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Performance Analysis of Extended Kalman filter and Unscented Kalman filter for Doppler Bearing Passive Target Tracking

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***Abstract*—Conventional passive bearing together with Doppler measurements is explored in Target motion analysis (TMA). This Doppler bearing tracking (DBT) approach offers one advantage compared to bearings-only tracking, that DBT does not require ownship maneuver. By using EKF and unscented Kalman filter (UKF), the target motion analysis is carried out. To obtain the convergence of the estimation fast the inclusion of range and bearing parameters is proposed in the target state vector. Doppler shifts are experienced by the harmonic signals at the ownship so that the frequency measurements can be explored to improve estimation accuracy. This paper deals with the performance evaluation of both EKF and UKF for under water tracking applications and the results for DBT are compared between extended Kalman filter and Doppler-bearing passive target tracking using unscented Kalman filter Monte Carlo simulations.**

***Keywords*- Estimation theory, Kalman based Filters, Sonar, Target tracking.**

1. INTRODUCTION

The topic of great interest with many applications in passive surveillance systems is the Bearings – only Tracking problem (BOT). Typically, two dimensional bearings only Target Motion Analysis (TMA) [1] is perhaps most familiar in the ocean environment. A moving observer monitors noisy sonar bearings to an acoustic source and processes these measurements to find out the parameters of target motion analysis by assuming the target to be travelling with uniform velocity. Determination of the trajectory of target solely from measurements of signals originating from the target is passive target tracking. The signals from target be a machinery noise, by increase in energy above the ambient at a certain bearing is the indication of the signals detection. The signal spectrum is mostly broadband but in some conditions it is limited. The harmonic components extracted from the source, and then Doppler shifts are experienced by the harmonic signals at the ownship so that the frequency measurements can be explored to improve estimation accuracy. To analyze the moving target, the Doppler shifts and bearing angles are used and it is termed as Doppler-bearing tracking (DBT) [2]. Compare to bearing-only tracking the Doppler-bearing tracking has the advantage i.e., to obtain target motion parameters Doppler-bearing tracking does not require the ownship to maneuver.

TMA is carried out for Doppler-bearing tracking and can be divided into two categories: Recursive method and Batch processing method. The recursive method based on the instrumental variables [3] and batch processing method such as maximum likelihood estimator [4]. By using search methods the solution is found out which are not suitable for real time applications. In practice, at each sample on every arrival of new measurement an improved estimate is required. For an unscented Kalman filter the inclusion of range, course and speed is proposed to track a target using frequency measurements and bearings and this algorithm is named as Doppler-bearing unscented Kalman filter (DBUKF) [5]. To calculate weapon preset parameter and to release a weapon on to target an ownship uses estimated target motion parameters.

In the frequency measurements the Doppler shift is described in terms of ownship speed, target and velocity of sound in water. The noise in the frequency measurements is not correlated with bearing measurements. The noise in the measurements is assumed by zero mean Gaussian. For every second the measurements are continuously available. Due to the number of frequency tonals the dimension of the state vector does not increase. Here state variable is defined by the sum of tonals and represented in the state vector.

2. EXTENDED KALMAN FILTER

Kalman Filter (KF) is one of the most widely used methods for linear tracking and estimation. The traditional Kalman Filter is optimal only when the model is linear. For a linear system the state estimation parameters like the mean and covariance can be exactly updated with the KF. The practical application of the KF is limited because most of the state estimation problems like tracking of the target are nonlinear.

The most common approach is linearize the nonlinear model to a certain extent before applying Kalman filter to the nonlinear model and is known as Extended Kalman Filter (EKF). And it works on the principal that a linearized transformation is approximately equal to the true nonlinear transformation. Therefore a Kalman filter that linearizes about the current mean and covariance is referred to as an extended Kalman filter or EKF. EKF can be used to track targets [3], but it introduces large errors in the estimated statistics for linearizing highly nonlinear processes and hence its applications are limited .So UKF [4] by means of UT was proposed for nonlinear models. But its application is limited only to Gaussian applications and does not applicable to general nongaussian models.

