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Some New-Irresolute and Closed Maps in Fuzzy Topological Spaces

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Abstract

In this paper weaker and stronger forms of fuzzy b-mappings are introduced and studied using the concept of fuzzy generalized b-closed sets which are named as fagb-continuous, fagb-irresolute and fagb-closed maps. Also $fgbT^*_{1/2}$ spaces, fuzzy b-locally closed sets, fb-homeomorphisms, fb*-homeomorphisms, fgb-neighbourhood and fgbq-neighbourhood are introduced and studied. Several interesting results are obtained.

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Keywords: Fb-homeomorphism, fb*-homeomorphism, fagb-irresolute, fagb-closed mappings, fuzzy b-locally closed set,$fgbT^*_{1/2}$ spaces, fgb-neighbourhood, fgbq-neighbourhood and fuzzy topological spaces

1 Introduction

Zadeh,in [5] introduced the fundamental concept of fuzzy sets. The study of fuzzy topology was initiated by Chang [4]. The theory of fuzzy topological spaces was subsequently developed by several researchers. The concept of fb-open sets and fgb-closed sets were introduced by Benchalli et al in [1]. In this paper, some results on fb-continuous, fb*-continuous, fb-open and fgb-mappings are obtained. Fb-homeomorphism, fb*-homeomorphism are introduced and some of their properties are proved. Further fuzzy b-locally closed sets are also introduced and characterizations are obtained. As an application to fgb-closed sets, new spaces, namely $fgbT^*_{1/2}$ are introduced and studied. Weaker and stronger forms of fgb-closed sets which are called as fuzzy approximately gb-irresolute and fuzzy approximately gb-closed mappings are introduced and studied. Some characterizations of $fgbT^*_{1/2}$ spaces in terms of fagb-irresolute and fagb-closed maps are obtained. Lastly fgb-neighbourhoods,fgbq-neighbourhoods are introduced and some of their properties are obtained.

2 Preliminary Notes

Throughout this paper (X,τ),(Y,σ) and (Z,ρ) (or simply X,Y and Z) mean fuzzy topological spaces.For a fuzzy set A of (X,τ), $Cl(A)$ and $Int(A)$ denote the closure and interior of A respectively.The family of all fb-open sets is denoted by $bO(X)$.The intersection of all fb-closed sets containing A is called fb-closure of A and is denoted by $bCl(A)$ and the union of all fb-open sets contained in A is called fb-interior of A and is denoted by $bInt(A)$.

Definition 2.1 [1] *A fuzzy set A in a fts (X,τ) is called*
(i)fb-open set iff $A \leq ((IntClA) \vee (ClIntA))$ and fb-closed set iff $A \geq ((IntClA) \wedge (ClIntA)))$.
(ii)fgb-closed if $bCl(A) \leq B$, whenever $A \leq B$ and B is fuzzy open.

Definition 2.2 *A mapping $f : (X,\tau) \to (Y,\sigma)$ is said to be*
(i)f-continuous [4] *if $f^{-1}(A)$ is f-open (f-closed) in X, for each f-open (f-closed) set A in Y.*
(ii)fb-continuous [2] *if $f^{-1}(A)$ is fb-open (fb-closed) in X, for each f-open (f-closed) set A in Y.*
(iii)fb^*-continuous [2] *if $f^{-1}(A)$ is fb-open (fb-closed) in X, for each fb-open (fb-closed) set A in Y.*
(iv)fb^*-open (closed) [2] *if for every fb-open (fb-closed) A in X, $f(A)$ is fb-open (fb-closed) set A in Y.*
(v)fgb-continuous [3] *if $f^{-1}(A)$ is fgb-open (fgb-closed) in X, for each f-open (f-closed) set A in Y.*

Definition 2.3 [3] *A fuzzy topological space (X,τ) is called a fuzzy $gbT_{1/2}$ space (briefly $fgbT_{1/2}$space) if every fgb-closed set in X is fb-closed.*

3 Fb-continuous and fb-open mappings

Some of the results on fb-continuous and fb-open mappings were obtained in [2].In this section few more results on these maps are proved.

Theorem 3.1 *A fuzzy set A of a fts (X, τ) is fgb-open iff $B \leq bInt(A)$, whenever B is f-closed set and $B \leq A$.*

Proof. Proof is straight forward.

