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# A Finite Element Inverse Method for the Design of Turbomachinery Blades

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## ABSTRACT

*We present a quasi-3D inverse method for the design of turbomachinery blades corresponding to a velocity distribution given arbitrarily. The theoretical aspect of the problem is first investigated, then the equations governing the quasi three-dimensional potential model are reviewed. The inverse method consists of solving the potential equation with Dirichlet boundary conditions on the profile, then modifying the profile iteratively until there is no mass flux through its surface. The convergence of the process is guaranteed by the preliminary theoretical study. The method is implemented using a finite element discretization, which relies on a mixed variational formulation involving two fields of unknowns: the velocity potential and the normal displacement of the profile.*

*Several results are shown on subsonic and transonic compressor profiles. The modified profiles are then validated with direct calculations such as quasi three-dimensional Navier-Stokes computations. These results illustrate the behavior of the method, in particular its robustness and its effectiveness.*

*The method was adapted to calculations on turbine blade profiles. Preliminary results are shown that illustrate an industrial use of the method on a subsonic profile.*

## Nomenclature

- $b$ : stream tube thickness
- $e$ : trailing edge gap
- $h$ : angular pitch between 2 consecutive profiles
- $m$ : arc length on a meridian line
- $M$ : relative Mach number
- $r$ : radius
- $s$ : curvilinear abscissa
- $V$ : absolute velocity
- $W$ : relative velocity
- $z$ : axial direction

- $\alpha$ : relative angle
- $\theta$ : polar angle
- $\rho$ : density
- $\omega$ : angular velocity
- $\Phi$ : velocity potential

- $\Omega$ : computational domain
- $\psi$ : test function associated to $\phi$
- $\xi$ : modification of the profile
- $\zeta$ : test function associated to $\zeta$

## Subscript

- $1$ : inlet
- $2$ : outlet
- $\infty$ : upstream condition

## 1. INTRODUCTION

The performances (aerodynamic load, efficiency) of a turbomachinery blade in a subsonic or transonic flow can be significantly increased by controlling the velocity (or pressure) distribution around the profile, thus monitoring the growth of the boundary layer along its surface. Two approaches can be adopted in order to obtain blades that achieve the desired characteristics: a "direct" approach, which consists in analyzing the flow around a given airfoil, then in modifying its shape if the velocity distribution on the profile is not satisfactory; or an "inverse" approach, in which one chooses a "target" velocity (or pressure) distribution from which the geometry is directly generated; the latter method is well suited to the designer's needs because it allows the direct use of the ideal velocity distribution from the boundary layer standpoint. Of course the resulting airfoil may or may not be physically or structurally sound. The purpose of this paper is to present one type of inverse method applied to the

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design of turbomachinery blades in quasi-three-dimensional transonic flows.

For a complete review of the two classes of method, we refer to MEAUZE [1] and SLOOF [2] ; we simply mention two authors who developed inverse methods that have provided us valuable ideas. In [3,4], VOLPE and MELNIK stressed the importance on the constraints exhibited by LIGHTHILL [5], and built an algorithm which produces isolated profiles. However it will be shown that their method is not suitable for the design of turbine blades and this paper will provide a proper formulation of the inverse problem.

In [6], Cedar and Stow proposed a finite element design and analysis method where the profile shape is iteratively modified using a transpiration model. Although successful in the design of quasi-three-dimensional turbomachinery blades, this method does not garantee the existence of a profile or the convergence of the modification process.

In this paper, we present a method whose theory has been studied and then implemented at ONERA [7]; its industrialisation, carried out at SNECMA, has led to the result thereafter presented.

## 2.A PROPER FORMULATION FOR THE INVERSE PROBLEM

The inverse problem for isolated profiles in incompressible flows was first formulated by LIGHTHILL [5 ] and more recently by VOLPE and MELNIK [3,4]. In order to get a well-posed inverse problem, they had to set three parameters free. They first chose to relax the upstream velocity ($q_\infty$) and the trailing edge gap ($\delta x$, $\delta y$) [4] ; then they slightly modified the algorithm by introducing two modification functions for the target velocity, so as to obtain closed profiles.

The inverse problem is different for quasi-three-dimensional cascade design. On the one hand, inlet and outlet velocities are linked to quantities computed in other stages, therefore these quantities cannot be set free. On the other hand, the prescribed distribution is defined on the pressure and the suction-side and given by two separate functions of the arc lengths $s_{pressure}$ and $s_{suction}$. The relative lengths of the two sides or, equivalently, the position of the stagnation point can therefore be considered to be the first necessary parameter.

Another difference is due to the axisymmetrical aspect of the problem we are dealing with. In planar problems, it is possible to set free both of the variables defining the trailing edge gap ($\delta x$ and $\delta y$). However, on axisymmetrical surfaces (parameters m and $\theta$), it is not possible to give physical interpretation to a trailing edge presenting a gap in the m-direction. We will therefore limit the study to profiles with suction and pressure trailing edge lying on the same coordinate $m_{te}$. The only variable characterizing the gap is its angle e chosen to be the second free parameter of the method.

