The Phase Shift in the Jumping Ring

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The popular physics demonstration experiment known as Thomson's Jumping Ring (JR) has been variously explained as a simple example of Lenz's law, or as the result of a phase shift of the ring current relative to the induced emf. The failure of the first-quadrant Lenz's law explanation is shown by the time the ring takes to jump and by levitation. A method is given for measuring the phase shift with results for aluminum and brass rings.

Why the First-Quadrant Explanation Fails

Perhaps the most common explanation for why the jumping ring jumps is that when current is first applied to the solenoid, a magnetic field is set up that induces a current in the ring. That ring current creates a magnetic field that opposes the changing field of the solenoid, according to Lenz's law.[1,2] The opposing poles of these two fields – solenoid and ring – cause the ring to be thrown upward. This argument uses just the first quarter-cycle, or first quadrant, of the applied alternating current and ignores the remainder of the cycle. The second explanation for why the ring jumps uses the entire cycle and says that the current in the ring is out of phase with the emf induced in the ring, and that this phase shift produces the net upward force on the ring.[3] Serway[4] suggested a variant of the first-quadrant explanation, namely that the ring jumps when a battery is connected to the solenoid. This impulsive explanation works only if a very large dc voltage, greater than 700 V, is applied, as shown recently by Tanner, et al.[5]

Since the experiment is always done with alternating current, the proper explanation must involve the complete cycle. Furthermore, the ring doesn't even jump but just "dances" if the iron core does not extend beyond the top of the solenoid, through the ring, contrary to what is implied by Feynman[1] and Griffiths.[2] Quinton[6] argued the case of the phase-shift explanation, but reference to the first-quadrant explanation persists in the literature.[7]

Several experimental facts show the failure of the first-quadrant explanation: (1) the time the ring takes to jump clear of the extended core is several periods of the applied alternating current, (2) under certain conditions the ring levitates on the core, and (3) if you hold the ring down you can feel the strong upward force. These all demonstrate that the force acting on the ring is not a one-shot impulse. As will be shown later, the force actually pulsates at twice the line frequency and averages over time to an essentially continuous net upward force.

This paper describes a kinematics experiment that permits determination of the time required for the ring to jump clear of the extended iron core. The setup is shown in Fig. 1. The motion may be broken into two parts: (1) motion along the core and (2) motion after the ring leaves the core. The justification for this is that the magnetic force approaches zero when the ring is above the core. this is discussed in more detail in the next section.

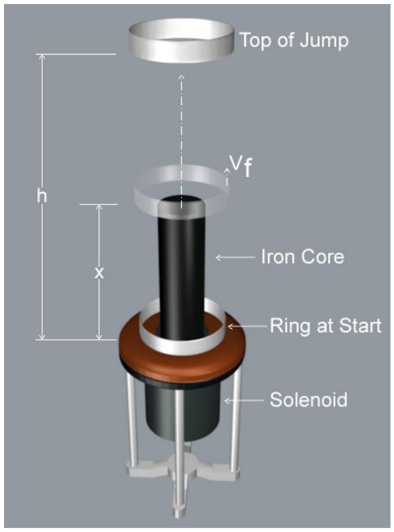

Fig. 1. Set up of kinematics experiment for the jumping ring. The ring jumps a distance h above the top of the solenoid. The iron core is extended a distance x beyond the top of the solenoid. The velocity as the ring leaves the core is v_f.

For the first part of the motion, the average velocity is $v_{ave} = v_f/2$, where v_f is the velocity with which the ring leaves the core and is given by:

$$v_f = \sqrt{2g(h-x)} \quad . \tag{1}$$

The average upward acceleration while the ring is on the core is given by

$$a_{ave} = \frac{2x}{t^2} = \frac{v_f{}^2}{2x} = g(\frac{h}{x} - 1) \, . \tag{2}$$

The time for the ring to jump is the distance, x, divided by the average velocity:

$$t = \frac{2x}{\sqrt{2g(h-x)}} \, . \tag{3}$$

The results of an experiment with a 2-cm high aluminum ring are given in Table I. The height was measured with a ruler held next to the apparatus and observing the maximum jump of the bottom of the ring. No significant variation (within 5%) of the jump height was observed from run to run with the same core extension. These results show that the time to jump off the core is much greater than the time of a quarter-cycle, 0.00417 s for the 60-Hz power used, even when the ring is cooled to 77 K.