Theorem 3.2 *Let $f : (X, \tau) \rightarrow (Y, \sigma)$.Then the following statements are equivalent.*
(i) f is fb-continuous
(ii) $f(bCl(A)) \leq Cl(f(A))$, for every fuzzy set A in X.

Proof.(i)⇒(ii)Let A be a fuzzy set in X.Then Cl(f(A)) is closed. Since f is fb-continuous, $f^{-1}(Cl(f(A)))$ is fb-closed in X and so $f^{-1}(Cl(f(A))) = bCl(f^{-1}(Cl(f(A))))$. Since $A \leq f^{-1}(f(A))$,we have $bCl(A) \leq bCl(f^{-1}(f(A))) \leq bCl(f^{-1}(Cl(f(A)))) = f^{-1}(Cl(f(A)))$. Hence $f(bCl(A)) \leq Cl(f(A))$.
(ii)⇒(i) Let B be a closed fuzzy set in Y. Let $A = f^{-1}(B)$, then by (ii),$f(bCl(f^{-1}(B))) \leq Cl(f(f^{-1}(B)))$, which implies $bCl(f^{-1}(B)) \leq f^{-1}(Cl(f(f^{-1}(B)))) \leq f^{-1}(Cl(B)) \leq f^{-1}(B)$.But $f^{-1}(B) \leq bCl(f^{-1}(B))$, so $f^{-1}(B) = bCl(f^{-1}(B))$. Hence $f^{-1}(B)$ is fb-closed in X.Therefore f is fb-continuous.

The proofs of the following five Theorems on composition are straight forward.

Theorem 3.3 *If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \rho)$ are both fb*-continuous, then $gof : (X, \tau) \rightarrow (Z, \rho)$ is fb*-continuous.*

Theorem 3.4 *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is fb^*-continuous and $g : (Y, \sigma) \rightarrow (Z, \rho)$ is f-continuous, then $gof : (X, \tau) \rightarrow (Z, \rho)$ is fb-continuous.*

Theorem 3.5 *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy open and $g : (Y, \sigma) \rightarrow (Z, \rho)$ is fb^*-open, then gof is fb^*-open.*

Theorem 3.6 *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \rho)$ be two mappings. If f is fb-continuous, onto and gof is fb^*-closed, then gof is fb-closed.*

Theorem 3.7 *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \rho)$ be two mappings. If f is fb^*-continuous, onto and gof is fb^*-closed, then g is fb^*-closed.*

Definition 3.8 *A bijective map $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy b-homeomorphism (briefly fb-homeomorphism) if f and f^{-1} are fb-continuous.*

Definition 3.9 *A bijective map $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy b^*-homeomorphism (briefly fb^*-homeomorphism) if f and f^{-1} are fb^*-continuous.*

Remark 3.10 *(i)Every f-homeomorphism is fb-homeomorphism.*
(ii) Every fb^*-homeomorphism is fb-homeomorphism.

The converses are not true as shown in following examples.

Example 3.11 *Let $X = Y = \{a, b\}$, $A = \{(a, 0), (b, 1)\}$, $B = \{(x, 0), (y, 1)\}$. Let $\tau = \{0, 1, A\}$, $\sigma = \{0, 1, B\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = a$ and $f(b) = b$ is fb-homeomorphism but not f- homeomorphism, since $f^{-1} : Y \to X$ is not f-continuous.*

Example 3.12 *Let $X = Y = \{a, b\}$,$A = \{(a, 1), (b, 0)\}$, $B = \{(a, 1), (b, 0)\}$. Let $\tau = \{0, 1, A\}$,$\sigma = \{0, 1, B\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = b$ and $f(b) = a$ is fb-homeomorphism but not fb^*-homeomorphism, since $f^{-1} : Y \to X$ is not fb^*-continuous.*

Theorem 3.13 *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective map.Then the following are equivalent:*
(i)f is fb- homeomorphism
(ii)f is fb-continuous and fb-open
(iii)f is fb-continuous and fb-closed