Finally, the last variable that characterizes the problem is the pitch-to-chord ratio : this quantity will be computed by the method and is the third parameter necessary to get a well-posed inverse problem.

## 3. GOVERNING EQUATIONS

### 3.1 The Quasi-Three-Dimensional Model

Under certain approximations, the study of a turbomachinery blading can be carried out using Wu's approach [8] : the general three-dimensional flow can be decomposed into a through-flow defining axisymmetrical stream surfaces around the wheel axis Oz, and a blade-to-blade flow on these surfaces. In this study, we suppose that the characteristics of the through-flow are known and given by the function r (z) defining a stream surface and the stream-tube thickness b (z). The computation is then carried out using the independant variables m, the arc length on a meridian line, and $\theta$ the polar angle around Oz. The continuity equation for a steady flow in a relative coordinate system rotating with the blade can be written as :

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left(\rho b r\left(\frac{\partial\Phi}{r\partial\theta}-\omega r\right)\right)+\frac{\partial}{\partial m}\left(\rho b r\frac{\partial\Phi}{\partial m}\right)=0 \qquad (1)$$

where $\Phi$ is the velocity potential.

The density is obtained from Bernoulli's equation in the relative axis system :

$$\rho=\rho_1\left[1+\frac{\gamma-1}{2}\frac{M_1^2}{W_1^2}\left(W_1^2-W^2-\omega^2\left(r_1^2-r^2\right)\right)\right]^{1/(\gamma-1)} \qquad \mathbf{(2)}$$

with

$$W^2=\left(\frac{\partial\Phi}{\partial m}\right)^2+\left(\frac{\partial\Phi}{r\partial\theta}-\omega r\right)^2 \qquad (3)$$

The indices 1 and 2 respectively denote the quantities relative to the inlet and to the outlet. The equation (1), with the expression of given by (2), and completed by proper boundary conditions, constitute the equation of the quasi-three dimensional potential model.

Even though this potential approach does not take into account the entropy variation through shocks, this model is valid in the case of compressible transonic fluid flows, shock-free or with weak shocks, that is flows where the relative Mach number does not exeed 1.4. The advantage of using such an approach is that potential calculations are very fast and consequently that the design process can be interactively carried out.

The boundary conditions and more particularly the ones applied on the profile determine the nature of the solution mode: direct or inverse.

### 3.2 The Computational Domain and the Conditions Applied far from the Profile

In order to take advantage of the periodicity of the problem, the computation is restricted to an area containing one profile. We denote h the angular pitch that separates two consecutive profiles. The profile is prolonged by a pseudo-wake, without lift and with a constant angular thickness equal to the trailing edge gap e. One must remember that both h and e will be the unknowns of the inverse determination.

A C-topology will be considered (fig.1) : the computational domain $\Omega$ surrounds both the profile and the wake issued from the trailing edge. This topology is well adapted to the determination of relatively thick leading edge profiles.

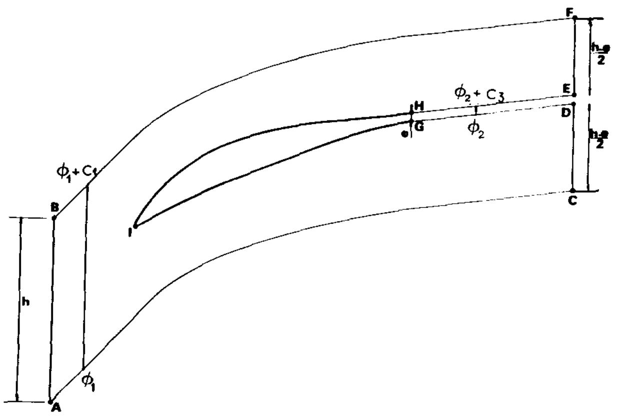


**Fig. 1: C topology definition**

Besides the conditions applied on the domain boundary, the solution of the problem requires the knowledge of certain quantities relative to the flow upstream and downstream. In order to be consistent with other steps of the engine design process, in particular with the matching of successive stages, the choice consists of prescribing of the upstream flow direction ($\alpha_1$) and velocity (given the relative Mach number $M_1$), and the downstream flow direction ($\alpha_2$). In the case where rotation of the blade occurs, besides the value of the whirl velocity $\omega$, another quantity (temperature or pressure) must be given in order to scale the parameter $\omega r_2 / w_1$. The upstream density $\rho_1$ can be normalized and chosen equal to unity.