Table I

Results of a kinematics experiment for an aluminum ring 2 cm high, 6 cm diameter and 3 mm thick, giving the final velocity leaving the core, the time to jump, and the average acceleration relative to the acceleration of gravity, g. See Fig. 1.

x (cm)	h (cm)	v_f (cm/s)	t (s)	a_{ave}/g
8	14	110	0.148	0.75
10	22	150	0.130	1.2
15	45	240	0.124	2.0
17	62	300	0.114	2.6
17*	180	570	0.060	9.6

***liquid nitrogen cooled**

In a recent paper[7] the suggestion was made that that a jump height of 4 m creates "some doubt" that the phase-difference explanation holds rather than a simple application of Lenz's law. Using the dimensions given in that paper, a jump of 1 meter takes 0.100 s, while the 4-meter jump takes 0.046 s to clear the core. Both clearly require more than a first-quadrant explanation.

The kinematics equations further reveal the fallacy of the first-quadrant explanation: If the ring jumps over even a 10-cm extended core in 4 ms, v_f would be 50 m/s, h would be over 120 m, and a_{ave} would be over 1200 g! Clearly such does not happen. The ring does not jump in one-quarter cycle. The average accelerations shown in Table I are just a few times the acceleration of gravity in the most extreme case.

Further evidence for the lack of any significant impulsive effect in the jump is shown by holding the ring down and then releasing it after power is applied: It jumps to the same height as when not held down at first.

Levitation is more evidence that an essentially continuous force acts on the ring.[8] Brass rings, for instance, levitate or "dance," and do not jump because of their higher resistivity and greater density. An aluminum ring will levitate near the top of the core if placed over the core with the power on. Placing the split ring on the levitating ring, thereby increasing only the mass but not the current, moves the levitation point lower on the core. This shows that the upward magnetic force is greater lower on the core and decreases toward the top. This was also the conclusion of J. Hall, who inverted the JR apparatus and measured the force exerted by the ring on a scale.[9] Tjossem and Cornejo also reported the force on the ring is greater at the bottom and decreases toward the top.[10] In addition, the latter authors found that the ring takes several cycles, or 0.10 s, to clear the core, using strobe photography. This is in agreement with the results of our kinematics experiment shown in Table I.

The Magnetic Force on the Ring

To investigate how the force varies along the core, both the emf in the ring and the radial component of the magnetic field have been studied as a function of position. The emf was determined using a split ring connected to an oscilloscope, measuring the peak-to-peak voltage, as shown in Fig. 2.

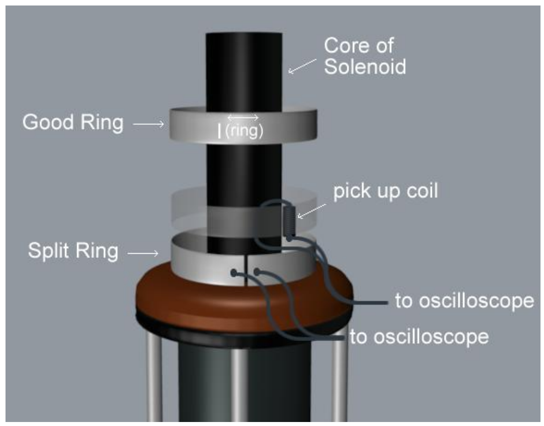

Fig. 2. Two rings are shown placed over the extended core of the JR apparatus: (1) the split ring placed at the bottom of the core to measure the emf in the ring; (2) the good ring which is lowered over a pickup coil and held while the emf from the pickup coil is observed on the oscilloscope.

The results, shown in Fig. 3, reveal: (1) for a given core extension, the emf decreases along the core, and (2) the emf at the bottom increases when more core is extended, corresponding to an increased current in the solenoid due to a decrease of its inductive reactance. .

