# Joint Entity Resolution 

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#### Abstract

Entity resolution (ER) is the process of matching records that represent the same real-world entity and then merging them. We consider the ER problem for two related datasets. In the datasets, a record in one can refer to a record in the other and an ER process running on one set can affect an ER process on the other. We formalize the joint ER model for datasets which reference each other by treating the match and merge functions as black boxes. We identify important properties for match and merge functions that, if satisfied, allow much more efficient ER. We provide four algorithms that run Entity Resolution for a pair of datasets. We show that our parallel algorithms require shorter runtime than naive alternate algorithms. We also introduce improvements for our parallel algorithms which result in fewer feature comparisons.


## 1 Introduction

Entity Resolution (ER) (sometimes referred to as deduplication) is the process of identifying and merging records judged to represent the same real-world entity. For example, two companies that merge may want to combine their customer records: for a given customer that dealt with the two companies they create a composite record that combines the known information.

Most of the previous work considered ER techniques for a single class dataset. However, in practice, many data integration tasks need to tackle complex information spaces where datasets of multiple classes and relationships between the datasets exist. In this paper, we assume

[^0]there are two data sets to be resolved. Those data sets are related, which means a record in one data set can refer to a record in the other data set. ER algorithm for those datasets is complex because a ER process for one dataset may affect other ER process.
For concreteness, in this paper we focus on a type of ER processing called generic pair-wise. In this case, a domain expert writes two functions, a match and a merge function. The pair-wise match rule $M\left(r_{1}, r_{2}\right)$ evaluates to true when two records $r_{1}$ and $r_{2}$ are determined to represent the same entity. If $M\left(r_{1}, r_{2}\right)$ is true, then a merge function is used to create the composite record $\left\langle r_{1}, r_{2}\right\rangle$. Note that after a merge we may identify new matches with other records. For example, the combined information in $\left\langle r_{1}, r_{2}\right\rangle$ may match with a third record $r_{3}$, while neither $r_{1}$ nor $r_{2}$ had enough information to generate a match with $r_{3}$.
The alternative to pair-wise ER is generally some type of global clustering strategy that groups records that are similar and are deemed to represent the same real-world entity [16, 4]. Briefly, pair-wise may be easier to implement since the domain expert only needs to consider two records at a time, and pair-wise may be more amenable to incremental and distributed processing [1]. Clustering approaches may yield more accurate results.
Several works [14, 15, 7, 5] have addressed algorithms which handles two datasets and they have focused on accuracy of matching and how to compare records. By contrast, we focus on performance and treat the functions for comparing and merging records as black-boxes. We will show efficient parallel algorithms for jointly comparing and merging $\left\langle r_{1}, r_{2}\right\rangle$ those data sets.

In summary, in this paper we make the following contributions:

- We formalize the generic ER problem for two related dataset (Section 3).
- We identify the ICAR properties of match and merge functions for this problem that lead to efficient strategies (Section 4.2).
- We present parallel ER algorithms for two related dataset (Section 5).
- We experimentally evaluate the algorithms using two sets of related data, one on papers and another on their authors. Our algorithms result in good performance in terms of the number of feature comparisons and runtime (Section 6).


## 2 Motivating Example

To illustrate joint ER, consider two data sets $X$ and $Y$, where $X$ is a set of books and $Y$ is a set of authors. Books in $X$ have attributes Title and WrittenBy. Authors in $Y$ have attributes Name, Affiliation and Publication. The two data sets are related because values in the WrittenBy attribute refer to $Y$ author names, and vice versa. For example, $X$ may contain the tuple, $\langle$ Database book, H.GarciaMolina〉 and Y may contain the tuple, 〈H.GarciaMolina, Stanford University, Database book $\rangle$.

In this example, the rules for determining when authors (or books) represent the same real world entity are interrelated. For instance, if two records in $X$ have identical title, or similar titles and were written by the same author, we can assume that they refer the same entity. Similarly, if two records in $Y$ have identical name, or similar name and identical affiliation and those authors wrote the same books, we can assume that they refer the same entity. For every record in $X$ we need to refer to $Y$ to find the affiliation of its author. Moreover, for every record in $Y$ we need to refer to $X$ to find the names of books its author wrote.

## 3 Model

A match rule $M_{I}$ determines if two records $r_{1}$ and $r_{2}$ in the record set $I$ refer to the same real-world entity. If
the records match, $M_{I}\left(r_{1}, r_{2}\right)=$ true, we denote this as $r_{1} \approx r_{2}$; otherwise, if $M_{I}\left(r_{1}, r_{2}\right)=$ false, we denote this $r_{1} \neq r_{2} . M_{I}$ results always true or false; we do not consider approximate matches (with an associated confidence value).

A merge rule $\mu_{I}$ merges two records in the record set $I$ into one. The function is only defined for matching records. The result of $\mu_{I}\left(r_{1}, r_{2}\right)$ is denoted $\left\langle r_{1}, r_{2}\right\rangle$.
Let $X$ and $Y$ be the sets of records to be resolved. We discuss entity resolution from the point of view of the $X$ records; the case for the $Y$ records in symmetrical. All the functions for $X$ that we define (which have an $X$ subscript) have a $Y$ analogue.

A record $r \in X$ has a field $F$ that refers to one or more $Y$ records. We refer to this field as $F_{X}(r)$. Note that $F_{X}(r) \subseteq Y$. (We could extend our model to include a confidence value with each link $s \in F_{X}(r)$, representing how confident we are that $s$ is the $Y$ record associated with $r$. These confidences could be taken into account by the match function when deciding if two records match. We do not discuss such an extension here.)