Then the quantities given upstream enable the computation of the downstream quantities (velocities $W_2$ and density $\rho_2$), using the mass conservation equation :

$$h\ b_1\ r_1\ \rho_1\ W_1 \cos\alpha_1 = (h-e)\ b_2\ r_2\ \rho_2\ W_2 \cos\alpha_2 \qquad (4)$$

We apply usual conditions on the parts of the boundaries other than the profile ; we suppose that these boundaries are taken far enough upstream and downstream so that the flow can be considered uniform and uniquely defined by the directions $\alpha 1$ and $\alpha 2$. We impose Neumann and periodicity conditions :

$$\rho b \frac{\partial\phi}{\partial n} = \rho_1 b_1\ W_1 \cos\alpha_1 \quad \text{on AB}$$

$$\rho b \frac{\partial\phi}{\partial n} = \rho_2 b_2\ W_2 \cos\alpha_2 \quad \text{on CD, EF} \qquad (5)$$

$$\phi_{BF} = \phi_{AC} + h\ r_1\ (W_1 \sin\alpha_1 + \omega r_1)$$

$$\phi_{HE} = \phi_{GD} + h\ r_1\ (W_1 \sin\alpha_1 + \omega r_1) - (h-e)\ r_2\ (\omega_2 \sin\alpha_2 - \omega r_2) \qquad (6)$$

## 4. THE INVERSE METHOD

### 4.1 Conditions on the Profile

The main difference between the direct and the inverse solution appears in the conditions which are applied on the profile : whereas a **Neumann** condition corresponding to a zero normal velocity is usually applied on the profile in direct computation, a **Dirichlet** condition is imposed in the inverse method. The tangential velocity can be expressed as :

$$W_s ds = W_\theta\ r d\theta + W_m\ dm = \left(\frac{\partial\Phi}{r\partial\theta} - \omega\ r\right) r d\theta + \frac{\partial\Phi}{\partial m}\ dm \qquad (7)$$

where s represents the curvilinear coordinate along the profile.
This equation can also be written

$$\frac{d\Phi}{ds} = W_0 + \omega\ r^2\ \frac{d\theta}{ds} \qquad (8)$$

In order to obtain the tangential velocity equal to the given velocity distribution $W_o$, the potential on the profile must satisfy the condition (9), obtained by the integration of (8) on the profile, starting from the leading edge (L.E.) :

$$\phi(M) = \phi(L.E.) + \int_{L.E.}^{M} (\ W_0 ds + \omega r(m)^2 d\theta\ ) \qquad (9)$$

The solution of the equations (1,2 and 5,6) with this Dirichlet condition (9) on the profile is called inverse solution and is the basis of the calculation method we introduce in this paper.

### 4.2 Profile Modification

The goal of the determination is to find a profile satisfying both constraints :

a) zero normal velocity
b) tangential velocity equal to $W_o$ given.

The solution of the direct problem (Neumann condition on the profile) satisfies (a) but not (b). The solution of the inverse problem (Dirichlet condition on the profile) leads to a flow that follows the prescribed tangential velocity on the profile (b), but does not satisfy the zero normal velocity on the blade walls (a). The residual normal velocity $W_n$ obtained is used to modify the profile in a transpiration model : the displacement of the blade is accounted for by injecting fluid through the original blade surface so that the new surface becomes a stream surface [4]. The normal displacement of the blade wall from the original to the modified shape can then be obtained by writing the mass conservation equation between two elements of length ds on the profile (Fig.2)

$$\rho b W_n\ ds = \rho b W_0\ \xi\ |_{s+ds} - \rho b W_0\ \xi\ |_s \qquad (10)$$

This leads to the differential equation for $\xi$:

$$\frac{d}{ds}(\rho b W_0 \xi) = \rho b W_n \qquad (11)$$

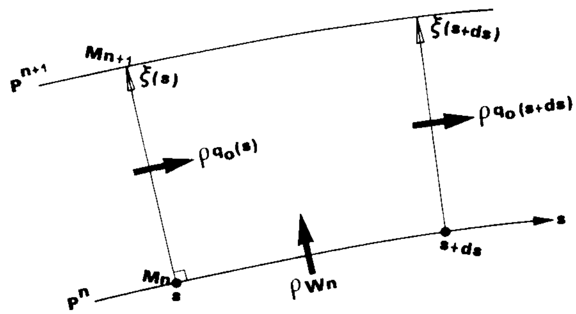


**Fig. 2 : Profile modification**

### 4.3 Inverse Design Algorithm

The inverse method consists therefore of a sequence of the following three-step iterations :

. Computation of the potential on the profile by integration of the prescribed velocity,
. Computation of the potential in the domain by solution of (1) with Dirichlet boundary condition on the profile,
. Computation of the normal displacement according to (11) and modification of the profile.

The first and third steps are one-dimensional integrations ; the second step is the solution of a bi-dimensional, second order, non linear partial differential equation that is chosen to be solved by a finite element method derived from a method which was developed for flows around isolated profiles [9], and is described below.