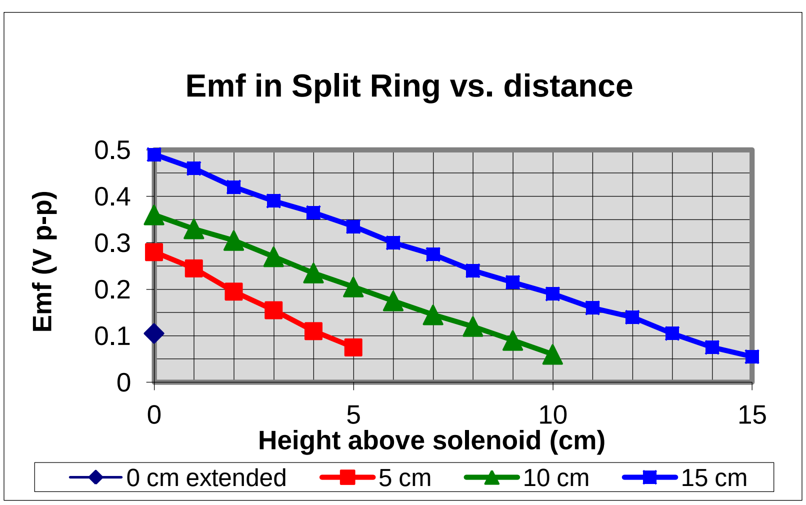

Fig. 3. Induced emf in a split ring vs. height above the solenoid for different lengths of core extended.

The magnetic field around the iron core has been studied both qualitatively and quantitatively. A map of the magnetic field pattern around the extended iron core was obtained using iron filings and is shown in Fig. 4. This picture was made by turning the JR apparatus on

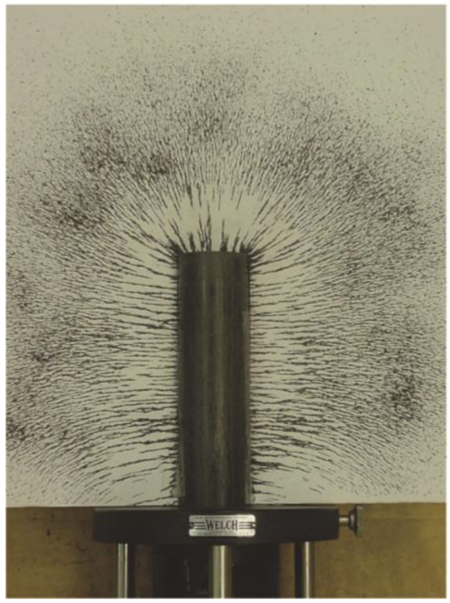

Fig. 4. Iron-filing map of the magnetic field around the extended core of the JR apparatus. The apparatus was laid on its side and alternating current used for this picture. Pictures made with dc were identical.

its side, taping a piece of paper to the core, and sprinkling iron filings around the area. Power was applied briefly with the pattern appearing as shown. Both dc and ac power gave exactly the same results. One of the features of this picture is that the field lines are perpendicular to the extended core over nearly its entire length. A similar feature was reported by Sumner and Thakkrar.[11]

Quantitative measurements of the horizontal and vertical components of the field were made using an F.W. Bell Gauss/Tesla meter, model 4048, which uses Hall probes. The measurements were made at a distance of 5 mm (the thickness of a meter stick) out from the core, which is about where the ring moves. The total field and angle off radial are shown in Fig. 5 for a core extension of 15 cm. there is some slight variation in the field strength along the core, but it falls off rapidly beyond the end of the core. The total magnetic field is radial to within a few degrees until near the end of the core, where the direction becomes more vertical. For example, the angle the field makes to the core is within 5^o from the bottom to about 3 cm from the end of the core. At 2 cm beyond the core, the off-radial angle of the field is about 50^o and the radial component of the field drops to about 30% of its value along the core. The description of the magnetic field given here is different from that of other authors, who assumed that the radial field resulted from "flaring out,"[9,10,12] "inhomogeneity,"[3] or "divergence"[13] of the total field.