When $F_{X}(r)$ contains a single element we omit the set brackets and simply write $F_{X}(r)=s(s \in Y)$. We use the notation $F_{X}^{-1}(r)$ to refer to the $Y$ records whose $F_{Y}$ field contains $r$. Note that $F_{X}^{-1}(r)$ can be implemented in a variety of ways. For instance, we can scan $Y$ searching for $r$ references, which would be expensive. We could also keep back pointers in $X$ records, i.e., materialize the $F_{X}^{-1}(r)$ values. The latter option would involve an update cost when $Y$ records change.
For set $X$, we have a match function $M_{X}\left(r_{1}, r_{2}\right)$ that returns true if records $r_{1}, r_{2} \in X$ represent the same real world entity. We assume that the match function is of the following form:

$$
M_{X}\left(r_{1}, r_{2}\right)=B_{X}\left(r_{1}, r_{2}\right) \vee\left[L_{X}\left(r_{1}, r_{2}\right) \wedge\right.
$$ $\left.C\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)\right]$

The "base" comparison function $B_{X}$ can determine if $r_{1}$ and $r_{2}$ match without consulting any $Y$ records. For example, if two products have almost identical names, prices and codes, then we assume they are the same, regardless of their manufacturer (represented by $Y$ records). If the $B_{X}$ function cannot determine that $r_{1}$ and $r_{2}$ match on their own, then the second part of the match function checks if they match due to common references to $Y$ records. For example, we may say that two products match if they are manufactured by the same company (the
$F_{X}$ fields point to a single common manufacturer), and if $L_{X}$ says the products names are roughly the same.

There are two important issues to discuss regarding the clause $C\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)$ in the match function.

First, there are two natural choices for the $C$ comparison function. One is a function $C_{1}$ that checks if all elements in the sets match. For example, say a book record $r_{1}$ in $X$ refers to two authors in $Y, s_{1}$ and $s_{2}$. Similarly, a second book $r_{2}$ refers to authors, i.e., $F_{X}\left(r_{2}\right)=\left\{s_{3}, s_{4}\right\}$. The condition $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)$ is true only if (a) $s_{1}$ equals either $s_{3}$ or $s_{4}$, and (b) $s_{2}$ equals the remaining $Y$ author.

$$
C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \equiv F_{X}\left(r_{1}\right)=F_{X}\left(r_{2}\right)
$$

The second natural condition, $C_{2}$ checks that a subset of $F_{X}\left(r_{1}\right)$ matches a subset of $F_{X}\left(r_{2}\right)$. In the previous example, if $s_{1}$ or $s_{2}$ equals either $s_{3}$ or $s_{4}$ then the condition is true.

$$
C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \equiv F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{2}\right) \neq \emptyset
$$

The second issue regarding the comparison function is how $F$ values are updated. In particular, during entity resolution we assume that as $Y$ records are merged, the $F_{X}$ values are updated. For instance, say $F_{X}\left(r_{1}\right)=s_{1}$ and $F_{X}\left(r_{2}\right)=s_{2}$. If $s_{1}$ and $s_{2}$ merge into a new record $s_{3}$, then $F_{X}\left(r_{1}\right)=F_{X}\left(r_{2}\right)=s_{3}$. This change may then make $r_{1}$ and $r_{2}$ match in set $X$. Note that these updates can be implemented in a variety of ways. For example, with an eager strategy, as soon as $s_{1}$ and $s_{2}$ merge, we can update all $X$ records in $F_{Y}^{-1}\left(s_{1}\right)$ and $F_{Y}^{-1}\left(s_{2}\right)$. With a lazy strategy, we do not update the $X$ records but instead keep a translation table $T$ that maps records to their current merged record. For instance, before the merge $T\left(s_{1}\right)=s_{1}$ and after the merge $T\left(s_{1}\right)=s_{3}$. In this lazy case, to perform the comparison $F_{X}\left(r_{1}\right)=F_{X}\left(r_{2}\right)$ in the match function, we actually perform $T\left(F_{X}\left(r_{1}\right)\right)=$ $T\left(F_{X}\left(r_{2}\right)\right.$ ) (where $T$ operates on a set in the obvious way).

When two $X$ records $r_{1}$ and $r_{2}$ merge, a composite record $r_{3}=\left\langle r_{1}, r_{2}\right\rangle$ is created by the merge function. The merge function creates an $F_{X}\left(r_{3}\right)$ value. For now we do not make any assumptions about $F_{X}\left(r_{3}\right)$.

However, the most natural value for $F_{X}\left(r_{3}\right)$ would be $F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)$.

Given match and merge functions for $X$, we can define the resolved set of records $E R_{X}(X)$ as done in [1]. We also have analogous functions for the $Y$ set and a definition of $E R_{Y}(Y)$.

We define the joint resolution of $X$ and $Y$ as an iterative process, where we alternate resolving one set and the other until we reach a fixed point.

```
ER(X, Y):
    done := false;
    X' := X; Y' := Y;
    while not done do
            [ newX := ER_X(X'); newY := ER_Y(Y');
                done := (newX = X') and (newY = Y')
                X' := newX; Y' := newY ]
    return (X', Y');
```

Note that the above is simply our definition of a correct joint resolution. In what follows we investigate efficient ways to perform such joint resolution (showing that the more efficient strategy yields the same correct results as the above definition). We also study properties of the match and merge functions that may make the resolution more efficient.

## 4 Properties

### 4.1 Basic Properties

We assume four basic properties called ICAR properties for $M_{I}$ and $\mu_{I}$ : idempotence, commutativity, representativity and associativity. (We use $I$ in $M_{I}$ and $\mu_{I}$ to represent either $X$ or $Y$.) These properties ware addressed in [2]. Idempotence says that any record matches itself, and merging a record with itself yields the same record. Commutativity says that, if $r_{1}$ matches $r_{2}$, then $r_{2}$ matches $r_{1}$. Additionally, the merged results of $r_{1}$ and $r_{2}$ should be identical regardless of the merge ordering. The meaning of the representativity property is that record $r_{3}$ obtained from merging two records $r_{1}$ and $r_{2}$ "represents" the original records, in the sense that any record $r_{4}$ that would have matched $r_{1}$ (or $r_{2}$ by commutativity) will also match
$r_{3}$. Intuitively, this property states that there is no "negative evidence": merging two records $r_{1}$ and $r_{2}$ cannot create evidence (in the merged record $r_{3}$ ) that would prevent $r_{3}$ from matching any other record that would have matched $r_{1}$ or $r_{2}$.