### 4.4. Mixed Variational formulation

In order to solve the equations (8,1,11) with a finite element method, we introduce a coupling of the three steps presented ; this leads to a mixed variational formulation involving two fields of unknowns, the potential and the normal displacement of the profile:

Find $\phi$ admissible defined in $\Omega$ and defined on the profile such that :

$$\int_\Omega \rho b \left[ \frac{\partial\phi}{\partial m}\frac{\partial\psi}{\partial m} + \left(\frac{\partial\phi}{r\partial\theta} - \omega r\right)\frac{\partial\psi}{r\partial\theta}\right] r dm d\theta - \int_{Profile} \psi d(\rho b W_0 \xi) = \int_{\Gamma_\infty} g \psi d\Gamma \quad \textbf{(12a)}$$

$$\int_{Profile} \rho b W_0 \varsigma \left(d\phi - \omega r^2 d\theta\right) = \int_{Profile} \rho b W_0^2 \, \varsigma \, d\Gamma \quad (12b)$$

for all $\psi$ admissible defined in $\Omega$ and $\xi$ defined on the profile.

Corresponding to each step described in the previous section, three weak formulations can be obtained from (12).

.Integration of the prescribed velocity :

Find $\phi_0$ defined on the profile such that

$$\int_{Profile} \rho b W_0 \varsigma \left(d\phi_0 - \omega r^2 d\theta\right) = \int_{Profile} \rho b W_0^2 \, \varsigma \, d\Gamma \quad (13)$$

for all $\xi$ defined on the profile
This represents a weak formulation of (8)

.Computation of the potential in the domain by solution of the potential equation with Dirichlet boundary condition on the profile :

Find $\phi$ defined in $\Omega$ such that $\phi = \phi_0$ on the profile and :

$$\int_\Omega \rho b \left[ \frac{\partial\phi}{\partial m}\frac{\partial\psi}{\partial m} + \left(\frac{\partial\phi}{r\partial\theta} - \omega r\right)\frac{\partial\psi}{r\partial\theta}\right] r\, dm\, d\theta = \int_{\Gamma_\infty} g\, \psi d\Gamma \quad (14)$$

for all $\psi$ defined in $\Omega$ and equal to 0 on the profile.

.Computation of the normal displacement :

Find $\xi$ defined on the profile such that :

$$\int_{Profile} \psi d(\rho b W_0 \xi) = \int_\Omega \rho b \left[ \frac{\partial\phi}{\partial m}\frac{\partial\psi}{\partial m} + \left(\frac{\partial\phi}{r\partial\theta} - \omega r\right)\frac{\partial\psi}{r\partial\theta}\right] r\, dm\, d\theta \quad \textbf{(15)}$$

for all $\psi$ defined on the profile and equal to 0 in $\Omega$/Profile.

This formulation is the proper formulation that gives meaning to $W_n$, and allows its computation in a weak sense compatible with the finite element method.

## 5. FINITE ELEMENT DISCRETIZATION

For the numerical implementation of the algorithm, we construct a mesh of quadrilateral elements in the computation domain and we introduce discrete finite-dimensional approximation spaces : $V^h$, set of piecewise bilinear continuous fonctions in $\Omega$ and $Q^h$, set of piecewise constant function on the profile. The discrete system used to compute the approximations of the solutions is obtained when one replaces the functions by their approximations in (12) or in (13-15). The solution of (13) is straighforward : it is equivalent to the integration of the given velocity and leads to nodal values of the potential by :

$$\phi^{i+1} = \phi^i + W_0^{i+½}\, \Delta s^{i+½} + \omega\, r^{i+½\, 2}\, \Delta\theta^{i+½} \quad (16)$$

where the indice i+1/2 denotes the value of the corresponding quantity evaluated between the nodes i and i+1. This integration of the potential is done separately on the pressure and on the suction sides of the profile from the stagnation point where the potential is supposed to be zero. This enables the calculation of the potential jump $\Delta\phi$ at the trailing edge between pressure and suction sides, followed by the calculation of the inter-blade pitch.

The variational formulation (14) leads to a set of non-linear equations solved with a mixed Newton/Fixed-Point algorithm [9]. For transonic flows the density is modified in a standard way in order to add an artificial viscosity term in supersonic region [9]. Linearized systems are solved with a preconditioned Conjugate Gradient algorithm.

Finally, concerning the integration of the displacements, we remark that, since the functions in $Q^h$ are not necessarily continuous, the discrete variational formulation is now taken in a distribution sense ; this leads to :

$$\rho b W_0 \xi \,|_i^+ - \rho b W_0 \xi \,|_i^- = \rho b W_n \,|_i \quad (17)$$

where the quantity in the right hand-side represents the mass flux through the node i; this can be computed by :

$$\rho b W_n |_i = \int_\Omega \rho b \left[ \frac{\partial \phi}{\partial m} \frac{\partial \psi_i}{\partial m} + \left(\frac{\partial \phi}{r\partial \theta} - \omega r\right) \frac{\partial \psi_i}{r\partial \theta} \right] r \, dm \, d\theta \quad (18)$$

In (18), $\psi^i$ is the test function equal to 1 at the node i and 0 at all other nodes. So, each of the profile element displacements $\xi_{i+1/2}$ are computed by successive application of (17) ; then, the node coordinates of the modified profile are obtained by averaging of the contribution of adjacent elements.

## 6. NUMERICAL RESULTS

All calculations were performed on a C grid with 10 x 117 points. The grid is first generated on the mean radius cylinder and projected on the stream line for the flow calculation.