Proof.(i)⇒(ii) Let f be fb-homeomorphism. Then f and f^{-1} are fb-continuous. To prove that f is fb-open. Let A be an f-open set in X. Then since $f^{-1} : Y \to X$ is fb-continuous $(f^{-1})^{-1}(A) = f(A)$ is fb-open in Y. Hence f is fb -open.
(ii)⇒(i) Let f be fb-continuous and fb-open. To prove that $f^{-1} : Y \to X$ is fb-continuous. Let A be a fuzzy open set in X. Then, $f(A)$ is fb-open in Y since f is fb-open,which implies $f(A) = (f^{-1})^{-1}(A)$ is fb-open in Y. Therefore $f^{-1} : Y \to X$ is fb-continuous. Hence f is fb-homeomorphism.
(ii)Let ⇒(iii) Let f be fb-continuous and fb-open. To prove that f is fb-closed. Let B be a f-closed set in X. Then $1-B$ is a f-open set in X. Since $f : X \to Y$ fb-open map, $f(1-B)$ is fb-open set in Y. Now $f(1-B) = 1-f(B)$ is fb-open set in Y, which implies $f(B)$ is fb-closed set in Y. Hence f is a fb-closed map.

Theorem 3.14 *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following are equivalent:*
(i) f is fb^*-homeomorphism
(ii) f is fb^*-continuous and fb^*-open
(iii) f is fb^*-continuous and fb^*-closed maps

Theorem 3.15 *If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \rho)$ are fb^*-homeomorphism then $gof : X \to Z$ is fb^*- homeomorphism.*

Proof. The proof is straight forward.

4 Fuzzy locally b-closed sets

Definition 4.1 *A fuzzy set A of a fts (X, τ) is said to be fuzzy locally b-closed if $A = U \wedge V$,where $U \in \tau$ and V is fb-closed.*

Theorem 4.2 *Let A be a fuzzy set in a fts (X, τ).Then A is fuzzy locally b-closed if and only if there exists a fuzzy open set U in X such that $A = U \wedge bCl(A)$.*

Proof. Let A be fuzzy locally b-closed set.Then $A = U \wedge V$, where U is fuzzy open and V is fb-closed. Then $A \leq U$ and $A \leq V.A \leq bCl(A) \leq bCl(V) = V$. Therefore $A \leq U \wedge bCl(A) \leq U \wedge bCl(V) = U \wedge V$.Hence $A = U \wedge bCl(A)$. Conversely, since bCl(A) is fb-closed and $A = U \wedge bCl(A)$. Hence A is fuzzy locally b-closed.

Theorem 4.3 *For a fuzzy set A of a fts (X, τ), the following are equivalent.*
(i)A is fb-closed,
(ii)A is fgb-closed and fuzzy locally b-closed.

Proof. (i)⇒(ii) Let A be fb-closed.Then $A = A \wedge X$, so that A is fuzzy locally b-closed. If U is a f-open set such that $A \leq U$ then $bCl(A) = A \leq U$. Hence A is fgb-closed.
(ii)⇒(i)Since A is fuzzy locally b-closed, there exists a f-open set U such that $A = U \wedge bCl(A)$. Now since $A \leq U$ and A is fgb-closed $bCl(A) \leq U$. Therefore $bCl(A) \leq U \wedge bCl(A) = A$. Hence A is fb-closed.

5 Fgb-closed sets and fgb-continuous mappings

Some of the results on fgb-closed sets and fgb-continuous mappings are already proved in [2],In this section some more results on such maps are obtained.

Theorem 5.1 *Every fuzzy closed set in (X, τ) is fgb-closed.*

Theorem 5.2 *If A is fgb-open set in a fts (X, τ) and $bInt(A) \leq B \leq A$ then B is fgb-open in (X, τ).*

Definition 5.3 *A fuzzy topological space (X,τ) is fuzzy $gbT^*_{1/2}$ space (briefly $fgbT^*_{1/2}$ space) if every fgb-closed set in X is f-closed.*

Remark 5.4 *A fuzzy topological space (X,τ) is fuzzy $gbT^*_{1/2}$ space if every fgb-open set in X is f-open.*

Theorem 5.5 *A fuzzy topological space (X,τ) is $fgbT*_{1/2}$ space if and only if every fuzzy set in (X,τ) is both f-open and fgb-open.*

Theorem 5.6 *Let $f:(X,\tau)\to(Y,\sigma)$ be an onto, fgb-irresolute and fb -closed map. If X is $fgbT^*_{1/2}$ space, then Y is $fgbT_{1/2}$ space.*

Proof. Let A be a fgb-closed set in Y. Since $f:X\to Y$ is fgb-irresolute, $f^{-1}(A)$ is fgb-closed set in X. As X is $fgbT^*_{1/2}$ space, by definition, $f^{-1}(A)$ is f-closed set in X. Also $f:X\to Y$ is fb-closed and onto, so $f(f^{-1}(A))=A$ is fb-closed in Y. Thus A is fb-closed in Y. Hence Y is also $fgbT_{1/2}$ space.