A. Unscented Kalman Filter

If the measurements are not available or not linear then the whole process becomes nonlinear [6]. When the model is only linear then traditional Kalman Filter is optimal. The state estimation problem like tracking of the target are nonlinear due to this the practical application of Kalman Filter is limited. The state estimation parameters like the mean and covariance can be exactly updated with the Kalman filter only when the system is linear. As we know the real systems that are inspiration for all these estimators like Kalman filter are governed by non-linear functions. So a non-linear version of the Kalman filter aka extended Kalman filter (EKF) is used [7]. This filter linearizes about the mean and covariance. The EKF provides first order approximations to the optimal prediction, optimal gain. But these approximations are not helpful always.

Where the non-linearity value is more it can even introduce large errors in the true posterior mean as well as in the covariance of the transformed random variable. This is not being a healthy approach to linearization might lead to sub-optimal performance and sometimes divergence of the filter. Hence the feasibility of the novel transformation of linear to nonlinear is adopted known as Unscented Kalman Filter (UKF). Unscented transformation (UT) is more accurate than linearization for propagating mean vector and covariance matrix through a nonlinear process which is explored for under water applications.

Set of points are used in UKF transform and propagates them through the actual nonlinear function, eliminating the linearization. To yield more accurate results, mean and covariance can be recalculated from the propagated points. Gaussian random variable uses 2N+1 sample points for N states. The UT approach provides advantage by treating the noise as a nonlinear function to account for non additive noises or non Gaussian for doing so firstly noise is propagated through the function by first augmenting the state vector including the noise sources. From the augmented state vector the sigma points are selected, where the noise values also includes. The result of any nonlinear effects of measurement noise and process are captured with the same accuracy the rest of state which increases the estimation accuracy in presence of additive noise and in noise free environment.

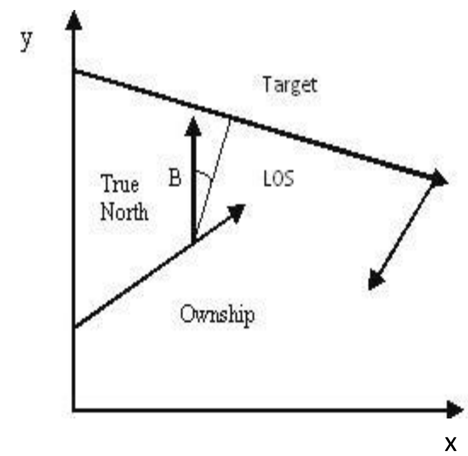

Figure.1. Target and observer encounter

3. MATHEMATICAL MODELLING

A. *State and measurement equations*

Let the target is assumed to be moving with uniform velocity as shown in fig1 and is defined the target state vector

$$X_s(k) = [\,\dot{x}(k)\;\; \dot{y}(k)\;\; R_x(k)\;\; R_y(k)\;\; F_s(k)\;\; W_x(k)\;\; W_y(k)]^T \qquad (1)$$

Where $\dot{x}(k)$ and $\dot{y}(k)$ are target velocity components and $R_x(k)$ & $R_y(k)$ denote the relative range components between observer and target.

The measured data consists of two types of measurements: frequency measurements and bearing measurements.

The bearing measurement, modeled as

$$B_m(k+1) = B(k) + \varsigma(k) \qquad (2)$$

Where $\varsigma(k)$ is the error in the measurement and is assumed to be zero mean Gaussian random variable with variance $\sigma^2(k)$.

Because of Doppler shift the tonals are constant in frequency; the frequency measured by the ownship is given by

$$f_m^{(i)}(k) = f_s^{(i)}(k)\left(1 + \frac{\dot{x}_r + \dot{y}_r}{C}\right) + \varsigma_f^{(i)}(k) \qquad (3)$$

Where $f_m^{(i)}(k)$ (i=1,2,…..,n) at k^{th}

instant denotes the measured frequency by the ownship, $f_s^{(i)}(k)$ (i=1,2,…..,n) at k[th] instant is the unknown constant source frequency, C is the speed of propagation of the signal, $\varsigma_f^{(i)}(k)$ is the Gaussian random variable.