Starting from the initial profile, three modifications are generally needed in order to ensure a good agreement between the prescribed Mach number distribution and the one corresponding to the generated profile. In fact, the code runs automatically in direct mode on the final profil output, the two different Mach number distributions are then directly comparable. Three profile modifications plus a direct calculation require about 15s on a IBM 3090 computer.

Four test cases are presented in order to illustrate the application of this inverse method. This technique has been checked with many stator and rotor airfoils that are representative of highly loaded compressors. The stream tube contraction and the stream tube radius variation are also given to show the quasi-three-dimensional effects.

This method was also tested on a turbine profile and the results are presented even though some additional work is needed for turbine applications.

### 6.1 Rotor hub section of an experimental compressor

The first example presented in this paper is the design of a very highly loaded compressor near the hub region. All the input parameters are obtained from a through-flow computation (inlet relative Mach number, flow angles and stream tube evolution). Fig.3 shows the compressor flow-path. The radius and streamtube thickness evolutions are also shown on fig.4.

The main design parameters are as follows

- . inlet Mach number : 0.95
- . inlet flow angle (deg) : 61.7
- . outlet flow angle (deg) :-2.5

The method was first initialized with a profile and the corresponding Mach number distribution obtained by the code run in analysis mode.

In order to reduce the peak Mach number with constant solidity and max thickness, the distribution was modified as shown on fig.6. After 3 modifications with an under relaxation factor of 0.2 at leading edge and 0.5 at trailing edge, the new profile was obtained with a different pitch angle and a different thickness law.

Fig.6 gives the Mach number contour lines calculated from the potential fields on the original and final profiles. It appears that the supersonic area has been slightly reduced whereas the thickness near the leading edge is larger. Finally a quasi-three-dimensional Navier-Stokes computation is performed in order to validate the final profile. This Navier-Stokes solver was developped jointly by SNECMA and ONERA in 1989 [11].

The final result (fig.7) shows good agreement between the prescribed and the calculated Mach number distributions, except on the suction side where the Navier-Stokes solver exhibits a small shock. The slight difference on the pressure side is probably due to the blockage effects of the boundary layer. On the whole however, the viscous analysis confirms the prescribed Mach number distribution.

### 6.2 Stator design

The second example concerns the design of a stator blade, from hub to tip. Three profiles at different span-wise locations illustrate the calculation. The main features we used for the computation are summarized in the following table.

| | Inlet Mach number | Flow angle $\alpha 1$ | $\alpha 2$ |
|---|---|---|---|
| . Hub | 0.83 | 47.9 | 14. |
| . Midspan | 0.68 | 44.5 | 9. |
| . Tip | 0.61 | 45. | 10.5 |

The prescribed Mach number distribution for the tip section is fully subsonic whereas a supersonic area appears in the two other cases (fig.8a). The purpose of this study was to limit the peak Mach number while keeping constant blading parameters such as the max thickness, the LE and TE thicknesses for example. The control of the suction side deceleration was also of great interest for the boundary layer behavior.

The profile given on fig.8b shows that the above remarks can be taking into account using the inverse method.

In order to further analyse the profile shapes, several Navier-Stokes calculations were performed. The resulting Mach number distributions are shown on fig 8c by dashed lines. The two curves are very similar near the leading edge where the boundary layer is very thin. But viscous effects become important near the trailing edge and the prescribed distribution should therefore be iterated on using the Navier-Stokes results.

The blade is then generated by stacking the resulting profiles on a 3D axis.

### 6.3 Hub section of a turbine stator

As previously mentioned, the use of this inverse method for turbine applications is recent.

We consider the case of a stator hub section where the three-dimensional effects are important due to a large variation of the streamtube thickness (outlet/inlet ratio equals 1.3). The aim is to eliminate the deceleration on the suction side resulting in this case from the use of a standard 2D design procedure. However, some characteristic dimensions of the blade profile are kept constant, such as chord and trailing edge thickness.

The computation is performed using an improved grid (fig.9) which better fits turbine blade geometries (large thickness, high camber, blunt leading and trailing edges). The inlet and exit flow angles are 31°4 and -61° respectively and the inlet Mach number is 0.424.

The velocity distribution on the initial blade and the target velocity distribution are shown on fig.10. After five blade modifications with an under-relaxation factor of 0.4 at the leading edge and 0.2 at the trailing edge, the velocity distribution converges to the required one with good accuracy. The resulting profile remains very smooth.

The velocity distribution recalculated by the method used in the analysis mode on the modified profile was then compared to the velocity distribution given by an Euler direct calculation (fig.11).

We observe good agreement between the two calculations, except in the trailing edge region. This discrepancy may arise because the Euler grid is finer in this region. In addition, the implementation of the Kutta condition in the potential method is not well suited to the case of a thick trailing edge combined with a highly skewed grid.