The proof of the following results are straight forward.

Theorem 5.7 *If $f:(X,\tau)\to(Y,\rho)$ is fb-continuous and $g:(Y,\sigma)\to(Z,\rho)$ is fgb-continuous then $gof:(X,\tau)\to(Z,\rho)$ is fb-continuous if Y is $fgbT^*_{1/2}$ space.*

Theorem 5.8 *If $f:(X,\tau)\to(Y,\sigma)$ is fg-continuous and $g:(Y,\sigma)\to(Z,\rho)$ is fgb-continuous then $gof:(X,\tau)\to(Z,\rho)$ is fg-continuous if Y is $fgbT^*_{1/2}$ space.*

Theorem 5.9 *Let $f:(X,\tau)\to(Y,\sigma)$ be fgb-continuous. Then f is f-continuous if X is $fgbT^*_{1/2}$ space.*

Theorem 5.10 *If $f:(X,\tau)\to(Y,\sigma)$ is a fgb- closed map and Y is $fgbT^*_{1/2}$ space, then f is a f-closed map.*

Theorem 5.11 *Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\rho)$ be two maps such that $gof:(X,\tau)\to(Z,\rho)$ is fgb^*- closed map.*
(i)If f is fgb-continuous and surjective, then g is fgb closed.
(ii)If g is fgb-irresolute and injective, then f is fgb* closed.

6 Fagb-irresolute and Fagb-closed mappings

Definition 6.1 *A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy approximately generalized b-irresolute (briefly fagb-irresolute) if $bCl(A)\le f^{-1}(B)$, whenever B is a fb-open set of Y,A is fgb-closed set of X and $A\le f^{-1}(B)$.*

Definition 6.2 *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy approximately generalized b-closed (briefly fagb-closed) if $f(B) \leq bInt(A)$, whenever A is fgb-open of Y, B is f-closed set in X and $f(B) \leq A$.*

Remark 6.3 *Clearly fb^*-continuous map is fagb-irresolute and fb^*-closed map is fagb-closed, but the reverse implication are not true.*

Example 6.4 *Let $X = \{a, b\}$ and $Y = \{x, y\}$. Define $A = \{(a, 0.3), (b, 0.4)\}$, $B = \{(x, 0.7), (y, 0.8)\}$.Let $\tau = \{0, 1, A\}$ and $\sigma = \{0, 1, B\}$. Then the map $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is fagb-irresolute but not fb^*-continuous.*

Example 6.5 *Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$. Define $A = \{(x, 0), (y, 0.3), (z, 0.2)\}$, $B = \{(a, 0), (b, 0.3), (c, 0.2)\}$, $C = \{(a, 0.9), (b, 0.6), (c, 0.7)\}$. Let $\tau = \{0, 1, A\}$ and $\sigma = \{0, 1, B, C\}$. Then the map $f : (X, \tau) \to (Y, \sigma)$ defined by $f(x) = a$ and $f(y) = b$,$f(z) = c$ is fagb-closed but not fb^*-closed.*

Theorem 6.6 *The mapping $f : (X, \tau) \to (Y, \sigma)$ is fagb-irresolute if $f^{-1}(A)$ is fb-closed in X for every fb-open set $A \in Y$.*

Proof. Let $B \leq f^{-1}(A)$, where $A \in bO(Y)$ and B be fgb-closed set in X.If $f^{-1}(A)$ is fb-closed in X then $bClB \leq bClf^{-1}(A) = f^{-1}(A)$. Thus $f : X \to Y$ is fagb-irresolute.

The converse is not true, in general. However,the following holds.

Theorem 6.7 *Let $f : (X, \tau) \to (Y, \sigma)$.If every fuzzy set in X is both fb-open and fb-closed sets, then f is fagb-irresolute if and only if $f^{-1}(A)$ is fb-closed in X for every $A \in bO(Y)$.*

Proof. Assume $f : X \to Y$ to be fagb-irresolute.Let $A \leq B$,where $B \in bO(X)$.Let every fuzzy set in X be both fb-open and fb-closed.So every fuzzy set of X is both fgb-closed and fgb-open in X. Hence for $A \in bO(Y)$,$f^{-1}(A)$ is fgb-closed in X.Since $f : X \to Y$ is fagb-irresolute we have $bClf^{-1}(A) \leq f^{-1}(A)$.But $f^{-1}(A) \leq bClf^{-1}(A)$.Therefore $bClf^{-1}(A) = f^{-1}(A)$. Hence $f^{-1}(A)$ is fb-closed in X.