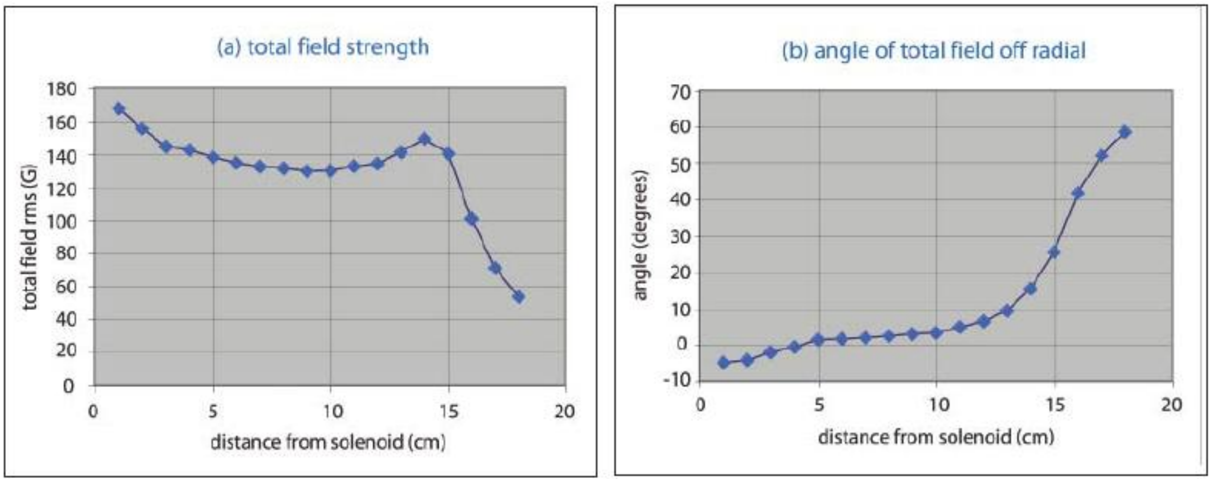

Fig. 5. Total field strength and angle off radial along the 15-cm extended iron core. Measurements of the horizontal and vertical components were made with a Gauss/Tesla meter and Hall probes positioned 5 mm from the iron core. The total field (a) and angle (b) were calculated from the components.

Since the magnetic force is the product of the current in the ring and the radial field, and because the field is almost perfectly radial along the core and varies only slightly, the magnetic force variation is dominated by the change in the emf. This means that as the ring moves up along the core, the magnetic force decreases due to the decreasing emf, approaching zero at the end of the core. This was the justification for dividing the motion into two parts in the kinematics experiment described in the previous section.

With more core extended, more current is drawn by the solenoid since it operates at a fixed ac voltage but has reduced overall impedance of solenoid plus core. The larger current produces a greater magnetic field in the core and hence a greater induced emf in the ring at the bottom of the core, as seen in Fig. 3. A minimum current, 2.8 A_{rms}, occurred with the core symmetrically positioned (about 5-6 cm out each end of the solenoid). The current increased quadratically with extended core, reaching 7.8 A_{rms} with 18 cm extended.

The Qualitative Role of the Phase Shift

We will now look at exactly how a phase shift in the induced current relative to the induced emf in the ring creates a net upward force. This is shown in Fig. 6. Fig. 6(a) shows the case of no phase shift; Fig. 6(b) shows the case of nonzero phase shift. In both cases, it is the *rate of change* of the flux through the ring that produces the emf (Faraday's law). The emf produces a current in the ring, which may be shifted in phase from the emf depending on the ratio of inductance and resistance of the ring.