- Idempotence: $\forall r ; r \approx r$ and $\langle r, r\rangle=r$
- Commutativity: $\forall r_{1}, r_{2}, r_{1} \approx r_{2}$ iff $r_{2} \approx r_{1}$, and if $r_{1} \approx r_{2}$, then $\left\langle r_{1}, r_{2}\right\rangle=\left\langle r_{2}, r_{1}\right\rangle$
- Associativity: $\forall r_{1}, r_{2}, r_{3}$ such that $\left\langle r_{1},\left\langle r_{2}, r_{3}\right\rangle\right\rangle$ and $\left\langle\left\langle r_{1}, r_{2}\right\rangle, r_{3}\right\rangle$ exist, $\left\langle r_{1},\left\langle r_{2}, r_{3}\right\rangle\right\rangle=\left\langle\left\langle r_{1}, r_{2}\right\rangle, r_{3}\right\rangle$
- Representativity: If $r_{3}=\left\langle r_{1}, r_{2}\right\rangle$ then for any $r_{4}$ such that $r_{1} \approx r_{4}$, we also have $r_{3} \approx r_{4}$.

In [2] we argue that many match and merge rules naturally satisfy the ICAR properties. However, now our match and merge function have specific components, so in the next sub-section we study what properties these components (i.e., $B, L, C$ ) must satisfy in order to meet the ICAR properties.

### 4.2 Properties in Two Datasets

Merge Function: To maintain representativity, $F_{X}\left(\left\langle r_{1}, r_{2}\right\rangle\right)$ must at least contain $F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)$; if the merge function removes any records in $F_{X}\left(r_{1}\right)$ or $F_{X}\left(r_{2}\right)$, representativity might no longer hold.

The proof of the above property, as well as other properties given in this section, are given in Appendix A.

Since it does not make sense for the merge function to add links not already in $r_{1}$ or $r_{2}$, to satisfy the above property we will assume that $F_{X}\left(\left\langle r_{1}, r_{2}\right\rangle\right)$ is equal to $F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)$.

Idempotence: For $M_{X}$ to be idempotent, $B_{X}$ must be idempotent. Note that this is regardless of $L_{X}$ and $C$.

Commutativity: If $B_{X}$ and $L_{X}$ are commutative, $M_{X}$ must be commutative. This property holds regardless of whether $C$ is implemented as $C_{1}$ or as $C_{2}$. (becasue both $C_{1}$ and $C_{2}$ are commutative.)

Associativity: As in the general case, the merge function $\mu_{I}$ should be associative.

Representativity: The conditions for representativity are different for $C_{1}$ and $C_{2}$. For $C_{2}$, if $B_{X}$ and $L_{X}$ are representative and $B_{X} \Rightarrow L_{X}$, then $M_{X}$ must be representative. Note that $B_{X} \Rightarrow L_{X}$ means $B_{X}$ is a stronger condition than $L_{X}$. For example, if $B_{X}\left(r_{1}, r_{2}\right)$ is $\operatorname{similarity}\left(r_{1}, r_{2}\right) \geq 0.9, L_{X}\left(r_{1}, r_{2}\right)$ could be $\operatorname{similarity}\left(r_{1}, r_{2}\right) \geq 0.8$. For $C_{1}$, in addition to the previous condition, the records must satisfy $B_{X} \Rightarrow C_{1}$. It is not possible to satisfy this last property in practice, since the content of the $F$ sets would have to be determined by the outcome of the $B_{X}$ function. Thus, the ICAR properties cannot be expected to hold if $C_{1}$ is used.
To summarize, for the ICAR properties to hold, the following conditions must be met:

- Merge: $F_{X}\left(\left\langle r_{1}, r_{2}\right\rangle\right)=F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)$
- Idempotence: $\forall r ; B_{X}(r, r)=$ true, $\langle r, r\rangle=r$
- Commutativity: $\forall r_{1}, r_{2}, B_{X}\left(r_{1}, r_{2}\right)=B_{X}\left(r_{2}, r_{1}\right)$, and $L_{X}\left(r_{1}, r_{2}\right)=L_{X}\left(r_{2}, r_{1}\right)$, and if $M_{X}\left(r_{1}, r_{2}\right)=$ true, then $\left\langle r_{1}, r_{2}\right\rangle=\left\langle r_{2}, r_{1}\right\rangle$.
- Associativity: $\forall r_{1}, r_{2}, r_{3}$ such that $\left\langle r_{1},\left\langle r_{2}, r_{3}\right\rangle\right\rangle$ and $\left\langle\left\langle r_{1}, r_{2}\right\rangle, r_{3}\right\rangle$ exist, $\left\langle r_{1},\left\langle r_{2}, r_{3}\right\rangle\right\rangle=\left\langle\left\langle r_{1}, r_{2}\right\rangle, r_{3}\right\rangle$
- Representativity: If $r_{3}=\left\langle r_{1}, r_{2}\right\rangle$ then
- For any $r_{4}$ such that $B_{X}\left(r_{1}, r_{4}\right)=$ true, we also have $B_{X}\left(r_{3}, r_{4}\right)=$ true
- For any $r_{4}$ such that $L_{X}\left(r_{1}, r_{4}\right)=$ true, we also have $L_{X}\left(r_{3}, r_{4}\right)=$ true.
- For any $r_{1}$ and $r_{4}$ such that $B_{X}\left(r_{1}, r_{4}\right)=$ true, we also have $L_{X}\left(r_{1}, r_{4}\right)=$ true
- In case $C_{1}$ is used, if $B_{X}\left(r_{1}, r_{4}\right)=$ true, then we mst have $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)=$ true.