The converse is proved in theorem 6.6.

Theorem 6.8 *A function $f : (X, \tau) \to (Y, \sigma)$ is fagb-closed if $f(A) \in bO(Y)$ for every fb-closed set A in X.*

Proof. Let A be fb-closed set in X and B be fgb-open set in Y such that $f(A) \leq B$. Therefore $bIntf(A) \leq bInt(B)$, which implies $f(A) \leq bInt(B)$. Thus $f : X \to Y$ is fagb-closed.

Theorem 6.9 *Let $f : (X, \tau) \to (Y, \sigma)$. If every fuzzy set in X is both fb-closed and fb-open ,then f is fagb-closed if and only if $f(B) \in bO(Y)$ for every fb-closed set B in X.*

Proof. Assume $f : X \to Y$ to be fagb-closed. Let every fuzzy set in Y be both fb-open and fb-closed. So every fuzzy set of Y are both fgb- closed and fgb-open in Y.Let B be fb-closed set in X.Then $f(B)$ is fgb-open in Y.Since $f : X \to Y$ is fagb-closed $f(B) \leq bInt(f(B))$. But $bInt(f(B)) \leq f(B)$. Therefore $bInt(f(B)) = f(B)$. Hence $f(B)$ is fb-open set in X.

Converse is proved in theorem 6.8.

Theorem 6.10 *If a map $f : (X, \tau) \to (Y, \sigma)$ is surjective,fb-continuous and fagb-closed, then the inverse image of each fgb-closed set in Y is fgb-closed in X.*

Proof. Let A be fgb-closed set in Y. Suppose $f^{-1}(A) \leq B$, where B is fuzzy open in X.Then $1 - B \leq 1 - f^{-1}(A)$ which implies $1 - f(B) \leq 1 - A$. Since f is fagb-closed, $1 - f(B) \leq bInt(1 - A)$, which implies $1 - f(B) \leq 1 - bCl(A)$.Thus $1 - B \leq 1 - (f^{-1}bCl(A)))$,which implies $f^{-1}bCl(A) \leq B$.Since f is fb-continuous,$f^{-1}(bCl(A))$ is fb-closed in X.We have $bCl(f^{-1}(A)) \leq bCl(f^{-1}(bCl(A))) = f^{-1}(bCl(A)) \leq B$.Thus $f^{-1}(A)$ is fgb-closed in X.

Theorem 6.11 *If $f : (X, \tau) \to (Y, \sigma)$ is fb-continuous and fagb-closed mapping, then $f(A)$ is fgb-closed set in Y for every fgb-closed set A of X.*

Proof. Let A be fgb-closed set in X. Let $f(A) \leq B$, where B is any f-open set in Y.Since f is fb-continuous , by definition $f^{-1}(B)$ is fb-open in X and $A \leq f^{-1}(B)$.Since A be fgb-closed set in X, we have $bCl(A) \leq f^{-1}(B)$.Thus $f(bCl(A)) \leq B$.Since f is fagb-closed,$f(bCl(A))$ is fgb-closed in Y and hence $bClf(A) \leq bCl(f(bCl(A)))= f(bCl(A)) \leq B$.Therefore $f(A)$ is fgb-closed in Y.

Theorem 6.12 *If a map $f : (X, \tau) \to (Y, \sigma)$ is fagb-irresolute and fagb-closed then for every fgb-closed set A of X,$f(A)$ is fgb-closed set of Y.*

Theorem 6.13 *Let $f : (X, \tau) \to (Y, \sigma)$,$g : (Y, \sigma) \to (Z, \rho)$ be two maps such that $gof : (X, \tau) \to (Z, \rho)$ then,*
(i)$g \circ f$ is fagb-closed, if f is fb^*-closed and g is fagb-closed.
(ii)$g \circ f$ is fagb-closed, if f is fab-closed and g is fb-open and g^{-1} preserves fgb-open sets.
(iii)$g \circ f$ is fagb-irresolute, if f is fagb-irresolute and g is fb^*-continuous.

