RECONSTRUCTION OF COMPRESSIVELY SAMPLED TEXTURE IMAGES IN THE GRAPH-BASED TRANSFORM DOMAIN

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ABSTRACT

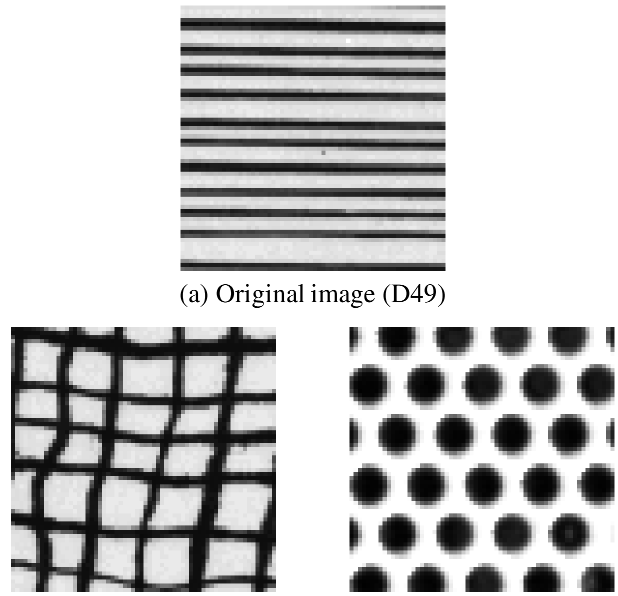
This paper addresses the problem of texture images recovery from compressively sampled measurements. Texture images hardly present a sparse, or even compressible, representation in transformed domains (e.g. wavelet) and are therefore difficult to deal with in the Compressive Sampling (CS) framework. Herein, we resort to the recently defined Graph-based transform (GBT), formerly introduced for depth map coding, as a sparsifying transform for classes of textures sharing the similar spatial patterns. Since GBT proves to be a good candidate for compact representation of some classes of texture, we leverage it for CS texture recovery. To aim, we resort to a modified version of a state-of-the-art recovery algorithm to reconstruct the texture representation in the GBT domain. Numerical simulation results show that this approach outperforms state-of-the-art CS recovery algorithms on texture images.

***Index Terms*—** Compressive sampling, texture, graph-based transform.

1. INTRODUCTION

The recently and widely explored Compressive Sampling (CS) theory provides a theoretical framework for reconstruction of a sparse signal from a restrained set of randomly acquired measurements. CS reconstruction procedures leverage the sparsity of the signal by resorting to suitably sparsifying transform of the signal itself. Unlike natural images, texture images hardly present a sparse, or even compressible, representation in transformed domains and are therefore difficult to deal with in the Compressive Sampling (CS) framework. The literature on textures focuses on the compact parametric representation of selected features sets [1] for classification or retrieval applications rather than on the definition of sparsity seeking transforms and redundant dictionaries.

Problems of texture classification given CS acquisitions are far more often tackled [2] than CS reconstruction. In fact, the lack of basis suited to compact texture representations actually limits the performance of CS reconstruction algorithms. In this paper, we leverage the Graph Based Transform (GBT) representation of texture images to reconstruct



(a) Original image (D49)

(b) Original image (D104) (b) Original image (1.5.02)

Fig. 1. Example of image textures [8].

textures from CS measurements. The GBT has been formerly introduced as an image dependent representation of images with sharp boundaries [3], and it has successfully been applied to the problem of depth map encoding. More in general, graph-based processing provides a fresh and promising approach to unsolved problems, ranging from multiview video coding [4] to network analysis [5]. In this paper, we observe that the GBT provides a sparse and compact representation of some classes of texture. Based in this novel observation, we leverage the GBT within a CS texture recovery procedure. We provide numerical simulation results assessing how algorithms encompassing GBT outperform state-of-the-art reconstruction procedures.