## 5 Resolution

### 5.1 Problem of Naive ER Algorithm

In section 3, we showed an algorithm which alternates running Entity Resolution on two data sets. That algorithm has two performance problems.

First, the algorithm of section 3 may need a large amount of time; each ER process must run to completion
before the next may begin. To improve response time, we will show the following algorithm, which runs Entity Resolution on X and Y concurrently.

Second, the algorithm may perform redundant comparisons. For example, say there are a records where $r_{1}, r_{2} \in X, s_{1}, s_{2} \in Y, F_{X}\left(r_{1}\right)=s_{1}, F_{X}\left(r_{2}\right)=$ $s_{2}, B_{X}\left(r_{1}, r_{2}\right)=$ false, $L_{X}\left(r_{1}, r_{2}\right)=$ true, $M_{Y}\left(s_{1}, s_{2}\right)=\mathrm{false}$. At the comparison in the first RSwoosh execution on $X$, records $r_{1}$ and $r_{2}$ do not match, because $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)$ is false. At the comparison in the next R-Swoosh execution on $X$ after doing so on $Y$, records $r_{1}$ and $r_{2}$ will not match because both $F_{X}\left(r_{1}\right)$ and $F_{X}\left(r_{2}\right)$ are not changed and $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)$ will be false. In the example, $M_{X}\left(r_{1}, r_{2}\right)$ is computed twice, and the computation is redundant. Section 5.4 shows a modification that eliminates redundancy.

### 5.2 Preliminaries

To explain the new algorithms, consider records $r_{1}, r_{2}, s_{1}, s_{2}$ where $r_{1}, r_{2} \in X$ and $s_{1}, s_{2} \in Y$. Furthermore, say $F_{X}\left(r_{1}\right)=\left\{s_{1}\right\}, F_{X}\left(r_{2}\right)=\left\{s_{2}\right\}$. Say we want to run ER on $X$ and $Y$.

We make the standard assumptions about our parallel computing model: we assume that no messages exchanged between processors are lost.

### 5.3 Running R-Swoosh in Parallel

Our Simple Parallel ER (SPER) algorithm consists of three code segments, shown as Algorithms 1, 2, 3.

Each processor for $X$ and $Y$ runs procedure $E R(I)$ detailed in Algorithms 1, where $I$ is the input set of records (either $X$ or $Y$ ). Except for lines 6, 25, 26, 29-31, $E R(I)$ is the Swoosh algorithm of [2], which assumes the ICAR properties. In this code, set $I^{\prime}$ contains the records that are tentatively in the result. Each record currentRecord in $I$ is considered in turn, and if it does not merge with any $I^{\prime}$ records, currentRecord is moved to $I^{\prime}$.

However, activity in the concurrent ER process may lead to a new local match (because the $F$ sets changed), and hence a record already in $I^{\prime}$ may have to be moved back to $I$ so it can be reconsidered in light of new information. In particular, it is possible for a match result $M_{I}\left(r_{1}, r_{2}\right)$ to change from false to
true when: $B_{I}\left(r_{1}, r_{2}\right)=$ false, $L_{I}\left(r_{1}, r_{2}\right)=$ true, $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)=$ false.

To inform the concurrent ER process of merges, the ER procedure sends UPDATE messages in line 25. Then, after each $I$ record is processed (line 29), procedure ER checks for received messages from its concurrent process by calling procedure UPDATE (Algorithm 2).

To check for changes in local $F$ sets, procedure ER calls function UPDATE after each $I$ record is processed (line 29). To illustrate function UPDATE, say in our running example $P_{Y}$ merges $s_{1}$ and $s_{2}$ to $s_{3}=\left\langle s_{1}, s_{2}\right\rangle$, so it sends $P_{X}$ an update messages which includes $F_{Y}^{-1}\left(\left\langle s_{1}, s_{2}\right\rangle\right)\left(=r_{1}, r_{2}\right)$ and the identity of $s_{3}$. When that message is received (line 3 of UPDATE) at $P_{X}$, if either $r_{1}$ or $r_{2}$ happen to be in $I^{\prime}\left(=X^{\prime}\right)$, they are moved back to $I(=X)$ since these records need to be compared again, in light of the new information. Also, in line XX the $F$ sets are updated locally using a lazy strategy.

The concurrent ER processes finish when neither one has new merged records to report and all outstanding UPDATE messages have been fully processed. The termination function (Algorithm 3) checks these conditions. When $P_{X}$ finished processing all records in $X$ (line 5), it sends a termination message with the number of messages it sent and retrieved. When $P_{Y}$ receives a termination message, it examines the contents and if it knows that all those messages were processed then it completes execution.

### 5.4 Enhanced Parallel ER Algorithm

We will show how the joint ER algorithm can be modified to improve efficiency by explaining how an ER process sends and reacts to messages.

First, we consider the possibility of reducing the number of record comparisons. When ER processes apply a Simple Parallel algorithm to the example given above, $P_{X}$ compares records in $X, r_{1}, r_{2}$, to all records in $X^{\prime}, r_{3}$, and moves the records in $X$ to $X^{\prime}$. Then, after receiving messages, $P_{X}$ moves $r_{1}$ and $r_{2}$ from $X^{\prime}$ back to $X$. Finally, $P_{X}$ compares $r_{1}$ and $r_{2}$ to $r_{3}$ again. However, only the comparison of $r_{1}$ and $r_{2}$ could match. All other comparisons between $r_{1}$ and $r_{3}$ will not match, and are redundant.

