_0)$$
 $v_0 \approx zc_0 / (z+1)(2.1.3.2)$

In the case when m_E is insignificantly small in relation to m_R an approximate expression is obtained:

$$z \approx v_0 / c_0$$
 $v_0 \approx z c_0$ (2.1.3.3)

b) The Doppler shift when E and R are approaching each other

Applying the formulae (2.1.2.4), we have:

$$z = [1 - v_0 / (c_0 + v_R)] - 1 \qquad z = -v_0 / (c_0 + v_R)$$
$$z = \frac{-v(m_E + m_R)}{c_0 (m_E + m_R) + v_0 m_E} \qquad v = \frac{-zc_0 (m_E + m_R)}{(z + 1)m_E + m_R}$$
(2.1.3.4)

In the case when m_R is insignificantly small in relation to m_E an approximate expression is obtained:

$$z \approx -v_0 / (c_0 + v_0)$$
 $v_0 \approx -zc_0 / (z+1)(2.1.3.5)$

In the case when m_E is insignificantly small in relation to m_R an approximate expression is obtained:

$$z \approx -v_0 / c_0$$
 $v_0 \approx -zc_0$ (2.1.3.6)

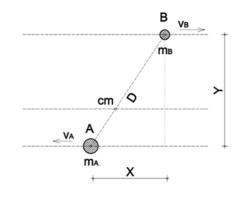
c) The Doppler shift in astronomy

When the Earth receives light signals from massive stars, the Earth's mass m_R is insignificantly small in relation to the mass of stars m_E , so the approximate expressions can be applied (2.1.3.2):

$$z \approx v_0 / (c_0 - v_0)$$
 and $v_0 \approx zc_0 / (z+1)$.

Table 2.1.3 offers a comparative description of the relationship between the Doppler shift z and the velocity v_0 calculated according to the relativistic Doppler Effect (STR) [5] and according to the formulae of MSR (Movement Shapes Reality).

It can be noticed that STR calculates a smaller Doppler shift for the same velocities, i.e. it calculates bigger velocities for the same Doppler shift. The bigger the velocity, the bigger the error in calculating the Doppler shift according to STR.



2.2.1. Light Signals Between Two Bodies in Motion

Let us think about the travel time of a light signal of the emitted speed of c_0 between bodies A and B, according to Figure 2.2.1. It has been accepted that the positive sign of the velocity is used when two bodies move away from each other. So, if the bodies approach each other, in the following formulae the sign before the velocity symbol should be altered.

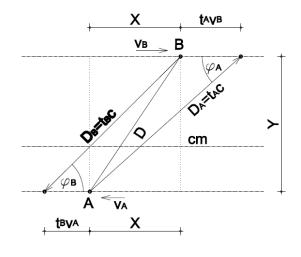


Figure 2.2.1.

Table 2.1.3.	The Doppler	shift in	astronomy	according	to STR and
MSR				_	

MSK				
v ₀ /c	z' (STR)	z (MSR)	$\Delta z=z-z'$	z/z'
0,01	0,0101	0,0101	0,0001	1,0050
0,02	0,0202	0,0204	0,0002	1,0101
0,05	0,0513	0,0526	0,0013	1,0257
0,10	0,1055	0,1111	0,0056	1,0528
0,20	0,2247	0,2500	0,0253	1,1124
0,30	0,3628	0,4286	0,0658	1,1814
0,40	0,5275	0,6667	0,1391	1,2638
0,50	0,7321	1,0000	0,2679	1,3660
0,60	1,0000	1,5000	0,5000	1,5000
0,70	1,3805	2,3333	0,9529	1,6902
0,80	2,0000	4,0000	2,0000	2,0000
0,90	3,3589	9,0000	5,6411	2,6794
0,93	4,2509	13,2857	9,0349	3,1254
0,96	6,0000	24,0000	18,0000	4,0000
0,98	8,9499	49,0000	40,0501	5,4749
0,99	13,1067	99,0000	85,8933	7,5534

2.2. The Motion of Two Bodies on Parallel Lines

Let us consider the mutual movement of two bodies A and B on parallel lines at the relative velocity of v_0 , according to Figure 2.2.

cm-the center of mass for m_A and m_B $v_0=v_A+v_B$

 $v_B = v_0 m_A / (m_A + m_B)$ $v_A = v_0 m_B / (m_A + m_B)$

Figure 2.2.