The structure of this paper is as follows. In Sect.2 we recall the Graph based Transform and we discuss its application to texture images. In Sect.3 we present a CS recovery procedure based on the CoSamp algorithm for texture reconstruction in the GBT domain, and in Sect.4 we provide numerical simulation results. Sect.5 concludes the paper.

2. TEXTURE IMAGES GRAPH BASED TRANSFORM

It is widely known that texture images are hardly compressible in conventional transform domain and the search for different redundant representation dictionary is an open issues. In [6], the authors observe that redundant dictionaries characterizing the local texture behavior have been used for texture synthesis [7]. Therein, the signal elements local neighborhood are assumed as the varying redundant dictionary, but, according to what reported in [6], such technique works well only if the sampling of the signal source is dense enough. Thereby, there is still need of devising sparsity enforcing linear transforms of texture images.

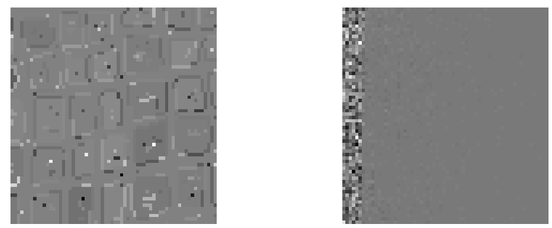
Herein, we discuss the issue of texture representation in the GBT domain. The GBT basis best matches images with sharp region boundaries, and it has been firstly applied to depth image coding. Since structured patterns due to abrupt luminance changes are observed also in the case of texture images, which are therefore hardly compressible in either the Discrete Cosine Transform or the wavelet domain, it is fair to ask whether they are compressible in the GBT domain.

Let us denote by $\boldsymbol{z}$ the $N \times 1$ dimensional vector of samples of the texture image, collected in lexicographical der. The GBT transform represents the signal as the sum N orthonormal basis vectors $\boldsymbol{u}_k$ related to the spatial str ture of the image $\boldsymbol{z}$. Specifically, the vectors $\boldsymbol{u}_k$ are selec as the eigenvectors of a symmetric matrix D related to the i age spatial discontinuities. In deeper detail, let L denote $N \times N$ binary matrix whose (i,j)-th element l_{ij} is non-z if and only if an edge is detected[1] between the i-th and j element of the image $\boldsymbol{z}$. Then D is computed as

$$D = \mathrm{diag}\left(\sum_{j=0}^{N-1} l_{0\,j}, \sum_{j=0}^{N-1} l_{1\,j} \cdots \sum_{j=0}^{N-1} l_{N-1\,j}\right) - L$$

The eigenvectors basis $\boldsymbol{u}_k,\ k = 0, \ldots N-1$ is ind related to the spatial structure occurring in the image $\boldsymbol{z}$, by means of the Laplacian D, in turns related to the adjacency matrix L. Notice that the adjacency matrix L is binary, and it is shared by several different images that may present similar patterns of spatial discontinuities, although they may present different luminance values. This closely resembles the case of images that, having the same covariance matrix, share the same optimal coding transform, namely the Karhunen-Loewe transform, regardless of their actual luminance values. In the following, we show by means of numerical examples that the GBT matrix may be calculated on a texture image and re-used for other textures images presenting similar spatial patterns. Under this respect, the GBT is a promising representation of texture images, since once the GBT basis has been computed on a particular texture, it is expected to be well-suited to texture images with similar spatial patterns. In the Section devoted to numerical simulation results we show by numerical examples the effectiveness of the GTB as a sparsifying transform of texture images, prior to demonstrate its application to CS recovery purposes.

[1] For the purpose of computing D, in our simulations we have estimated the binary matrix L by thresholding the output of a Sobel operator.



(a) GBT basis element (b) GBT coefficients

Fig. 2. Example of texture representation in the GBT domain (D104).