To avoid this redundant comparison, we will show how a process can react to messages more efficiently. Just be-
fore moving $X^{\prime}$ records back to $X, P_{X}$ runs R-Swoosh for the records which refer to merged $Y$ records. If by doing so, $P_{X}$ produces new records, then $P_{X}$ adds them to $X$. If instead $P_{X}$ deletes records, then it deletes them from $X^{\prime}$. In the process, $P_{X}$ will not retrieve messages. This algorithm is shown in Algorithm 4. In the example given above, after $P_{X}$ pops the update messages, $P_{X}$ runs R-Swoosh for $r_{1}, r_{2}$, and produces $\left\langle r_{1}, r_{2}\right\rangle . P_{X}$ then adds $\left\langle r_{1}, r_{2}\right\rangle$ to $X$, and deletes $r_{1}$ and $r_{2}$ from $X^{\prime}$. Note that $P_{X}$ does not compare $r_{1}$ and $r_{3}$ again.

Second, we consider the possibility of reducing the number of feature comparisons in a record comparison. We use a cache to reduce the number of feature comparisons. When records do not match, they may be compared again. However, once an ER process compares the records, the evaluation of $B_{I}$ and $L_{I}$ will not change so the process will not need to be repeated. In our algorithm, if $L_{I}$ is false, an ER process memoizes the combination of records in a table. Before comparing records, if an ER process finds the combination of records in the table, it knows the evaluation is false without computing $B_{I}$ and $L_{I}$. Algorithm 5 is called as a match function from Algorithm 1 and update the cache named $N O M A T C H_{I}$. Algorithm 6 is called as a match function from Algorithm 4 and use the cache.

## 6 Experiments

We implemented the Alternate ER algorithm given in section 3 and the Simple Parallel ER algorithm, the Enhanced Parallel ER Algorithm, and the Enhanced Parallel ER algorithm with cache given in section 5 . We used the cora citation data set. We compared the number of merges performed and messages sent by the four algorithms while varying the thresholds of $B_{I}$ and $L_{I}$. We also compared the number of comparisons by and runtime of the algorithms while varying the thresholds of $B_{I}$ and $L_{I}$. Finally, we conducted scalability tests.

### 6.1 Experimental Setting

We ran our experiments on a citation dataset. We retrieved two data sets from the cora dataset: author and paper. The author dataset includes an ID, an author name, an institution, and a paper title, which refers to paper


Figure 1: Merges and Messages Plotted Against $L_{I}$ for the Paper Dataset
records. The paper dataset includes an ID, a title, a venue, and an author name, which refers to author records. $B_{I}$ and $L_{I}$ evaluate the lexical similarity of each attribute other than ID and compare the similarity to a threshold. The $B_{I}$ threshold is larger than $L_{I}$ threshold so that $B_{I}$ and $L_{I}$ meet the representativity condition, $B_{I} \rightarrow L_{I}$, then they satisfy ICAR properties. We fixed $k=2$, $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \equiv F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{2}\right) \neq \emptyset$. We implemented $F_{X}$ for author records as a set of IDs of paper records whose title is the exactly same as the title in the author record. An approximate match between the titles in the author and paper records might be required, however we do not talk in this paper. This may be a future work. We also implemented $F_{X}$ for paper records as a set of IDs of author records whose author name is the exactly same as the author name in the paper record.

The algorithms were implemented in Java, and our experiments were run on four 2.4 GHz Intel Core processor with 4GB of memory.

### 6.2 Merges and Messages

We measured the number of merges performed in and messages sent by a ER process by varying the thresholds of either $B_{I}$ or $L_{I}$.
Figure 1 and 2 shows the number of merges and messages while varying the $L_{I}$ threshold. When we varied the $L_{I}$ threshold, we fixed the $B_{I}$ threshold to 1.0. Figure 3 shows the number of merges while varying the $B_{I}$ threshold. When we varied the $B_{I}$ threshold, we fixed the $L_{I}$ threshold to 0.7. Lower thresholds of $B_{I}$ and $L_{I}$ resulted


Figure 2: Merges and Messages Plotted Against $L_{I}$ for the Author Dataset


Figure 3: Merges and Messages Plotted Against $B_{I}$ for the Paper Dataset
in a higher number of merges because processes produced more new records. The number of messages corresponded to the number of merges because when records were merged processes sent messages. Relatively few termination messages were observed. The fact that the Enhanced Parallel algorithm sent more messages than the alternate algorithm seems inefficient however it resulted in a shorter runtime. We will explain why below.

### 6.3 Comparisons and Runtimes

We measured the number of feature value comparisons for each algorithm and runtimes of the four algorithms by varying the $L_{I}$ thresholds for both the author and paper datasets. We started measuring the runtime when a ER process start R-Swoosh algorithm, and we finished measuring it when a ER process end by termination protocol


Figure 4: Feature Comparisons for the Paper Dataset


Figure 5: Feature Comparisons for the Author Dataset
which we showed. We fixed $B_{I}$ threshold to 1.0 in this experiment.
Figure 4 shows the number of feature comparisons for paper records. In general, the number increased for lower thresholds because lower thresholds produced more new composite records. However, when the threshold was high, more than 0.8 , the number of feature comparisons decreased for lower thresholds in the alternative algorithm. The reason why is that processes run R-Swoosh repeatedly for larger record sets. The Enhanced Parallel ER algorithm performed $40.1 \%$ of the comparisons of the alternate algorithm for a high threshold (0.98). For lower thresholds, the relative advantage over the alternate algorithm decreased. However it still performed only $70.3 \%$ of the comparisons of the alternate algorithm for a low threshold (0.6).
Figure 5 shows the number of feature comparisons for author records. Notice that the number of feature com-


Figure 6: Runtime for the Paper Dataset


Figure 7: Runtime for the Author Dataset
parisons for the Simple Parallel ER algorithm was much larger than for the other algorithms. The reason why is that processes repeated comparisons every time records matched. The plot also shows the Enhanced Parallel ER algorithm reduced redundant comparisons; the number of comparisons was between 23.5 and $40.3 \%$ of the Simple Parallel algorithm's.