The measurement vector Z is denoted by

$$Z=[B_m(k) \quad F_m(k)] \quad (4)$$

The target state dynamic equation is given by:

$$Xs(k+1) = \Phi(k+1/k)\, Xs(k) + b(k+1) + w(k) \quad (5)$$

Where Φ(k+1/k) is the transient matrix, b (k+1) is the deterministic vector and w(k) is the plant noise.
The transient matrix is given by:

$$\Phi(k+1/k)=\begin{bmatrix} 1 & 0 & 0 & 0 & ts & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & ts & 0 \\ ts & 0 & 1 & 0 & \frac{ts^2}{2} & 0 & 0 \\ 0 & ts & 0 & 1 & 0 & \frac{ts^2}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Where t is sample time and b(k+1) is given by:

$$b(k+1)=\begin{bmatrix} 0\ 0 - \left(x(k+1) - x(k)\right) \\ -\left(y(k+1) - y(k)\right) 0\ 0\ 0 \end{bmatrix}^T \quad (7)$$

w(k) is a zero mean Gaussian random variable with

$$E[w(k)\, w(k)^T]=Q\delta_{kj}$$

Where δ_{kj} =1 if k=j
=0 if k≠j

B. *Extended Kalman Filter Algorithm*

The familiar extended Kalman filter equations are given by

Prediction:

State: $X(k+1/k) = \Phi(k+1/k)X(k/k)+b(k+1)$ (8)

Covariance: $P(k+1/k)= \Phi(k+1/k)P(k/k)\Phi^T(k+1/k)+Q(k+1)$ (9)

Kalman Gain:

$$G(k+1)=P(k+1/k)\, H^T(k+1) [H(k+1)P(k+1/k)\, H^T(k+1) + R(k+1)]^{-1} \quad (10)$$

Correction:

State:

$$X(k+1/k+1) =X(k+1/k)+G(k+1)[Z(k+1)-\hat{Z}(k+1)] \quad (11)$$

Covariance:

$$P(k+1/k+1) = [I - G(k+1)H(k+1)]P(k+1/k)[I - G(k+1)H(k+1)]^T + GRG^T \quad (12)$$

Where Q (k+1) is covariance of the plant noise, $\hat{Z}(k+1)$ is the estimated measurement and H (K+1) is the measurements matrix at time (k+1)[th] instant.

Consider the non-linear system

$$X_{k+1}=f(x_k,u_k,w_k) \quad (13)$$

$$Z_k=h(x_k,v_k) \quad (14)$$

K is the time index, $x_{k,}$ & u_k are the state and input to the system. And w_k, v_k are white Gaussian, independent random processes with zero mean and covariance matrix.

System equations:

X_{k+1}=f (x_k,u_k,w_k) (15)

Z_k=h (x_k,v_k) (16)

Nominal Trajectory:

$\bar{x}_{k+1}$ =f ($\bar{x}_k$,u_k) (17)

$\bar{y}_k$ =h ($\bar{x}_k$) (18)

at each time step ,compute the following partial derivatives, evaluated at the nominal state

A_k=f ' ($\bar{x}_k$,u_k) (19)

C_k=h '($\bar{x}_k$) (20)

The derivatives in the above equation are taken with respect to x_k An example of partial derivative matrices is given later in this article

Define Δy_k *as the* difference between the actual measurement y_k and the nominal measurement:

$\Delta y_k = y_k - \bar{y}_k$

$= y_k - h(\bar{x}_k)$ (21)

Execute the following Kalman equations:

$K_k = P_k C_k^T (C_k P_k C_k^T + R)^{-1}$

$\Delta\hat{x}_{k+1} = A_k \Delta\hat{x}_k + K_k(\Delta y_k - C_k \Delta\hat{x}_k)$

$P_{k+1} = A_k (I - K_k C_k) P_k A_k^T + Q$

$\hat{x}_{k+1} = \bar{x}_{k+1} + \Delta\hat{x}_{k+1}$

C. *Unscented Kalman Filter Algorithm*

The Unscented Kalman Filter uses 2n+1 scalar weights, which can be calculated as

$w_o^{(m)}$= λ / (n+λ)

$w_o^{(c)}$ = [λ /(n+λ)]+(1-v^2+β^2)

$w_j^{(m)}$=$w_j^{(c)}$=1/ 2(n+λ) (22)

Where *j= 1,2,……2n* and $\lambda = (a^2-1)*n$ is a scaling parameter, *a* denotes the spread of the sigma points around the mean and *β* is used to incorporate the prior knowledge of the state distribution. *β=2* is used for Gaussian distribution.