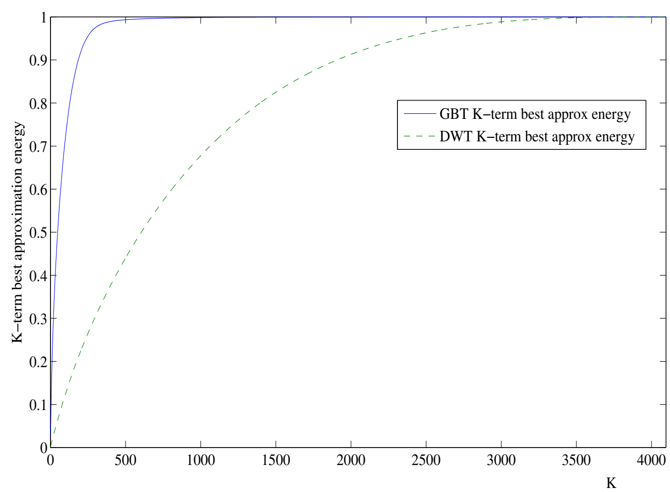


Fig. 3. Normalized energy captured by the best K-term approximation for the GBT and DWT coefficients (D104).

In the next Section, we devise a reconstruction algorithm which is a modified version of the CoSamp algorithm [11] to be applied for recovery of the texture image in the GBT domain.

3. COSAMP ALGORITHM FOR TEXTURE RECONSTRUCTION IN THE GBT DOMAIN

Let us consider the $N \times 1$ dimensional vector $\boldsymbol{z}$ built by the samples of the texture image. We assume that the texture is S-sparse in the GBT domain. The observation vector $\boldsymbol{y}$ is built

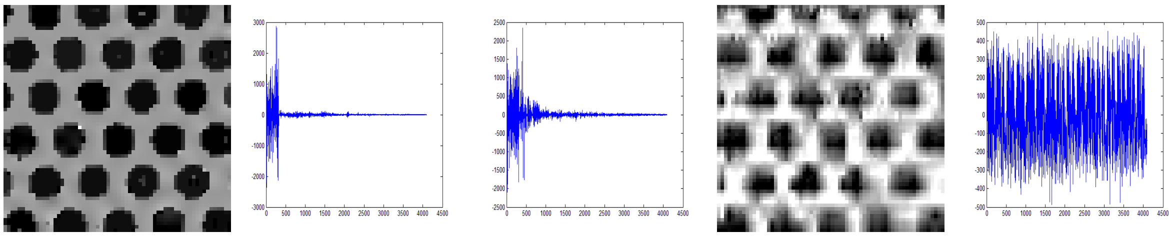


Fig. 4. Texture reconstruction in the GBT domain (1.5.02), from left to right: reconstructed texture , GBT coefficients, GBT coefficients evaluated using a GBT transform from a different texture realizazion, reconstruction using [16], DWT coefficients.

by M random projections of the vector $\boldsymbol{z}$:

$$\boldsymbol{y} = \Phi \boldsymbol{z} \tag{1}$$

being Φ a $M \times N$ matrix of Gaussian entries. CS theory dictates conditions for reconstructing the signal provided that $M \geq f(S)$, with $f(S)$ a suitable function of the signal sparsity, and still assuring $M \ll N$. If the signal is sparse in the GBT N-dimensional orthonormal basis vectors $\boldsymbol{u}_k$, i.e.

$$\boldsymbol{z} = \sum_{n=0}^{N-1} \zeta_k \boldsymbol{u}_k, \;\; \zeta_k = \boldsymbol{u}_k^{\mathrm{T}} \boldsymbol{z},$$

being $\boldsymbol{\zeta} = [\zeta_0 \dots \zeta_{N-1}]^{\mathrm{T}}$ an S-sparse vector, the observation model in (1) is written by introducing the basis matrix $U = [\boldsymbol{u}_0 | \dots | \boldsymbol{u}_{N-1}]$ so as to obtain

$$\boldsymbol{y} = \Phi U \; \boldsymbol{\zeta} \tag{2}$$

With these positions, the CS measurements $\boldsymbol{y}$ are expressed as collected by means of a sensing matrix ΦU from the vector $\boldsymbol{\zeta}$ that, although non-sparse, is compressible, i.e. its energy is carried by a reduced number of coefficients.