a) The light signal from the body A towards the body B

While a light signal travels from A towards B at the speed c during a particular time t_A , the body B covers the distance of $\Delta X_B = t_A v_B$ in that time, so the light signal covers the distance of $D_A = t_A c_0 = (X+t_A v_B)/\cos\varphi_A$. Solving this for t_A , it is obtained as follows:

$$t_{A} = X / (c_{0} \cos \varphi_{A} - v_{B}) \qquad (2.2.1.a)$$

b) The light signal from the body B towards the body A

While a light signal travels from B towards A at the speed c during a particular time t_B , the body A covers the distance $\Delta X_A = t_B v_A$ in that time, so the light signal covers the distance of $D_B = t_B c_0 = (X + t_B v_A)/\cos\varphi_B$. Solving this for t_B , it is obtained as follows:

$$t_{\rm B} = X / (c_0 \cos \varphi_{\rm B} - v_{\rm A})$$
 (2.2.1.b)

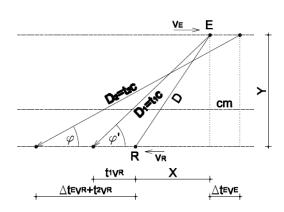
c) The relationship of the signal from A to B and from B to A

Comparing the travel times of t_A and t_B of light signals from the body A to the body B and from the body B to the body A, it is perceived that they are different, i.e. $t_A \neq t_B$ (except for the bodies of identical masses $m_A=m_B$ when both $v_A=v_B$ and $\phi_A=\phi_B$):

$$t_{\rm A} / t_{\rm B} = (c_0 \cos \varphi_{\rm B} - v_{\rm A}) / (c_0 \cos \varphi_{\rm A} - v_{\rm B}) (2.2.1.c)$$

2.2.2. Time Intervals of Two Bodies in Motion

Let us consider the time difference (interval) between two successively emitted light signals from the body B and the time difference (interval) of receiving these same signals by the body A, according to the Figure 2.2.



The total distance covered by the body R from the emission of the first signal to the reception of the first signal amounts to $\Delta X_1 = t_1 v_R$.

Let us define the travel time from the body E to the body R of the second light signal emitted from the body E after particular time Δt_E in relation to the first signal.

During the time Δt_E the body E covers the additional distance of $\Delta X_E = \Delta t_E v_E$ and the body R covers the additional distance of $\Delta X_{RI} = \Delta t_E v_R$.

While the light signal travels from the body E to the body R during the time t_{2} , the body R covers the additional distance of ΔX_{R2} = t_2v_R ; thus, the total distance of moving away of the bodies E and R on the motion line (x-axis) from the emission of the first signal to the reception of the second signal amounts to:

$$\begin{split} \Delta \mathbf{X}_2 &= \Delta \mathbf{X}_{\mathrm{E}} + \Delta \mathbf{X}_{\mathrm{R}1} + \Delta \mathbf{X}_{\mathrm{R}2} \\ &= \Delta \mathbf{t}_{\mathrm{E}} \mathbf{v}_{\mathrm{E}} + \Delta \mathbf{t}_{\mathrm{E}} \mathbf{v}_{\mathrm{R}} + \mathbf{t}_{2} \mathbf{v}_{\mathrm{R}}. \end{split}$$

Since $v_E+v_R=v_0$, the previous expression becomes: $\Delta X_2 = \Delta t_E v_0 + t_2 v_R$ so

$$\Delta X_2 - \Delta X_1 = \Delta t_E v_0 + (t_2 - t_1) v_R$$

Applying the law of sine to Figure 2.2.2, we have:

$$(t_2 - t_1)c_0 / \sin\beta = \left[\Delta t_E v_0 + (t_2 - t_1)v_R\right] / \sin\alpha$$

Changing

$$\alpha = \pi / 2 + (\varphi' - \varphi) / 2 \qquad \sin \alpha = \cos \left[(\varphi' - \varphi) / 2 \right]$$
$$\beta = \pi / 2 - (\varphi' + \varphi) / 2 \qquad \sin \beta = \cos \left[(\varphi' + \varphi) / 2 \right]$$

in the previous expression and solving for (t_2-t_1) , we have:

$$t_2 - t_1 = \Delta t_E \frac{v_0 \left(1 - \tan\frac{\varphi}{2}\tan\frac{\varphi}{2}\right)}{c_0 \left(1 + \tan\frac{\varphi}{2}\tan\frac{\varphi}{2}\right) - v_R \left(1 - \tan\frac{\varphi}{2}\tan\frac{\varphi}{2}\right)}$$

In order to simplify the text, all 'B' marks will be replaced by 'E' (emitter), and all 'A' marks will be replaced by 'R' (receiver), according to Figure 2.2.2.