Proof. (i)Let B be fb-closed set in X and A be fgb-open set in Z such that $(gof)(B) \leq A$. Since $f : X \to Y$ is fb^*-closed $f(B)$ is fb-closed

in Y. Since $g : Y \rightarrow Z$ is fagb-closed $g(f(B)) \leq bIntA$ which implies $(g \circ f)(B) \leq bIntA$.Hence $g \circ f$ is fagb-closed.
(ii)Suppose B is fb-closed in X and A is fgb-open in Z for which $(gof)(B) \leq A$.Hence $f(B) \leq g^{-1}(A)$. Now $g : Y \rightarrow Z$ is fb^*-open and g^{-1} preserves fgb-open sets, so $f(B) \leq bIntg^{-1}(A)$.Therefore $(g \circ f)(B) = g(f(B)) \leq g(bInt(g^{-1}(A))) \leq bInt(g(g^{-1}(A)) \leq bInt(A)$ .Therefore $g \circ f$ fagb-closed map.
(iii)Suppose B is fgb-closed in X and $A \in Z$ for which $B \leq (g \circ f)^{-1}(A)$. Since $g : Y \rightarrow Z$ is fb^*-continuous $g^{-1}(A)$ is fb-open in Y.Since $f : X \rightarrow Y$ is fagb-irresolute that implies $bCl(B) \leq f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$.Hence $g \circ f$ is fagb-irresolute.

Theorem 6.14 *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map such that*
(i)If the fb-open and fb-closed sets in X coincide,then $f : X \rightarrow Y$ is fagb-irresolute if and only if $f : X \rightarrow Y$ is fb^*-continuous.
(ii)If the fb-open and fb-closed sets in Y coincide, then $f : X \rightarrow Y$ is fagb-closed if and only if $f : X \rightarrow Y$ is fb-closed.

Theorem 6.15 *Let (X, τ) be a fts.Then the following statements are equivalent:*
(i) (X, τ) is $fgbT_{1/2}$ space.
(ii) For every fts (Y, σ) and every map $f : (X, \tau) \rightarrow (Y, \sigma)$, f is fagb-irresolute.

Proof. (i)⇒(ii)Let A be a fgb-closed subset in X and suppose that for $B \in bO(Y)$,$A \leq f^{-1}(B)$. Since X is $fbT_{1/2}$-space by definition, every fgb-closed set is fuzzy closed.Hence A be a fgb-closed subset of X.Therefore $bCl(A) \leq f^{-1}(B)$.Hence $f : X \rightarrow Y$ is fagb-irresolute.
(ii)⇒(i)Let A be a fgb-closed subset of X. Let $f : X \rightarrow Y$ be the identity map, where $\sigma = \{0, A, 1\}$. So A is fgb- closed in X and fb-open in Y. Since $f : X \rightarrow Y$ is fagb-irresolute $A \leq f^{-1}(A)$. Hence it follows that $bCl(A) \leq A$. Hence A is fb-closed.Every fgb-closed set is fb-closed in X, hence X is $fgbT_{1/2}$ space.

Theorem 6.16 *Let (Y, σ) be a fts.Then the following statements are equivalent:*
(i) (Y, σ) is $fgbT_{1/2}$ space.
(ii) For every fts (X, τ) and every map $f : (X, \tau) \rightarrow (Y, \sigma)$,f is fagb-closed.

7 Fgb-neighbourhoods and fgbq-neighbourhoods

Definition 7.1 *Let A be a fuzzy set in fts X and x_p is a fuzzy point of X, then A is called fuzzy generalized b-neighbourhood (briefly fgb-neighbourhood) of x_p if and only if there exists a fgb-open set B of X such that $x_p \in B \leq A$.*

Definition 7.2 *Let A be a fuzzy set in fts X and x_p is a fuzzy point of X, then A is called fuzzy generalized b-q-neighbourhood (briefly fgbq-neighbourhood) of x_p if and only if there exist a fgb-open set B such that $x_p q B \leq A$.*

Theorem 7.3 *A is fgb-open set in X if and only if for each fuzzy point $x_p \in A$, A is a fgb-neighbourhood of x_p.*

Theorem 7.4 *If A and B are fgb-neighbourhood of x_p then $A \wedge B$ is also a fgb-neighbourhood of x_p.*

Theorem 7.5 *Let A be a fuzzy set of a fts X.Then a fuzzy point $x_p \in bCl(A)$, if and only if every fgbq-neighbourhood of x_p is quasi- coincident with A.*

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