These plots also shows that the use of a cache did not affect the number of feature comparisons. The difference between the Enhanced Parallel ER algorithm with and without cache was between 95 and $100 \%$.
Figures 6 and 7 show runtime. The graphs are nearly identical because our termination protocol required both ER processes to terminate at the same time. For large thresholds (0.98), the Enhanced Parallel ER algorithm required $30.9 \%$ of the runtime of the alternate algorithm. For thresholds as low as 0.6 , the runtime was still almost $50.8 \%$. Our Enhanced Parallel algorithm was sig-


Figure 8: Scalability Test of Feature Comparisons for the Paper Dataset


Figure 9: Scalability Test of Feature Comparisons for the Author Dataset
nificantly faster in terms of runtime.

### 6.4 Scalability

We conducted scalability tests for the alternate algorithm and for the Enhanced Parallel ER Algorithm. We measured the number of feature comparisons and runtimes by varying the number of input record both for paper and author records. Input records were selected randomly from a paper dataset containing 1037 records and an author dataset containing 780 records. The $B_{I}$ and $L_{I}$ thresholds were fixed to 1.0 and 0.8 respectively.

Figure 8 and 9 shows the number of comparisons made by each algorithm. The plot shows that the Enhanced Parallel ER algorithm performed fewer comparisons than the alternate algorithm. For very few number of input records near 400, the Enhanced Parallel ER algorithm


Figure 10: Scalability Test of Runtime for the Paper Dataset


Figure 11: Scalability Test of Runtime for the Author Dataset
performed very fewer comparisons than the alternate algorithm, $42.6 \%$ for paper records and $66.8 \%$ for author records. For larger number of record more than 700, the improvement rates were almost same around $50 \%$ for paper records and $70 \%$ for author records.

Figure 10 and 11 shows rumtimes of each algorithm. The plots shows that the Enhanced Parallel ER algorithm performed faster than the alternate algorithm. For very few number of input records near 400, the Enhanced Parallel ER algorithm performed very faster than the alternate algorithm, $27.2 \%$ for paper records and $21.3 \%$ for author records. For larger number of record more than 700, the improvement rates were almost same around $35 \%$ for both records.

## 7 Related Work

Entity Resolution has been studied under various names including record linkage [17], merge/purge [12], deduplication [18], reference reconciliation [8], object identification [19], and more (see [21, 11] for recent surveys).

Most approaches focus on matching: accurately finding records that represent the same entity, using a variety of techniques such as Fellegi and Sunter's probabilistic linkage rules [9], Bayesian networks [20], or clustering [16] [4]. Our approach encapsulates the behavior of such complex decision processes into a Boolean match function that decides whether two records represent the same entity or not. Iterative approaches [3] [8] identify the need for a feedback loop that compares merged records in order to discover more matches. Our ER algorithm provides a general framework where match and merge are black-box functions.

In the parallel computing literature, $[1,13]$ introduced a parallel algorithm for single dataset based on the Swoosh algorithm. $[6,10]$ used parallel algorithms as well.

Several works $[14,15,7,5]$ have addressed algorithms which handles two datasets. [14] introduced a parallel algorithm for two datasets. However the datasets it handles belong to same class. By contrast, our target datasets belong to different classes. [15, 7, 5] presented collective models for different datasets. However their research focused on accuracy rather than runtime. Although they demonstrated that making multiple ER decisions collectively can provide better accuracy than historical approaches, however it is believed that the approach is expensive.

## 8 Future Work

Future work might proceed in several directions. First, one might consider how to run ER for three or more datasets. Second, one might consider more flexible implementation of $C\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)$ which can handle similarity of confidence values, because $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \equiv F_{X}\left(r_{1}\right)=F_{X}\left(r_{2}\right)$ may be too strict and $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \equiv F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{2}\right) \neq \emptyset$ may be too lax. Finally, one might investigate approximate reference between two datasets.

## 9 Conclusion

In this paper, we formalized the problem of Entity Resolution for two related different datasets. In the datasets, a record in one can refer to a record in the other and an ER process affect to other ER process.

We also provided four algorithms that run ER jointly for two datasets. The alternate algorithm is the most straightforward, but it results in long runtimes. The Simple Parallel algorithm is efficient in terms of runtime, but it performs more redundant comparisons than the alternate algorithm. The Enhanced Parallel algorithm is an efficient way to reduce the number of comparisons and runtime. In our experiments, it required $31 \%$ of the runtime of the alternate algorithm. Adding a cache to the Enhanced Parallel algorithm did not affect its performance.

Finally, we presented four important properties for match and merge functions for two datasets that lead to significantly more efficient ER. We argued that these properties should guide the development of match and merge functions.

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## A Appendix: Proofs

Proposition 1. Idempotence: If $B_{X}$ and $\mu_{X}$ are idempotent, then idempotence holds.

Proof. If we know $B_{X}(r, r)=$ true, then we know $M_{X}(r, r)=$ true. $\left\langle F_{X}(r), F_{X}(r)\right\rangle=F_{X}(r) \cup F_{X}(r)=$ $F_{X}(r)$. Therefore, because $M_{X}(r, r)=$ true and $\langle r, r\rangle=r$ by definition, the idempotence holds.

Proposition 2. Commutativity: If $B_{X}$ and $L_{X}$ and $\mu_{X}$ are commutative, then commutativity holds.

Proof. For $C_{1},\left(F_{X}\left(r_{1}\right)=F_{X}\left(r_{2}\right)\right)=\left(F_{X}\left(r_{2}\right)=\right.$ $F_{X}\left(r_{1}\right)$ ). Similarly, for $C_{2}, F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{2}\right)=F_{X}\left(r_{2}\right) \cap$
$F_{X}\left(r_{1}\right)$, so $C_{1}$ and $C_{2}$ are always commutative, then $M_{X}$ are commutative. In terms of merge, $F_{X}$ is commutative because $F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)=F_{X}\left(r_{2}\right) \cup F_{X}\left(r_{1}\right)$. Therefore the records are always commutative.

Proposition 3. Associativity: If $\mu_{X}$ is associative, the records are associative.