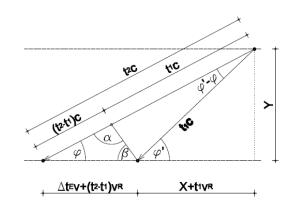


Figure 2.2.2

that is, introducing the relation $k = \frac{\tan\left(\frac{\varphi}{2}\right)}{\tan\left(\frac{\varphi}{2}\right)}$

$$t_{2} - t_{1} = \Delta t_{E} \frac{v_{0} \left(1 - k \tan^{2} \frac{\varphi}{2}\right)}{c_{0} \left(1 + k \tan^{2} \frac{\varphi}{2}\right) - v_{R} \left(1 - k \tan^{2} \frac{\varphi}{2}\right)} \quad (*)$$

Since the time interval between the reception of the first and second signal equals

$$\Delta t_{\rm R} = \Delta t_{\rm E} + \left(t_2 - t_1\right)$$

we have:

$$\Delta t_R = \Delta t_E \left[1 + \frac{v_0 \left(1 - k \tan^2 \frac{\varphi}{2} \right)}{c_0 \left(1 + k \tan^2 \frac{\varphi}{2} \right) - v_R \left(1 - k \tan^2 \frac{\varphi}{2} \right)} \right]$$

that is:

$$\Delta t_{R} = \gamma \Delta t_{E} \quad v = v_{R} + v_{E} / \gamma \quad v = v_{0} + v_{E} (1 - 1 / \gamma)$$

$$\gamma = 1 + \frac{v_0 \left(1 - k \tan^2 \frac{\varphi}{2}\right)}{c_0 \left(1 + k \tan^2 \frac{\varphi}{2}\right) - v_R \left(1 - k \tan^2 \frac{\varphi}{2}\right)} (2.2.2.1)$$

Applying the law of sine to Figure 2.2.2, we have:

$$\frac{D_2}{\sin\varphi'} = \frac{D_2 - (t_2 - t_1)c_0}{\sin\varphi}$$
$$(t_2 - t_1) = \frac{D_2}{c_0} \left(1 - \frac{\sin\varphi}{\sin\varphi'}\right) = \frac{D_2}{c_0} \left[1 - \frac{\tan\frac{\varphi}{2} \left(1 + \tan^2\frac{\varphi'}{2}\right)}{\tan\frac{\varphi'}{2} \left(1 + \tan^2\frac{\varphi}{2}\right)}\right]$$

Equalizing the previous expression with the expression (*) and solving for k, we have:

$$k = \frac{\left[\left(c_0 + v_R + \frac{v_0 c_0 \Delta t_E}{D_2} \right) \tan^2 \frac{\varphi}{2} \right]}{-\left(c_0 - v_R - \frac{v_0 c_0 \Delta t_E}{D_2} \right)}$$

$$k = \frac{\left[\left(c_0 - v_R - \frac{v_0 c_0 \Delta t_E}{D_2} \right) \tan^2 \frac{\varphi}{2} \right]}{2(c_0 + v_R) \tan^2 \frac{\varphi}{2}}$$

$$+ \frac{\left\{ \left[\left(c_0 - v_R - \frac{v_0 c_0 \Delta t_E}{D_2} \right) \tan^2 \frac{\varphi}{2} \right]^2 \right]}{2(c_0 - v_R^2) \tan^2 \frac{\varphi}{2}} \right]$$

Introducing the change $1/\Delta t_E = f_E$ (the frequency of the emitted signal), we have:

$$k = \frac{\left[\begin{pmatrix} c_0 + v_R + \frac{v_0 c_0}{D_2 f_E} \end{pmatrix} \tan^2 \frac{\varphi}{2} - \left(c_0 - v_R - \frac{v_0 c_0}{D_2 f_E} \right) \right]}{\left[\left\{ \left[\left(c_0 + v_R + \frac{v_0 c_0}{D_2 f_E} \right) \tan^2 \frac{\varphi}{2} \right]^2 \right] + \left\{ \sqrt{\left[\left[\left(c_0 - v_R - \frac{v_0 c_0}{D_2 f_E} \right) \tan^2 \frac{\varphi}{2} \right]^2 \right] + 4(c_0^2 - v_R^2) \tan^2 \frac{\varphi}{2}} \right]} \\ k = \frac{\left[2 \left(c_0 + v_R \right) \tan^2 \frac{\varphi}{2} \right] + 4(c_0^2 - v_R^2) \tan^2 \frac{\varphi}{2} \right]}{2(c_0 + v_R) \tan^2 \frac{\varphi}{2}} \\ \left[2 \left(c_0 + v_R \right) \tan^2 \frac{\varphi}{2} \right] + k \tan \left(\frac{\varphi}{2} \right) \quad \phi' = 2 \tan \left[k \tan \left(\frac{\varphi}{2} \right) \right] (2.2.2.2)$$

The above mentioned shows that the relationship between the time intervals Δt_R and Δt_E does not depend only on the angle φ and velocity v_0 but also on the distance between E and M (D₂) and on the time interval of the emitted signals, i.e. on the frequency of the emitted signal f_E .