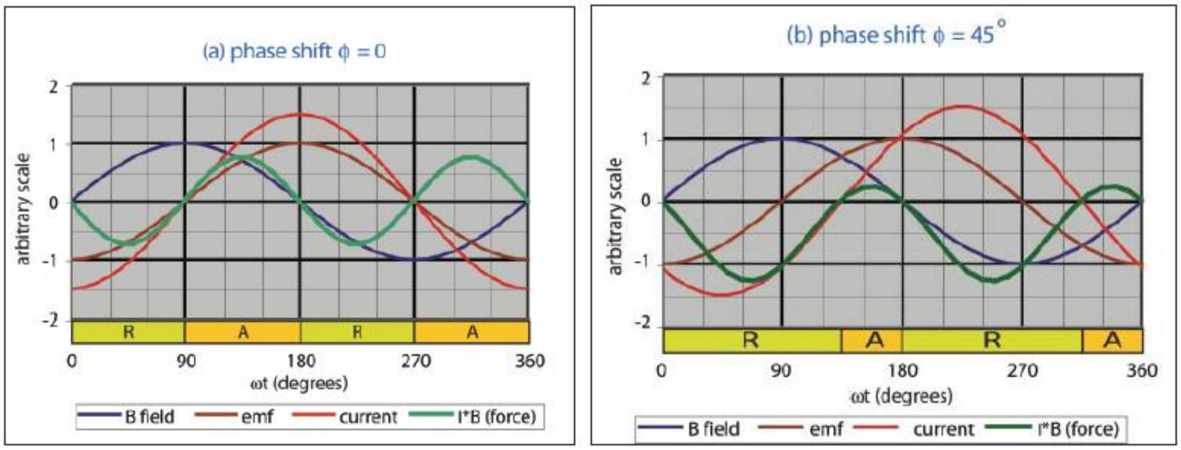

Fig. 6. The radial magnetic field, the induced emf, the current in the ring, and the force on the ring for (a) zero phase shift and (b) 45^{o} phase shift of the ring current relative to the induced emf. The symbols R and A refer to times of repulsion and attraction, respectively. Over each half-cycle the force averages to zero in (a) but to a net repulsion in (b).

The force on the ring is the result of the ring current interacting with the radial magnetic field. The iron-filing map of the magnetic field around the core, shown in Fig. 4, reveals that the field where the ring moves is radially outward over most of the length of the core. The direction of the force, given by the right-hand rule, may be upward or downward depending on the direction of the current in the ring relative to the radial field at any given instant.

In Fig. 6(a), where the current and emf are in phase, the attractive force and the repulsive force average to zero in every half-cycle. In Fig. 6(b), when the current *lags* the emf, the repulsive force is greater than the attractive force in magnitude, and lasts for a larger fraction of each half-cycle. This results in a net upward force, which may cause the ring to jump or levitate. The net force depends on the amount of the lag, or the phase shift. If the phase shift is zero the net force is zero.

The phase shift arises from the total impedance of the ring and includes both resistive and reactive parts. For an inductive circuit,

$$tan\varphi = \frac{X_L}{R} = \frac{\omega L}{R}$$

where L and R are the inductance and resistance of the ring, and ω is the angular frequency.[14]

The Mathematics of the Jump

We will now investigate how the force depends on the phase shift. Taking the magnetic field in the iron core as the reference of time, we can write

$$\mathbf{B}_{\text{core}}(t) = \mathbf{B}_{\text{core,max}} \sin \omega t \ . \qquad (4)$$

The induced emf in the ring is given by:

$$E(t) = -\frac{AdB_{core}}{dt} = -A\omega B_{core,\max} \cos \omega t = -E_{\max} \cos \omega t \tag{5}$$

A is the cross-sectional area of the core and $E_{\max}$ is the maximum emf in the ring at any given vertical position, as shown in Fig. 3.

The force on the ring at any position along the core depends on the current and the radial magnetic field, $\boldsymbol{B}_r$, and may be written as:

$$\mathbf{F}(t) = I_{ring}(t)\boldsymbol{\ell} \times \mathbf{B}_r(t) \tag{6}$$

where $\boldsymbol{\ell}$ is a vector along the current path in the ring. As seen in Fig. 4, the field, $\mathbf{B}_r$, felt by the ring current, is essentially radially outward, perpendicular to the ring, over most of its travel.

Since $\mathbf{B}_r$ and $\mathbf{I}_{ring}$ are perpendicular, we need be concerned only with the magnitudes, B_r and I. For the case of zero phase shift, $I_{ring} = I_{max} cos\omega t$. This gives

$$F(t) = F_{\max} \cos \omega t \sin \omega t \qquad \text{[no phase shift]} \tag{7}$$

$$= \frac{F_{\max}}{2} (\sin 2\omega t) \ , \tag{8}$$

where $F_{\max}$ is the maximum force at any position of the ring and incorporates $I_{max} = E_{max}/R$, the circumference of the ring and the magnitude of B. Thus the force alternates between being repulsive and attractive (at any position) with twice the line frequency, as seen in Fig. 6(a). Averaged over one period, T, we have

$$F_{ave} = \frac{F_{\max}}{2T} \int_0^T \sin(2\omega t)dt = 0 \qquad \text{[no phase shift]} \tag{9}$$