However, the general shape of the velocity distribution on the modified blade is satisfactory, showing the great interest for the turbine designer to have such a quasi 3D inverse method. Moreover, this test demonstrates the efficiency and the robustness of the method.

### 6.4 Example with a poor initialization

The last example (fig.12) presented in this paper involves large changes in the profile from the initialization. Starting with a geometry with a relative max thickness of 3% and a pitch angle of 25°, the code converges to a new profile with a thickness of 7% and a pitch angle of 3°. After one iteration a very large displacement is observed but the calculation remains stable.

This example (and others presented in [10]) proves the robustness of the method. From a practical point of view, of course, we would generally use as input an existing profile with aerodynamic features close to the desired ones, which is a good way to include previous experience in the design procedure and thus reduce delays.

## 7. CONCLUSION

A quasi-3D blade-to-blade inverse method is described. The equations governing the potential model are solved using a mixed finite element discretization. The potential field and the profile displacement are jointly determined. The interest of the method resides in its potential to correctly define sub- or transonic profiles taking into account the streamtube thickness and the radius variation.

Four examples of application at SNECMA are shown concerning compressor or turbine airfoils. In view of analysing several inverse outputs, different Navier-Stokes or Euler computations were performed which confirm the global behavior of those profiles.

This code is robust, efficient, fast and is routinely used in the SNECMA design procedure. The next step will certainly be the coupling with a boundary layer calculation in order to correctly predict the viscous effects after the design procedure.

**AKNOWLEDGEMENT**

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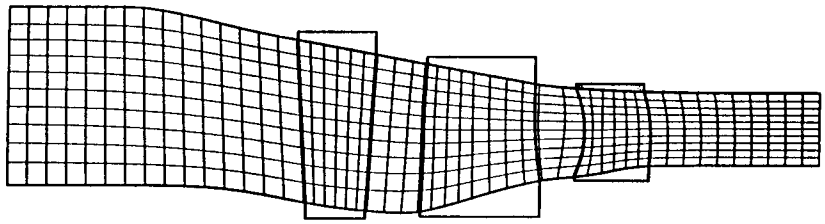

Fig. 3 : Through-flow calculation

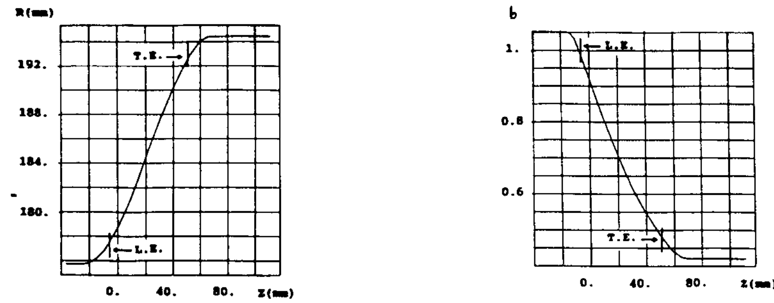


Fig. 4 : Radius and thickness stream-tube evolutions

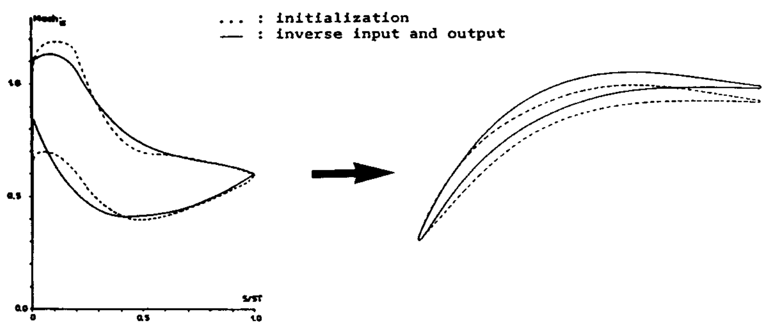


Fig.5 : Required Mach number distribution and final profile after 3 modifications