In the case of long distances and high signal frequency,

the member $\frac{v_0 c_0}{D_2 f_E}$ can be disregarded.

2.2.2.1. Time Intervals for $\phi \ll \pi/2$

In the case of long distances for the angle $\varphi << \pi/2$ and the small velocity v, an insignificantly small difference of the angles φ' and φ ($\varphi' \approx \varphi$) is obtained, so $k \approx 1$ and the formula (2.2.2.1) is simplified to the approximate formula:

$$\Delta t_{R} = \gamma \Delta t_{E} \quad v' = v_{R} + v_{E} / \gamma \quad v' = v_{0} + v_{E} (1 - 1 / \gamma)$$
$$\gamma = 1 + \frac{v_{0} \cos \varphi}{c_{0} - v_{R} \cos \varphi} = \frac{c_{0} + v_{E} \cos \varphi}{c_{0} - v_{R} \cos \varphi} (2.2.2.1.1)$$

 $v_E \cos \varphi$ is the vector projection of velocity v_E on the line ER and $v_R \cos \varphi$ is the vector projection of velocity v_R on the line ER, i.e. these are collinear vectors on the line ER so the expression (2.2.2.1.1) is simplified to (2.1.2.1):

$$\gamma = 1 + \frac{v_L}{c_0 - v_{PL}} = \frac{c_0 + v_{EL}}{c_0 - v_{RL}} \qquad (2.2.2.1.1a)$$

where v_L, v_{EL} and v_{RL} are the components of the velocities v_0, v_E and v_R on the line of the light signal ER.

In the case when m_R is insignificantly small in relation to m_E an approximate expression is obtained:

$$\gamma = c_0 / (c_0 - v_0 \cos \varphi), \Delta t_R = \Delta t_E$$

$$c_0 / (c_0 - v_0 \cos \varphi), v = v_0$$
(2.2.2.1.2)

In the case when m_E is insignificantly small in relation to m_{R_c} an approximate expression is obtained:

$$\gamma = (c_0 + v_0 \cos \varphi) / c_0, \Delta t_R = \Delta t_E, (c_0 + v_0 \cos \varphi) / c_0, v = v_0 c_0 / (c_0 + v_0 \cos \varphi)$$
(2.2.2.1.3)

a) intervals for $\varphi = 0$

For $\varphi = 0$, $\cos \varphi = 1$ so the equation (2.2.2.1.1) becomes:

$$\begin{split} \gamma &= (\mathbf{c}_0 + \mathbf{v}_E) / (\mathbf{c}_0 - \mathbf{v}_R), \Delta t_R = \Delta t_E, \\ (\mathbf{c}_0 + \mathbf{v}_E) / (\mathbf{c}_0 - \mathbf{v}_R), \quad (2.2.2.1.4) \\ \mathbf{v} &= \mathbf{v}_R + \mathbf{v}_E (\mathbf{c}_0 - \mathbf{v}_P) / (\mathbf{c}_0 + \mathbf{v}_E). \end{split}$$

which is identical to the expression (2.1.2.1).

In the case when m_R is insignificantly small in relation to m_E , the expression (2.2.2.1.4) becomes:

$$\gamma = c_0 / (c_0 - v_0), \Delta t_R = \Delta t_E,$$

$$c_0 / (c_0 - v_0), v = v_0$$
(2.2.2.1.5)

which is identical to the expression (2.1.2.2).

In the case when m_E is insignificantly small in relation to m_R , the expression (2.2.2.1.3) becomes:

$$\gamma = (c_0 + v_0) / c_0, \Delta t_R = \Delta t_E, (c_0 + v_0) / c_0, v = v_0 c_0 / (c_0 + v_0).$$
 (2.2.2.1.6)

which is identical to the expression (2.1.2.3).

b) the reception of the signal from Cosmos

When the Earth receives light signals from massive stars, the Earth's mass m_R is insignificantly small in relation to the mass of stars m_E , so the expressions (2.2.2.1.2) and (2.2.2.1.5) can be applied.

c) the intervals in the case of approaching bodies

The previous expressions have been derived for the situation of the bodies moving away from each other. As mentioned above, if the bodies approach each other, the sign in front of the velocity symbol should be changed in the previous expressions.