If there is an inductive *lag* of the current relative to the emf, i.e., a nonzero phase shift as seen in Fig. 6(b), the current may be written as

$$I_{ring}(t) = I_{\max} \cos(\omega t - \varphi) \qquad \text{[phase shift } \varphi\text{]} \tag{10}$$

where I_{max} now is

$$I_{max} = \frac{E_{max}}{Z} = \frac{A\omega B_{core,max}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{sin\varphi}{L} AB_{core,max}$$

The average force then becomes:

$$F_{ave} = (\frac{\sin\varphi}{L})\frac{F_{\max}}{T}\int_0^T \cos(\omega t-\varphi)\sin\omega t dt = \frac{F_{\max}}{2L}\sin^2\varphi \quad (11)$$

Thus, for a finite phase shift, the net force depends on the $\sin^2$ of φ,[10,12,15] and is zero if φ is zero. Of course, F_{max} is a function of the height above the solenoid because of the variation of E_{max}, seen in Fig. 3.

Measuring the Phase Shift

Measuring the phase shift of the ring current relative to the induced emf requires simultaneously observing the magnetic field passing through the ring and the current in the ring. A dual-trace oscilloscope was used for this purpose, with the magnetic field detected by a split ring (or single-turn loop of #10 wire) located at the bottom of the core and the ring current detected by a small pickup coil (0.10 mH) mounted directly on the core. The pickup coil was placed far enough away so that the large ring current (as much as several hundred amps[16]) did not interfere with the split ring. The basic setup is shown in Fig. 2. The solid ring was pushed over the pickup coil and held briefly, while observing shifts in phase of the pickup coil signal relative to that of the split-ring. These measurements must be made quickly -- within a few seconds -- due to heating of the ring.

The required phase shift, φ , is related to the phase difference between E1, the split-ring emf, and E2, the pickup coil emf, with an extra 90^o, as will now be shown. Taking E1 as the reference for time on the oscilloscope:

$$E_1 = E_{1,max}\sin(\omega t). \quad (12)$$

The ring current is shifted in phase from E1 by φ and may be written as:

$$I = I_{max}\sin(\omega t - \varphi). \quad (13)$$

The magnetic field produced by the ring current is in phase with that current, and can be written as $B = B_{max}\sin(\omega t - \varphi)$. The emf, E2, of the pickup coil is the negative derivative of the magnetic flux through the pickup coil, and may be written as:

$$E_2 = -E_{2,max}\cos(\omega t - \varphi)$$

$$= -E_{2,max}\sin(90^0 - \omega t + \varphi)$$

This finally can be written as:

$$E_2 = E_{2,max} \sin[\omega t - (90^o + \varphi)]. \quad (14)$$

The phase difference between E1 and E2 is seen from comparing Eqs. (12) and (14) to be $(90^o + \varphi)$. This is found on the oscilloscope from the zero-crossing point of E2 relative to the starting point of E1, as shown in Fig. 7.

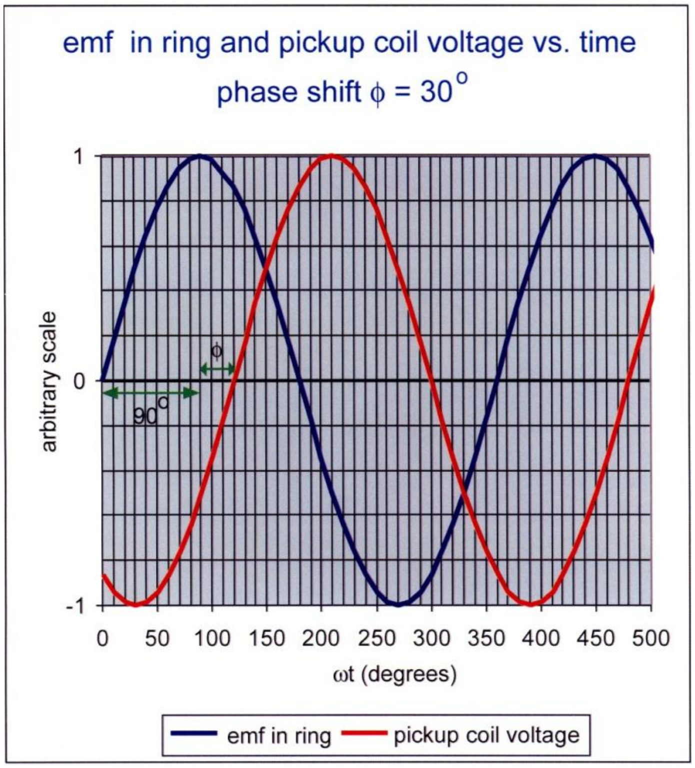

Fig. 7. Ring emf, E1, and pickup coil emf, E2, vs. time for a phase shift of 30°, as it would appear on the oscilloscope screen. The difference of zero-crossing points is $(90^o + \varphi)$.