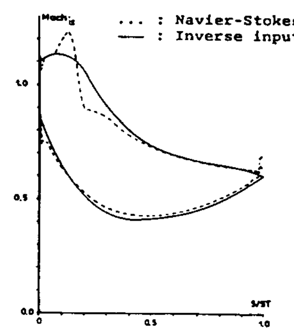


Fig.7 : Direct calculation

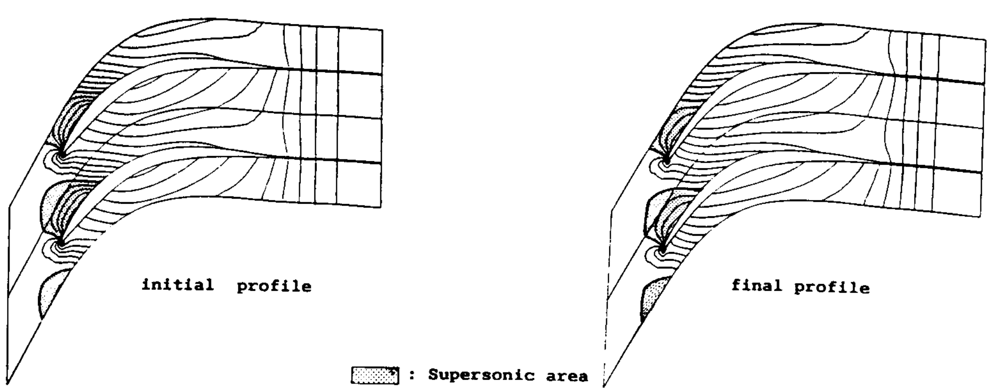


Fig. 6 : Mach number countour lines

**hub section** **midspan section** **tip section**

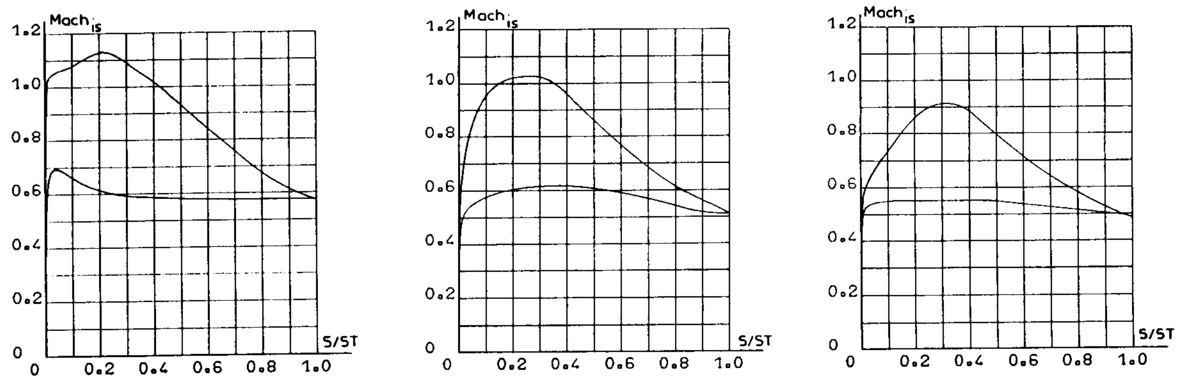


Fig. 8a : Prescribed Mach number distribution

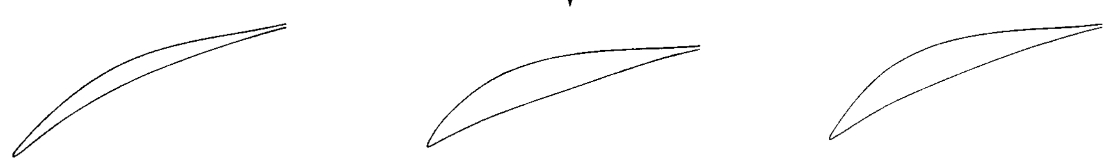

Fig. 8b : Profile definition

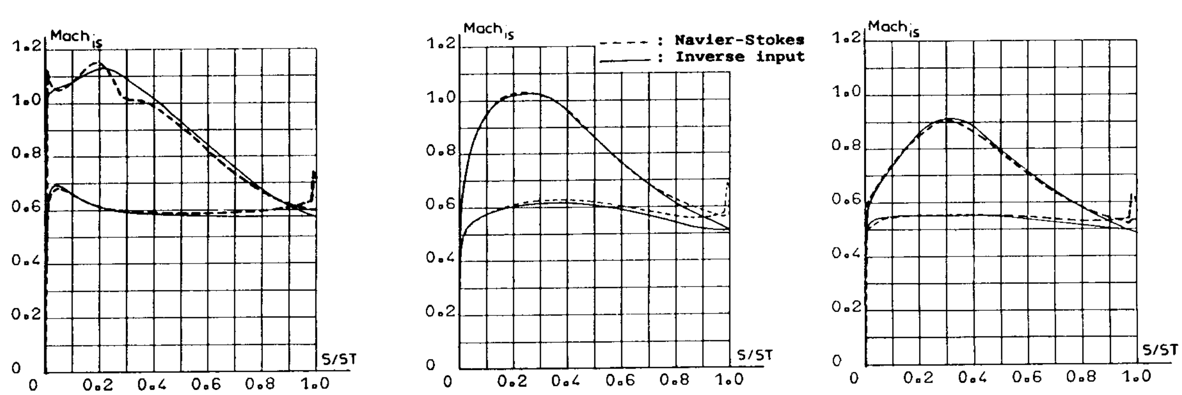


Fig. 8c : Comparison of Navier-Stokes calculation with the inverse input