Let us point out that the GBT transform basis depends on image dependent patterns, which are shared by textures of the same class. Thereby, in the implementation of a a CS texture recovery procedure, firstly CS texture classification stage is needed to identify the texture class, then the GBT basis matching the so found class is applied, and finally the CS recovery algorithm is applied.

Based on the compact texture representation $\boldsymbol{\zeta}$ provided in the GBT domain, several state-of-the-art algorithms could be applied. Herein, we resort to the well-known CoSamp algorithm [11], and we adapt it to account for the compressible, yet non exactly sparse nature of the texture GBT.

The state-of-the-art CoSamp reconstruction procedure defined for an S-sparse signal iteratively performs i) the estimation of the non-zero coefficients of the signal, in the sparsifying domain, and ii) the estimation of the non-zero coefficients' indexes, i.e. of the signal support itself. In a nut-shell, at each iteration the CoSamp pursue a better approximation of the non zero image coefficients by locally pseudo-inverting either the sensing matrix Φ, if the image is sparse in the spatial domain, or the transformed matrix ΦU, if the image is sparse in the basis U. Once a given set of coefficients has been estimated, the largest ones are retained as an initial estimate for the next iteration. Besides, the indexes of the estimated samples identify the current estimate of the signal support.

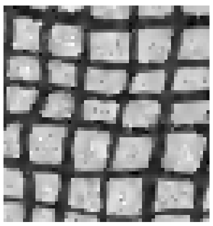
Herein, we adapt the CoSamp algorithm to capitalize on the GBT coefficients structure, in which the first coefficients always play a dominant role in the image reconstruction. We modify the algorithm so as to assure that, at every iteration, the S-dimensional support estimated by the conventional CoSamp is refined by finding its intersection with the set of the $N/4$ lower index coefficients of the GBT of the texture image. The support reduction plays a key role to enable convergence of the CoSamp algorithm, and it is theoretically legitimated since it is a particular instance of the so-called model-based recovery, whose convergence is demonstrated and discussed in depth in [10].

4. NUMERICAL SIMULATIONS

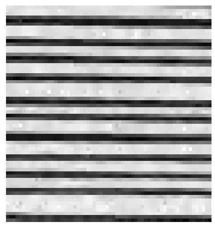
In this Section, we first discuss the role of GBT as a sparsifying transformation for texture images by means of numerical examples, and then we present the performance of application of the modified CoSaMP algorithm in the GBT domain.

Fig.1 shows three textures from the database [8], and Figs. 2-3 exemplify the GBT representation of the texture in Fig.1(b) and compare it with a wavelet based representation. Specifically, Fig.2(a) provides an example of GBT basis element found for the texture in Fig.1(b), and Fig.2(c) shows the texture transform. We clearly appreciate that most of the energy of the transformed image is carried by the first coefficients, and the texture is compactly represented in the GBT domain. The motivation is found in that, generally speaking, the GBT basis is related to the image structure, so assuring a compact image representation. In the case of textures, where many actually different images have similar appearance, the GBT basis is rather related to the entire class of images sharing the same basic visual patterns.

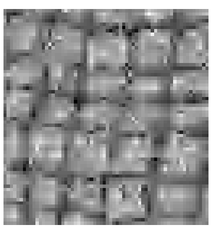
For the sake of comparison, in Fig. 3 we plot the normalized energy of the best K-term approximation obtained using the coefficient vectors of the GBT and wavelet transform domain (this latter obtained by the Daubechies wavelet transform), respectively. We recognize that, although non sparse, the texture is highly compressible in the GBT domain in that the image can be approximated by keeping only a reduced coefficients' set formed by the larger ones; in other words, as clarified in [10], the texture GBT coefficients, sorted in magnitude decreasing order, decay to zero faster than the wavelet coefficients. This observation motivates us to leverage this compressibility for reconstruction of compressively sampled texture images.