The implementation of standard Unscented kalman filter consists of the following steps:

1. Starting from the initial conditions the calculation of 2n+1 sigma points is

X(k)=x(0) and P(k)=p(0)

X(k)=[x_s(k)x_s(k)+$\sqrt{(n+\lambda)P(k)}$$x_s$(k)-$\sqrt{(n+\lambda)P(k)}$] (23)

2. Transformation of sigma points through the process model using in equation (5)

3. The prediction of state estimate at k+1 with measurements upto k is given as:

X(k+1/k)= $\sum_{j=0}^{2n} w_j^{(m)}$X(j,k+1/k) (24)

4. The process noise is independent and additive, then the predicted covariance is given as:

P(k+1/k)= $\sum_{j=0}^{2n} w_j^{(c)}$ [X(j,k+1/k)-X(k+1/k)]* [X(j,k+1/k)-X(k+1/k)]^T+Q(k) (25)

5. With the predicted mean and covariance the updation of the sigma points are given as:

X(k+1/k)=[X(k+1/k) X(k+1/k)+$\sqrt{(n+\lambda)P(k+1/k)}$ X(k+1/k)-$\sqrt{(n+\lambda)P(k+1/k)}$] (26)

6. The transformation of each of the predicted

points is propagated through the measurement model equations (6)

7. The prediction of measurements is given as:
$Y(k+1/k)= \sum_{j=0}^{2n} w_j^{(m)} Y(k+1/k)$ (27)

8. The measurement noise is independent and also additive, then the innovation covariance is given as:
$P_{yy}=\sum_{j=0}^{2n} w_j^{(c)} [Y(j,k+1/k)Y(k+1/k)]*[Y(j,k+1/k)Y(k+1/k)]^T+R(k)$ (28)

9. Then the cross covariance is given as:
$P_{xy}=\sum_{j=0}^{2n} w_j^{(c)} [X(j,k+1/k)X(k+1/k)]*[Y(j,k+1/k)Y(k+1/k)]^T + R(k)$ (29)

10. The kalman gain is to be calculated as :
$K(k+1)=P_{xy}*P_{yy}^{-1}$ (30)

11. The state estimated is given as:
$X(k+1/k+1)= x(k+1/k)+ K(k+1)(y(k+1/k+1)-y(k+1/k))$ (31)
Where y(k) is the true measurement

12. Estimated error covariance is to be given as:
$P(k+1/k+1)=P(k+1/k)-K(k+1)*P_{yy}*K(k+1)^T$ (32)

5. RESULTS

The target state vector is initialized as:
$X(0,0)=[\dot{x}(k)\,\dot{y}(k) r\sin(B_m(0)) r\cos(B_m(0)) F_s(k) W_x(k) W_y(k)]^T$ (33)

where $F_m(0)$ and $B_m(0)$ the initial Doppler frequency measurements and bearing measurements.

The covariance matrix is initialized as: P(0/0) is a diagonal matrix of covariance matrix with the elements are given as
$P(0/0)=diagonal(4X(i)^2/12)$ (34)
Where i=1,2,……10

All frequency measurements and bearing measurements are corrupted by the additive zero mean Gaussian noise. Due to against a number of geometries the performance of this algorithm is evaluated. Here all angles are to be considered with respect to True north, positive clockwise. Comparison of Doppler-bearing extended kalman filter and Doppler-bearing unscented kalman filter target tracking estimate results are shown below for scenario1:

Scenario-1 Initial Range=1800 meters, Initial Bearing =50.00degress, Initial Position of the Observer=0.0, Standard deviation in Bearing=0.17 degrees, Standard deviation in frequency=0.33Hz, Target frequency=800.00Hz.