Proof. $\left\{\left\{F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)\right\} \cup F_{X}\left(r_{3}\right)\right\}=\left\{F_{X}\left(r_{1}\right) \cup\right.$ $\left.\left\{F_{X}\left(r_{2}\right) \cup F_{X}\left(r_{3}\right)\right\}\right\}$, then $F_{X}$ is associative, so if the $\mu_{X}$ is associative, the records are always associative.

Proposition 4. Representativity: For $C_{2}$, if $B_{X}$ and $L_{X}$ are representative and $B_{X} \Rightarrow L_{X}$, then representativity holds. For $C_{1}$, if $B_{X}$ and $L_{X}$ are representative and $B_{X} \Rightarrow L_{X}$ and $B_{X} \Rightarrow C_{1}$, then representativity holds.

If $r_{3}=\left\langle r_{1}, r_{2}\right\rangle$ then for any $r_{4}$ such that $B_{X}\left(r_{1}, r_{4}\right)=$ true, we also have $B_{X}\left(r_{3}, r_{4}\right)=$ true, and for any $r_{4}$ such that $L_{X}\left(r_{1}, r_{4}\right)=$ true, we also have $L_{X}\left(r_{3}, r_{4}\right)=$ true, and for any $r_{1}$ and $r_{4}$ such that $B_{X}\left(r_{1}, r_{4}\right)=$ true, we also have $L_{X}\left(r_{1}, r_{4}\right)=$ true and in the case of $C_{1}$, we also have $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)=$ true.

Proof. We begin by showing $C$ is representative. To do that, we must consider two cases. For $C_{2}$, if $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)=$ true, $F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{4}\right) \neq$ $\emptyset$ and $F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{4}\right) \subseteq F_{X}\left(r_{1}\right) . \quad F_{X}\left(r_{3}\right)=$ $F_{X}\left(r_{1}\right) \cup F_{X}\left(r_{2}\right)$, then $F_{X}\left(r_{1}\right) \subseteq F_{X}\left(r_{3}\right)$, so $F_{X}\left(r_{1}\right) \cap$ $F_{X}\left(r_{4}\right) \subseteq F_{X}\left(r_{3}\right)$. Then $F_{X}\left(r_{4}\right) \cap F_{X}\left(r_{3}\right) \neq \emptyset$ and $C_{2}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)=$ true. We will consider for $C_{1}$ later.

We now consider four cases in turn.
If $B_{X}\left(r_{1}, r_{2}\right)$ and $B_{X}\left(r_{1}, r_{4}\right)$ equal to true, then $B_{X}\left(r_{3}, r_{4}\right)$ equal to true, because $B_{X}$ is representative. Then, $r_{3} \approx r 4$.
If $L_{X}\left(r_{1}, r_{2}\right) \quad$ and $\quad C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \quad$ and $L_{X}\left(r_{1}, r_{4}\right)$ and $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)$ equal to true, then $L_{X}\left(r_{3}, r_{4}\right)=$ true, because $L_{X}$ is representative. $C_{2}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)$ also equal to true, as we explain above. Then, $r_{3} \approx r 4$.
If $\quad B_{X}\left(r_{1}, r_{2}\right)$ and $L_{X}\left(r_{1}, r_{4}\right)$ and $C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)$ equal to true, then $L_{X}\left(r_{1}, r_{2}\right)=$ true, because $B_{X} \Rightarrow L_{X} . L_{X}\left(r_{3}, r_{4}\right)$ and $C_{2}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)$ equal to true because of same reason as previous explanation. Then, $r_{3} \approx r 4$.

If $\quad L_{X}\left(r_{1}, r_{2}\right) \quad$ and $\quad C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \quad$ and $B_{X}\left(r_{1}, r_{4}\right)$ equal to true, then $L_{X}\left(r_{1}, r_{4}\right)=$ true, because $B_{X} \Rightarrow L_{X} . L_{X}\left(r_{3}, r_{4}\right)$ and $C_{2}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)$ equal to true because of same reason as previous explanation. Then, $r_{3} \approx r 4$.

We now consider four cases for $C_{1}$.
If $B_{X}\left(r_{1}, r_{2}\right)$ and $B_{X}\left(r_{1}, r_{4}\right)$ equal to true, then $B_{X}\left(r_{3}, r_{4}\right)$ equal to true, because $B_{X}$ is representative. Then, $r_{3} \approx r 4$.

If $L_{X}\left(r_{1}, r_{2}\right) \quad$ and $\quad C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \quad$ and $L_{X}\left(r_{1}, r_{4}\right)$ and $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)$ equal to true, then $L_{X}\left(r_{3}, r_{4}\right)=$ true, because $L_{X}$ is representative. $C_{1}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)$ also equal to true, because $F_{X}\left(r_{3}\right)=F_{X}\left(r_{1}\right) \cap F_{X}\left(r_{2}\right)=F_{X}\left(r_{1}\right)=F_{X}\left(r_{4}\right)$. Then, $r_{3} \approx r 4$.

If $B_{X}\left(r_{1}, r_{2}\right)$ and $L_{X}\left(r_{1}, r_{4}\right)$ and $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)$ equal to true, then $L_{X}\left(r_{1}, r_{2}\right)=$ true, because $B_{X} \Rightarrow L_{X}$, and then $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right)=$ true, because $B_{X} \Rightarrow C_{1}$. $L_{X}\left(r_{3}, r_{4}\right)$ and $C_{1}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)$ equal to true because of same reason as previous explanation. Then, $r_{3} \approx r 4$.