Measurements of the phase shift for several aluminum and brass rings are given in Table II.

Table II

Measured values of phase shift, φ, for several rings. The uncertainty is about $\pm 3^o$.

(a) Aluminum rings, 3 mm thick and 6.0 cm mean diameter.

Ring height (mm)	Phase shift, φ (degrees)
5	25
10	35
17	45
20	50
30	65

(b) Brass rings, 1 mm thick and 6.0 cm mean diameter.

Ring height (mm)	Phase shift, φ (degrees)
12.5	20
15.5	22
47	28

Since the phase shift is given by

$$tan\varphi = \frac{\omega L}{R}$$

and R varies inversely with the height of the ring, we expect that $tan\varphi$ should vary in direct proportion to the height, provided L is independent of height. Figure 8 shows $tan\varphi$ versus ring height for aluminum and confirms this linear dependence. For brass there are too few points to draw any conclusion.

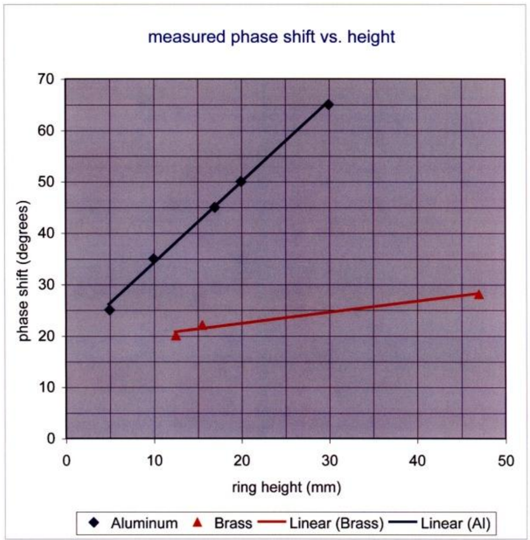

Fig. 8. Measured phase shift vs. ring height for aluminum and brass rings. The straight lines are best fits to the data points. Uncertainty in each value is about $\pm 3^o$.

Churchill and Noble estimated a phase shift of "nearly 60^{o}" for the phase shift of a "typical" aluminum ring.[3] No details were provided either on the dimensions of the ring or how the estimation was made. Tjossem and Cornejo reported phase shifts for a range of frequencies with 60^o at 60 Hz.[10] Both of these estimates are consistent with our results.

Comments and Conclusions

We have shown that the first-quadrant explanation fails to account for several key features of the jumping ring experiment. A correct explanation must account for the entire cycle, recognizing that the jump requires multiple cycles and that there is a phase shift of the ring current relative to the induced emf. A method has been presented by which the phase shift can be measured. For rings of various heights, a phase shift of 25 to 60^{o} has been measured for aluminum rings and 20 to 30^{o} for brass rings.

The authors believe that an important outcome of this work is a clearer picture of the magnetic field around the extended core of the jumping ring apparatus. The magnetic field where the ring move is perpendicular to the core over most of the ring's travel while it is on the core. These radial magnetic field lines emerging from the sides of the core result in a decreasing magnetic flux through the center of the core, which in turn leads to the observed decrease in the emf in the ring along the core. This results in a decreasing force on the ring as it moves along the length of the core.

The phase-shift explanation of the jumping ring has been available in the literature for several decades and actually goes back to Elihu Thomson in 1887. It is our hope that this work will lead to a better understanding of this popular demonstration.

Acknowledgments

We wish to thank the referee for helpful suggestions, including plotting $tan\varphi$versus ring height in Fig. 8 and for pointing out that Thomson mentioned the phase shift explanation in his original work.

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PACS codes: 01.50.My, 01.50.Pa, 41.00.00

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