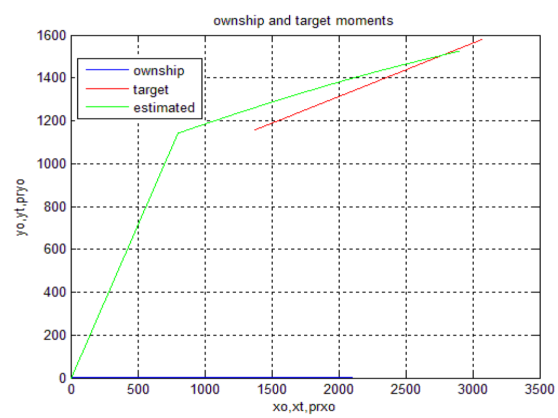

Figure.2. Doppler- bearing passive target tracking using extended kalman filter result

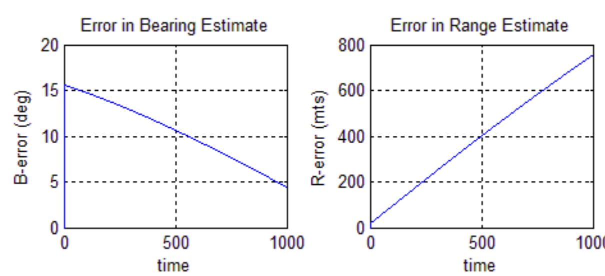

Figure.3. Doppler- bearing passive target tracking using extended kalman filter error estimate results

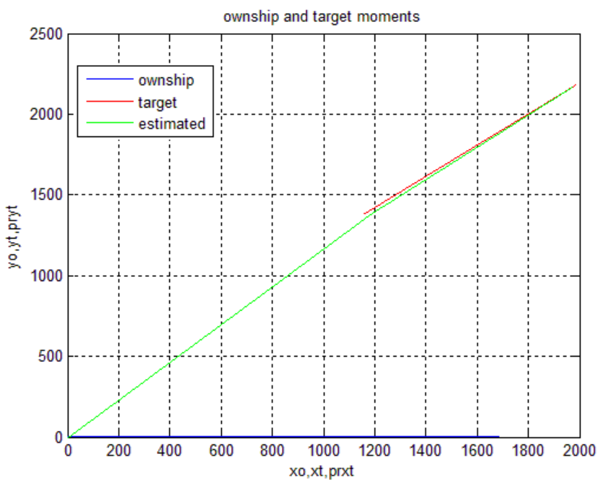

Figure.4. Doppler- bearing passive target tracking using unscented kalman filter result

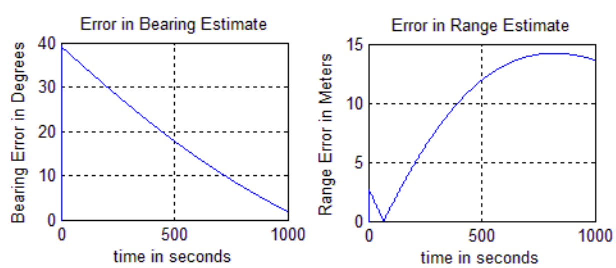

Figure.5. Doppler- bearing passive target tracking using unscented kalman filter error estimate results

6. LIMITATIONS OF THE ALGORITHM

In some cases, there will not applicable changes in the bearing measurements from beginning to end of the process. In frequency measurements the changes depends on the bearing rate, hence the changes in the frequency measurement is negligible. So , in these cases the convergence of solution is not possible, unless the ownship maneuvers in such a way that is there will be an applicable changes in bearing measurements. When the signal-to-noise ratio (SNR) is sufficiently high then the sonar can listen to a target. When the signal-to-noise ratio becomes less, then auto-tracking of the target fails; the measurements are not available continuously when the sonar tracks the target in manual mode. The bearings measurements available in manual mode are highly inconsistent and not useful for good tracking of the target. In this algorithm, it is assumed for good tracking continuity is maintained over the simulation period

7. CONCLUSIONS

In this paper, an approach without using ownship maneuver in passive target tracking is proposed to estimate target motion parameters by using EKF and UKF. It is very difficult to find out a number of maneuvers until the required accuracy in the estimated target motion parameters is achieved. But in many situations, only a single sensor is available for under water tracking application. So, to obtain the target motion parameters without using ownship maneuver the DBT is right applicable. Fig 2 to 5 shows the estimated mean square errors of Doppler-bearing Target tracking .we can ensure from the results, that A Doppler- Bearing only passive Target tracking using unscented Kalman filter has been shown to provide more accuracy than the Doppler-Bearing only passive Target tracking using Extended Kalman filter. In this approach target estimation is before converged by using unscented kalman filter compare to extended Kalman filter.

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