(a) PSNR=19.5 (proposed)

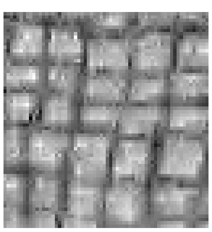


(b)PSNR=29.5 (proposed)

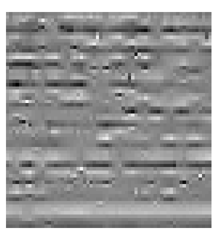


(c) PSNR=12.51 [10]

(d) PSNR=11.25 [10]



(e) PSNR=11.75 [16]



(f) PSNR=12.80 [16]

Fig. 5. Texture reconstruction in the GBT domain.

We now present a few numerical simulation results obtained by CS of the textures D104 and D49 [8]. From each of the high resolution texture we have taylored a 64×64 detail, corresponding to $N = 4096$. We have then evaluated the CS measurement vector $\boldsymbol{y}$ using a Gaussian entries matrix Φ with $M = 1512$. Finally, we apply the modified CoSamp using $S = 1024$ and restricting the iteratively estimated support to the $N/4$ lower indexes coefficients.

To the best of the author knowledge, no procedures in the literature deal with the reconstruction of compressively sampled texture. For comparison sake, we report also the reconstruction results obtained by the algorithms in [16], [10], that have been selected since they present very good performances on a wide class of natural images. The first exploits a representation of the texture in the DWT domain and relies on the assumptions that the transformed coefficients assume a typical wavelet tree structure. The second applies the general model-based recovery approach in [10] assuming a block based DWT coefficient structure.

We report the recosntructed images and their Peak Signal to Noise Ratio PSNR $= N \cdot 255^2 \left(\sum_{n=0}^{N-1} (z_n - \hat{z}_n)^2\right)^{-1}$ in Fig.5. From these results we recognize that the proposed GBT based recovery algorithm better approximates the original images. Results using different textures or increasing the number of measurements M confirm the main trends herein discussed (see Fig.4), as well as the performance ranking of the reconstruction algorithms.

To sum up, the GBT provides a promising representation domain for texture images, and can be included in state-of-the-art recovery algorithms. Further performance improvement are envisaged by the adoption of compact texture generation model [13] as well as of Bayesian CS recovery [14]. Furthermore, as far as application of GBT-based approach to compound images comprising different textures is concerned, it is worthy observing that, until now, GBT has been applied in compression problems in order to transform image patches using a locally optimized basis. Such approach implies that the transform matrix U is block diagonal. Turning to the CS acquisition model in in (2), we recognize that the impact of the U matrix structure on the CS measurements depends on the matrix Φ. Interesting developments are envisaged when the matrix Φ is block diagonal as well. This occurs, for instance, when the CS acquisition occurs in clustered networks, as analyzed in [18]. If the clustering matches the underlying image structures, the actual sensing matrix ΦU is block diagonal as well, and this is expected to lead to more effective reconstruction procedures. A detailed analysis of these challenging issues is left for further studies. Support reduction improves the convergence characteristics of CS on compressible signals, as discussed in [9], where a hierarchical extension of the CS algorithm is proposed.

5. CONCLUSION

In this paper we have investigated the reconstruction of texture images from compressively sampled measurements exploiting the texture compressibility the GBT domain. We have shown that the GBT offers promising results for a compact representation of texture images to be leveraged by CS recovery algorithms. Future research directions stem on these results. More in depth investigation is needed to relate the GBT transform, computed from local sample image characteristics, to the statistical characteristics of the image discontinuities, namely edges, often characterized by binary or complex processes [17].

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