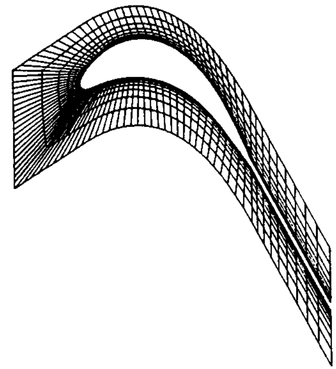

**Fig. 9 : computational grid**

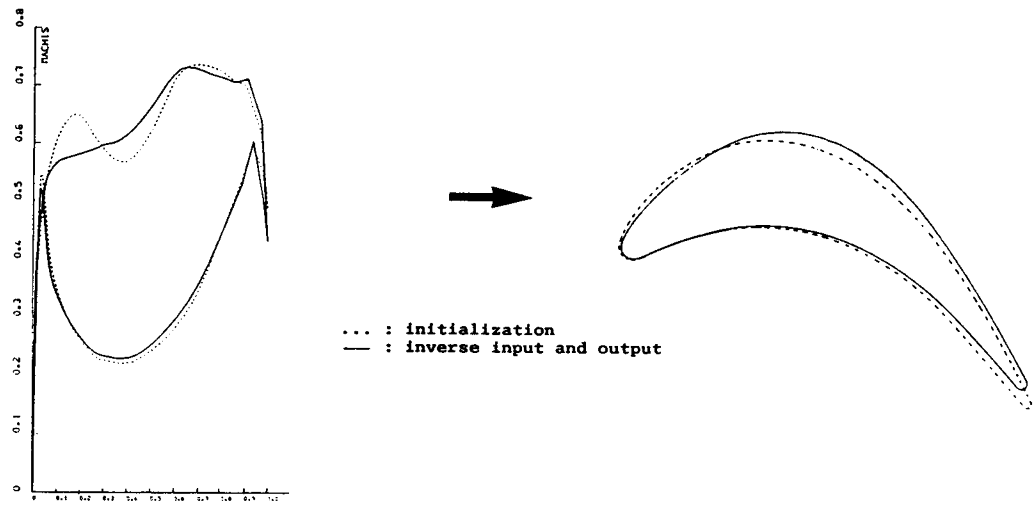


**Fig. 10 : required Mach number distribution and final profile after five modifications**

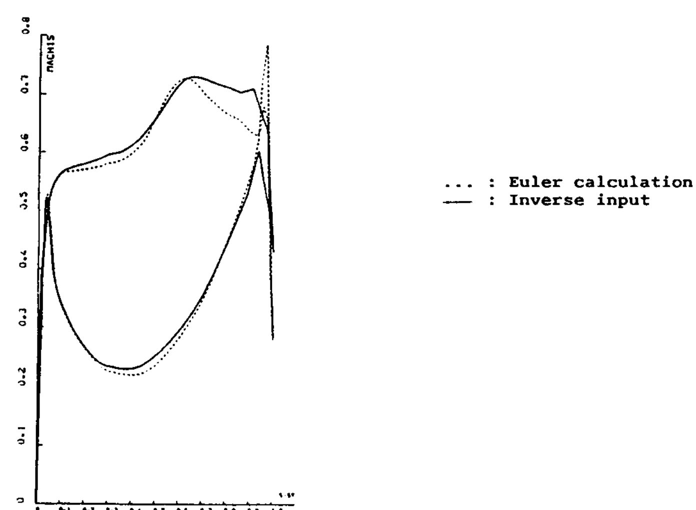


**Fig. 11 : Euler analysis**

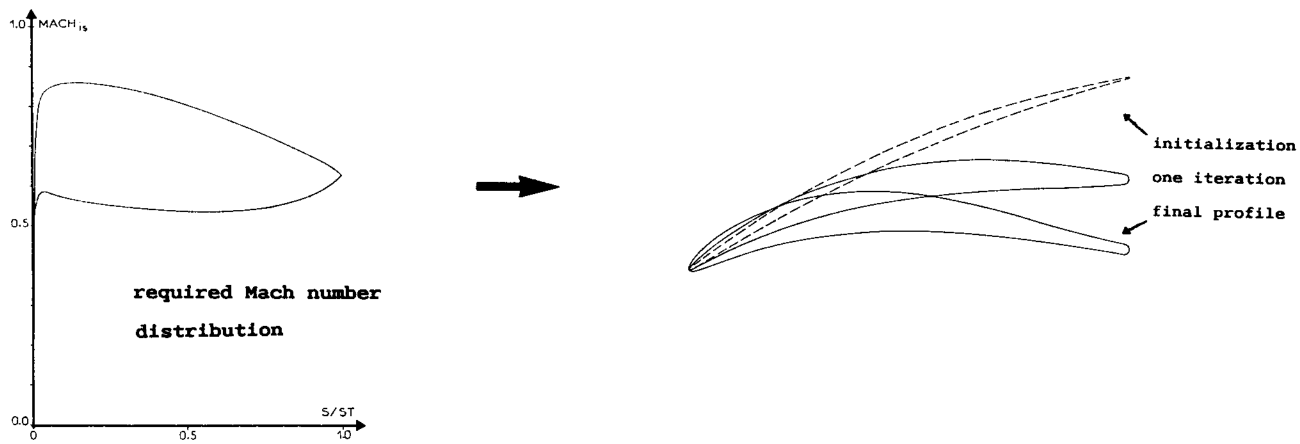


**Fig. 12 : subsonic example with a poor initialization**

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