If $\quad L_{X}\left(r_{1}, r_{2}\right) \quad$ and $\quad C_{2}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{2}\right)\right) \quad$ and $B_{X}\left(r_{1}, r_{4}\right)$ equal to true, then $L_{X}\left(r_{1}, r_{4}\right)=$ true, because $B_{X} \Rightarrow L_{X}$, and then $C_{1}\left(F_{X}\left(r_{1}\right), F_{X}\left(r_{4}\right)\right)=$ true, because $B_{X} \Rightarrow C_{1} . \quad L_{X}\left(r_{3}, r_{4}\right)$ and $C_{1}\left(F_{X}\left(r_{3}\right), F_{X}\left(r_{4}\right)\right)$ equal to true because of same reason as previous explanation. Then, $r_{3} \approx r 4$.

As we mentioned above, in all case where $r_{3}=\left\langle r_{1}, r_{2}\right\rangle$ and $r_{1} \approx r_{4}$, we always have $r_{3} \approx r_{4}$, because we showed in all cases, $r_{1} \approx r_{4}$.

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```
input: record set \(X\) and \(Y\)
output: a set \(X^{\prime}\) and \(Y^{\prime}\) of records, \(X^{\prime}=E R(X)\),
\(Y^{\prime}=E R(Y)\)
call \(E R(X)\) and \(E R(Y)\) together
\(E R(I)\) :
\(I^{\prime} \leftarrow \emptyset\), sentCount \({ }_{I}\), recievedCount \({ }_{I} \leftarrow 0\)
loop
    if \(I \neq \varnothing\) then
        currentRecord \(\leftarrow\) a record from \(I\)
        remove current Record from \(I\)
        \(b u d d y \leftarrow\) null
        for all records \(r^{\prime}\) in \(I^{\prime}\) do
            result \(\leftarrow M_{I}\) (currentRecord, \(r^{\prime}\) )
            if result \(=\) true then
                buddy \(\leftarrow r^{\prime}\)
                    exit for
            end if
        end for
        if \(b u d d y=\) null then
            add currentRecord to \(I^{\prime}\)
        else
            \(r^{\prime \prime} \leftarrow\langle\) currentRecord, buddy \(\rangle\)
            remove buddy from \(I^{\prime}\)
            add \(r^{\prime \prime}\) to \(I\)
            SEND
                            (UPDATE,
            \(F_{I}^{-1}\left(r^{\prime \prime}\right),\left(\right.\) currentRecord, buddy, \(\left.\left.r^{\prime \prime}\right)\right)\)
            sentCount \(_{I}++\)
        end if
    end if
    UPDATE()
    if TERMINATION ()\(=\) true then
        return \(I^{\prime}\)
    end if
end loop
```

Algorithm 1: Parallel Joint ER Algorithm

```
UPDATE():
while UPDATE messages exist do
    pop UPDATE message }
    (oldRecord 1, oldRecord }\mp@subsup{\mp@code{N}}{2}{}\mathrm{ , newRecord)
    T(oldRecord}\mp@subsup{d}{1}{})\leftarrow\mathrm{ newRecord
    T(oldRecord}\mp@subsup{)}{2}{})\leftarrow\mathrm{ newRecord
    recievedCount }\mp@subsup{I}{}{+}
    add I'\prime}\cap\mp@subsup{I}{}{\prime}\mathrm{ to }
    remove }\mp@subsup{I}{}{\prime\prime}\cap\mp@subsup{I}{}{\prime}\mathrm{ from }\mp@subsup{I}{}{\prime
end while
```

Algorithm 2: Simple Update Algorithm

```
UPDATE():
while UPDATE messages exist do
        pop UPDATE message \(\rightarrow \quad I^{\prime \prime}\),
        (oldRecord \({ }_{1}\), oldRecord \({ }_{2}\), newRecord)
        \(T\left(\right.\) oldRecord \(\left._{1}\right) \leftarrow\) newRecord
        \(T\left(\right.\) oldRecord \(\left._{2}\right) \leftarrow\) newRecord
        recievedCount \({ }_{I}++\)
        \(E R\left(I^{\prime \prime} \cap I^{\prime}\right) \rightarrow I^{\prime \prime \prime}\), with no UPDATE
        add \(I^{\prime \prime \prime}-I^{\prime}\) to \(I\)
        remove \(\left(I^{\prime \prime} \cap I^{\prime}\right)-I^{\prime \prime \prime}\) from \(I^{\prime}\)
    end while
```

Algorithm 4: Enhanced Update Algorithm

```
TERMINATE():
if \(I \neq \emptyset \vee\) UPDATE messages exist then
    return false
end if
SEND (TERMINATE, sentCount \({ }_{I}\),
receivedCount \(_{I}, P_{J}\) )
loop
    Wait for messages
    if UPDATE messages exist then
        return false
    end if
    if TERMINATE messages exist then
        pop TERMINATE message
        sentCount \(_{J}\), receivedCount \(_{J}\)
        if sentCount \(_{J}=\) receivedCount \(_{I} \wedge\)
        receivedCount \(_{J}=\) sentCount \(_{I}\) then
            return true
        end if
    end if
end loop
```

Algorithm 3: Termination Protocol

```
\(M_{I}\left(r_{1}, r_{2}\right)\) :
if \(B_{I}\left(r_{1}, r_{2}\right)=\) true then
    return true
else if \(L_{I}\left(r_{1}, r_{2}\right)=\) false then
    add \(r_{1}\) to NOMATCH \(H_{I}\left(r_{2}\right)\)
    return false
else if \(C_{k}\left(F_{I}\left(r_{1}\right), F_{I}\left(r_{2}\right)\right)=\) true then
    return true
else
    return false
end if
```

Algorithm 5: Update Cache

```
\(M_{I}\left(r_{1}, r_{2}\right)\) :
if \(\quad r_{1} \quad \in \quad \operatorname{NOMATCH}_{I}\left(r_{2}\right) \mid r_{2} \quad \in\)
NOMATCH \(H_{I}\left(r_{1}\right)\) then
    return false
else
    return \(M_{I}\left(r_{1}, r_{2}\right)\) without \(N O M A T C H_{I}\)
end if
```

Algorithm 6: Match Function with Cache


[^0]:    *Visiting from NEC Corporation