2.2.2.2. TIME INTERVALS FOR $\phi = \pi/2$

For the angle φ near $\pi/2$, the formula (2.2.2.1) is used after the angle φ' has been calculated according to the formulae (2.2.2.2).

For the angle $\varphi = \pi/2$, $tg(\varphi/2) = 1$ so the expression (2.2.2.1) becomes:

$$k = \frac{v_{R} + \frac{v_{0}c_{0}\Delta t_{E}}{D_{2}} + \sqrt{\left[\left(v_{R} + \frac{v_{0}c_{0}\Delta t_{E}}{D_{2}}\right)^{2}\right]}{(c_{0} + v_{R})^{2}}$$

$$= \frac{v_{R} + \frac{v_{0}c_{0}}{D_{2}f_{E}} + \sqrt{\left(v_{R} + \frac{v_{0}c_{0}}{D_{2}f_{E}}\right)^{2} + c_{0}^{2} - v_{R}^{2}}}{(c_{0} + v_{R})} (2.2.2.2.1)$$

$$\tan\left(\frac{\varphi}{2}\right) = k, \varphi' = 2\operatorname{atan}(k)$$

In the case when m_R is insignificantly small in relation to m_E , the expression (2.2.2.2.1) becomes:

$$k = v \frac{1 + \frac{c_0 \Delta t_E}{D_2} + \sqrt{\left(1 + \frac{c_0 \Delta t_E}{D_2}\right)^2 + \frac{c_0^2}{v_0^2} - 1}}{(c + v_0)}$$
$$= v \frac{1 + \frac{v_0 c_0}{D_2 f_E} + \sqrt{\left(1 + \frac{c_0}{D_2 f_E}\right)^2 + \frac{c_0^2}{v_0^2} - 1}}{(c + v_0)}}{(c + v_0)} \quad (2.2.2.2.2)$$
$$\tan\left(\frac{\varphi'}{2}\right) = k, \varphi' = 2 \operatorname{atan}(k)$$

In the case when m_E is insignificantly small in relation to m_R , the expression (2.2.2.2.1) becomes:

$$k = \frac{v_0 \Delta t_E}{D_2} + \sqrt{\left(\frac{v_0 \Delta t_E}{D_2}\right)^2 + 1}$$
$$= \frac{v_0}{D_2 f_E} + \sqrt{\left(\frac{v_0}{D_2 f_E}\right)^2 + 1} \qquad (2.2.2.2.3)$$
$$\tan\left(\frac{\varphi'}{2}\right) = k, \varphi' = 2\operatorname{atan}(k)$$

When it comes to cosmic distances, the member $\frac{v_0 c_0}{D_2 f_E}$

can be disregarded, so for $\varphi = \pi/2$ we have k=1, γ =1.

The previous expressions have been derived for the situation of the bodies moving away from each other. As mentioned above, if the bodies approach each other, the sign in front of the velocity symbol should be changed in the previous expressions.

2.2.3. The Doppler Shift

The Doppler shift z is defined in the formula:

$$z = \Delta f / f_{\rm R} = (f_{\rm E} - f_{\rm R}) / f_{\rm R} = f_{\rm E} / f_{\rm R} - f_{\rm R}$$

1

where:

 $\begin{array}{ll} f_{\rm E} & {\rm emitted \ frequency} & f_{\rm E} = 1/\Delta t_{\rm E} \\ f_{\rm R} & {\rm observed \ frequency} & f_{\rm p} = 1/\Delta t_{\rm R} \\ \Delta f & {\rm the \ difference \ between \ the \ emitted \ and \ observed} \\ \Delta f = f_{\rm R} - f_{\rm E} \end{array}$

$$f_{\rm E} / f_{\rm R} = (1 / \Delta t_{\rm E}) / (1 / \Delta t_{\rm R}) = \Delta t_{\rm R} / \Delta t_{\rm E} = \gamma$$

$$z = \Delta t_{\rm R} / \Delta t_{\rm F} - 1 = \gamma - 1 \qquad (2.2.3)$$

Applying the expression (2.2.2.1.) to the formula (2.2.3), general formulae are obtained:

For E and R moving away from each other:

$$z = \frac{v_0 \left(1 - k \tan^2 \frac{\varphi}{2}\right)}{c_0 \left(1 + k \tan^2 \frac{\varphi}{2}\right) - v_R \left(1 - k \tan^2 \frac{\varphi}{2}\right)}$$
(2.2.3.1)
$$v_0 = z c_0 \frac{\left(1 + k \tan^2 \frac{\varphi}{2}\right) (m_E + m_R)}{\left(1 - k \tan^2 \frac{\varphi}{2}\right) \left[m_E \left(z + 1\right) + m_R\right]}$$

For E and R approaching each other:

$$z = \frac{-v\left(1 - k\tan^2\frac{\varphi}{2}\right)}{c_0\left(1 + k\tan^2\frac{\varphi}{2}\right) + v_R\left(1 - k\tan^2\frac{\varphi}{2}\right)}$$

$$v_0 = -zc_0\frac{\left(1 + k\tan^2\frac{\varphi}{2}\right)(m_E + m_R)}{\left(1 - k\tan^2\frac{\varphi}{2}\right)\left[m_E\left(z + 1\right) + m_R\right]}$$
(2.2.3.2)

2.2.3.1. Angle φ<< π/2

In the case of long distances for angle $\varphi << \pi/2$ and small velocity v_0 , an insignificantly small difference of the angles φ' and φ ($\varphi' \approx \varphi$) is obtained, so $k \approx 1$ and the formulae (2.2.3.1) and (2.2.3.2) are simplified to the approximate formulae:

For E and R moving away from each other:

$$z = \frac{v_0 \cos \varphi}{c_0 - v_R \cos \varphi}$$
$$v_0 = \frac{zc_0}{\left(1 + z \frac{m_E}{m_E + m_R}\right) \cos \varphi}$$
(2.2.3.1.1)

For E and R approaching each other:

$$z = \frac{-v_0 \cos \varphi}{c_0 + v_R \cos \varphi}$$
$$v_0 = \frac{-zc_0}{\left(1 + z \frac{m_E}{m_E + m_R}\right) \cos \varphi}$$
(2.2.3.1.2)

If m_R is insignificantly small in relation to m_E , then $v_R \approx v$ and $v_E \approx 0$, so the expressions (2.2.3.1.1) and (2.2.3.1.2) become:

For E and R moving away from each other:

$$z = v_0 \cos\varphi / (c_0 - v_0 \cos\varphi)$$

$$v_0 = zc_0 / \lceil (z+1) \cos\varphi \rceil$$
(2.2.3.1.3)

For E and R approaching each other:

$$z = -v_0 \cos\varphi / (c_0 + v_0 \cos\varphi)$$

$$v_0 = -zc_0 / [(z+1)\cos\varphi]$$
(2.2.3.1.4)

If m_E is insignificantly small in relation to m_R , then $v_E \approx v$ and $v_R \approx 0$, so the expressions (2.2.3.1.1) and (2.2.3.1.2) become:

For E and R moving away from each other:

$$z = v_0 \cos \varphi / c_0$$

$$v_0 = z c_0 / \cos \varphi$$
(2.2.3.1.5)

For E and R approaching each other:

$$z = -v_0 \cos \varphi / c_0$$

$$v_0 = -zc_0 / \cos \varphi$$
(2.2.3.1.6)

For $\varphi = 0$ (longitudinal Doppler shift) the equations (2.2.3.1.3) to (2.2.3.1.6) are simplified to the equations (2.1.3.2) and (2.1.3.3), that is (2.1.3.5) and (2.1.3.6).

2.2.3.2. Angle $\phi = \pi/2$

For angle φ near $\pi/2$ the formula (2.2.3.1) is used after the angle φ' has been calculated according to the formula (2.2.2.2).

For the angle $\varphi = \pi/2$ the formula (2.2.3.1) is used after the angle φ' has been calculated according to the formula (2.2.2.2.1).

If one of the masses is insignificantly small in relation to the other mass and the angle $\varphi = \pi/2$ (transversal Doppler shift), the formulae (2.2.2.2.2) and (2.2.2.3) are used for calculating the angle φ' .

When it comes to cosmic distances, the member $v_0 c_0$

 $\frac{v_0 c_0}{D_2 f_E}$ can be disregarded, so for $\varphi = \pi/2$ we have k=1,

γ=1, z=0.

2.2.4. Exchange of Information with the Satellite

Let us consider the communication between the Earth and the satellite through a light signal.

Since the satellite's mass m_s is insignificant in relation to the Earth's mass m_E , the Earth can be considered to be at rest ($v_E=0$) while the satellite moves at the speed of $v_s=v_0$.

However, the communication with the satellite is not performed from the center of the Earth's mass but from the surface 6.400km distant from the center of the Earth's mass and the Earth rotates around its axis (tangential velocity on the equator around 0,465km/sec). Thus, in the communication with the satellite we should also take into account the rotational velocity of the point from which the communication is carried out, as well as the satellite's trajectory in relation to the direction of the Earth's rotation, i.e. the projection of the rotational velocity on the line of the satellite's movement during the communication.

The growth of the satellite's distance and the deviation of the signal's angle of $\pi/2$ in relation to the current direction of rotational velocity lead to the decrease of the influence of the Earth's rotation so it can be disregarded in the case of long distances and small angles of signals. Also, when the point from which the communication with the satellite is performed is in the vicinity of one of the Earth's rotation can be disregarded.

This chapter will deal with the situation when the angle of the signal is $\varphi \ll \pi/2$ and when the influence of the Earth's rotation can be disregarded.

Communication with the satellite includes sending the signal from the Earth towards the satellite and its returning to the Earth at specific intervals, according to Figure 2.2.4.1.

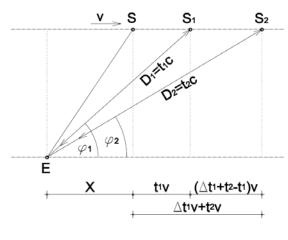


Figure 2.2.4.1.

We will observe two signals which, at the interval $\Delta t_{1,}$ are emitted from the Earth (E) towards the satellite (S) and returned to the Earth.

While the first signal takes time t_1 to reach S from E moving at the speed of c_0 , the satellite covers the additional distance of $\Delta X_1 = t_1 v_0$. Returning from S to E (having in mind that E is motionless), the return signal covers the same distance t_1c_0 as the emitted signal.

After particular time Δt_1 the second signal is emitted. During the same time, S covers the additional distance of ΔX_{2} = $\Delta t_1 v_0$. While the second signal reaches E from S during the time t_2 , the satellite covers the additional distance of ΔX_3 = $t_2 v_0$. Returning from S to E (since E is motionless), the return signal covers the same distance $t_2 c_0$ as the emitted signal.

$$t_2 - t_1 = \Delta t_1 \frac{v_0 \left(1 - k \tan^2 \frac{\varphi_1}{2}\right)}{c_0 \left(1 + k \tan^2 \frac{\varphi_1}{2}\right)}$$
(**)

Since the travel time of the first signal from E to S and back is $T_1=2t_1$ and the travel time of the second signal from E to S and back is $T_2=2t_2$, the time interval between the reception of the first signal and the second signal equals

$$\Delta \mathbf{t}_2 = \Delta \mathbf{t}_1 + 2(\mathbf{t}_2 - \mathbf{t}_1)$$

so we have:

$$\Delta t_P = \Delta t_1 \left[1 + 2 \frac{v_0 \left(1 - k \tan^2 \frac{\varphi_1}{2} \right)}{c_0 \left(1 + k \tan^2 \frac{\varphi_1}{2} \right)} \right] \quad (2.2.4.1)$$

Since the communication with satellites is performed at high frequencies (about 3GHz) and that the situation of long satellite distance and small velocity v_0 (in relation to the signal speed c_0) is being considered, the formula (2.2.4.1) for the signal angle $\varphi_1 \ll \pi/2$ is reduced to the approximate formula:

$$\Delta t_2 = \Delta t_1 [1 + 2v_0 \cos \varphi / c_0] \qquad (2.2.4.2)$$

Introducing the expression (2.2.4.2) in the formula (2.2.3), approximate formulae for the Doppler shift are obtained.

For E and S moving away from each other:

$$z = 2v_0 \cos \varphi / c_0$$

$$v_0 = zc_0 / 2\cos \varphi$$
(2.2.4.3)

For E and S approaching each other:

$$z = -2v_0 \cos \varphi / c_0$$

$$v_0 = -zc_0 / 2\cos \varphi.$$
(2.2.4.4)

2.2.4.1. Pioneer 10 Anomaly

Two spacecrafts, Pioneer 10 and 11, which were launched in 1972 and 1973, do not behave according to the calculations based on the relativistic formulae and on the basis of the measured Doppler shift [6].

Pioneer 10 should move at the speed of 12,2 km/sec. However, on the basis of the measured Doppler shift according to the relativistic calculations, the spacecraft moves at the increasingly slower speed so that each year it covers the distance which is smaller than expected by 8000 km.

Table 2.2.4.1 represents the difference of the Doppler shift calculated according to STR (special theory of relativity) and according to MSR (motion shapes reality). The calculation is performed for the leaving and returning (coming) signal on the basis of the approximate values of the signal angle (φ_1) in relation to the line of the spacecraft's movement.

The calculation according to MSR is carried out on the basis of the formula (2.2.4.3):

$$z = 2v_0 \cos \varphi / c_0.$$

STR is used as the basis for calculating the Doppler shift z':

$$z' = [(c + v * \cos \varphi)^2 / (c^2 - v^2)] - 1$$

Table 2.2.4.1.									
12,2 3,0E+05		The Doppler shift for Pioneer 10 leaving and coming signal							
φ (°)	cos φ	According to STR (z')	According to MSR (z)	∆z=z-z'	Δv , km/sec	ΔD, km/year			
20,7	0,9354	7,6086E-05	7,6083E-05	-3,101E-09	-2,486E-04	-7.840			
17,6	0,9532	7,7529E-05	7,7526E-05	-3,156E-09	-2,484E-04	-7.832			
14,7	0,9673	7,8674E-05	7,8671E-05	-3,201E-09	-2,482E-04	-7.827			
12,5	0,9763	7,9409E-05	7,9405E-05	-3,230E-09	-2,481E-04	-7.825			
10,8	0,9823	7,9896E-05	7,9893E-05	-3,250E-09	-2,481E-04	-7.824			
9,6	0,9860	8,0198E-05	8,0194E-05	-3,262E-09	-2,481E-04	-7.824			
0	1,0000	8,1337E-05	8,1333E-05	-3,308E-09	-2,481E-04	-7.823			
	$\begin{array}{c} 3,0E+05\\ \hline \phi \ (^{\circ})\\ \hline 20,7\\ \hline 17,6\\ \hline 14,7\\ \hline 12,5\\ \hline 10,8\\ \hline 9,6\\ \end{array}$	$\begin{array}{c c} 3,0E+05 \\ \hline \phi \left(^{\circ} \right) & \cos \phi \\ \hline 20,7 & 0,9354 \\ \hline 17,6 & 0,9532 \\ \hline 14,7 & 0,9673 \\ \hline 12,5 & 0,9763 \\ \hline 10,8 & 0,9823 \\ \hline 9,6 & 0,9860 \\ \hline \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $			

The table shows that, for the given speed, the Doppler shift calculated according to STR is bigger than the one calculated by MSR. The difference indicates the annual "lags" which extremely precisely correspond to the measured values. Thus, it is not an anomaly but the incorrect "relativistic" calculation.

2.3. On the Speed of Light

When talking about the speed of light, a difference should be made between the speed of light (photon) in relation to the body E emitting the light and the speed of light reaching the body R which moves in relation to the body E.

The speed of light is determined by the system of fixed emitters and receivers [7].

Thus, the postulate on the constant speed of light refers to the constant speed of the emitted light, independently of its frequency (the speed of the emitted light of any frequency is the same).

According to the analyses in chapter 2.2, when determining the speed of light reaching the body R in the system of two bodies E (light emitter) and R (light receiver) which are moving in relation to each other, the mutual motion of these two bodies should be taken into account.

When the photon moves from the body E at the speed of light c_0 towards the body R, this body R escapes from the photon at the speed v according to (2.2.1), (2.2.1a) and

(2.2.1b). Thus, the speed of the photon reaching the body R is reduced by the escape speed of the body R, i.e. the speed of the photon (c) which has reached the body R, according to the classical law on velocity addition, amounts to:

$$c = c_0 - v, v = v_R + v_E / \gamma, v = v_0 - v_E(\gamma - 1) / \gamma$$
 (2.3.1)

where v_0 , v_E , v_R and v are collinear components of the velocities on the line ER, according to (2.2.2.1.1) and (2.2.2.1.1a).

This change of the speed of light is recorded as the change of frequency (the speed of photon oscillation), i.e. as the Doppler shift.

Since the Doppler shift is $z=\gamma-1$, i.e. $\gamma=z+1$, by changing in (2.3.1) we have:

$$v = v_0 - v_E z / (z+1), c = c_0 - v_0 + v_E z / (z+1) (2.3.2)$$

The comments in chapter 2.2. illustrate that the difference of the velocities v_0 and v in the function of the relationship of the masses of the emitter m_E and the receiver m_R is:

 $v_E = v_0 m_R / (m_E + m_R)$ $v_R = v_0 m_E / (m_E + m_R)$ $v_0 = v_E + v_R$ $v_E = v_0 - v_E z / (z+1)$

If m_R is insignificantly small in relation to m_E , then $v_R \approx v$ and $v_E \approx 0$, so the expressions (2.3.2) become:

$$v = v_0, c = c_0 - v_0$$
 (2.3.3)

If m_E is insignificantly small in relation to m_R , then $v_E \approx v$ and $v_R \approx 0$, so the expressions (2.3.2) become:

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