The Impact of Population Ageing on Technological Progress and TFP Growth, with Application to United States: 1950-2050 (Working Paper)

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Yusuf Izmirlioglu

University of Pittsburgh, Department of Economics

Abstract

I examine the effect of age-distribution of the society on economic growth through technological progress. I build a multisector economy model that involves population pyramid. I characterize the steady-state of the model for low and high population growth rate. Higher population growth rate yields faster TFP and output growth in the long-run. I analyze dynamic behavior of the economy. I calibrate the model for United States, 1950-2000 and using the estimated parameters I make predictions about the impact of population ageing on economic growth.

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Key words: Population Ageing, Demographic Transition, TFP Growth, Technological Progress, Economic Growth Forecast, United States

1 Introduction

Today's developed countries have been experiencing an important process: Population Ageing. According to United Nations 2006 World Population Prospects, world population will continue to age until 2050. Recently, economists have been investigating the effect of population ageing on global economy and living standards in the future. In this respect studies are trying to find the impact on international capital flows, migration, social security expenditures and fiscal policy response. All of these studies, however, have assumed a critical element of economic growth as exogenous: Technological Progress

Since significant amount of growth performance both across countries and within the country over time is attributable to TFP growth, this assumption is not very realistic. Moreover, Kremer(1993) shows the link between rate of technological progress and population growth in the long-run. But his model consists of agricultural sector only and does not include the age-structure of the society, thus it is not suitable for examining TFP growth in the short-run or post industrial revolution multisector economy. Therefore in this paper, I examine the impact of population ageing on economic growth through the channel of technological progress and knowledge production. For this purpose I establish a model of economic growth and technological progress that incorporate demographic dynamics. In particular I extend the R&D based model of economic growth, presented in Jones(1995).

The model in Jones (1995) consists of the research firms, intermediate good producing sector and the final good producing sector. Labor allocation across sectors is determined by wage equality condition. Under this setup, the economy exhibits constant TFP growth at the steady-state. To implement the idea of demographic structure, I realize a society which is composed of 1-year age cohorts and I allow productivity of individuals depend on both their age and occupation. So human capital in any sector now becomes a function of age-distribution of employees in that sector. Individuals of each cohort decide to work in either research sector or final-good producing sector considering their lifetime wage earnings.

To analyze the model, I characterize the steady-state values, growth rates and the impulse response for low population growth rate and high population growth rate. I fit the model to the United states data between 1950 and 2000. Having estimated parameters, I perform forecasts for TFP and income growth of United States until year 2050.

The remainder of this paper is organized as follows: Section 2 reviews the literature. Section 3 presents the model. Section 4 explains calibration of the model. Section 5 demonstrates steady-state and impulse response dynamics. Section 6 shows forecast results for United States. Section 7 concludes.

2 Related Literature

For long time, economists have tried to explain the underlying reasons for fertility decline. The leading papers of this field are Leibenstein (1963), Becker and Lewis (1973) and Caldwell (1976). Later studies deal with the economic consequences of the demographic transition and attempt to predict its effect on future economic growth. In particular D'Albis (2007), Fougere and Merette(1999), Tosun(2003) and Storesletten (2000) examine the age structure of capital holders and workers, intergenerational equity, international capital flows and migration respectively. Fehr, Jokisch and Kotlikoff (2004) develop a three-region dynamic general equilibrium model life-cycle model to forecast the effect of general and skilled immigration during demographic transition. The three regions in the paper are US, Japan and the European Union. Fehr, Jokisch and Kotlikoff (2005) extend this model by adding China as the fourth region.

All of these papers take TFP growth rate as exogenous. As far as modelling the relationship between population and knowledge production is concerned, Kremer(1993) shows the positive link between rate of technological progress and population growth in the long-run. However his model consists of agricultural sector only and does not include the age-structure of the society. Moreover Kremer's model is not suitable for examining short-term TFP growth or TFP growth in the multisector economy. Jones (1995) and Jones (2004) provide a model of endogenous technological progress that is consistent with the empirical data. He shows that a multisector decentralized economy model can explain the constant TFP growth rate of United States despite increasing share of science, technology and engineering labor force since 1950. However Jones' two papers do not consider the change in the demographic structure either. To best of my knowledge, this is the first study that builds a framework for investigating the connection between the age-distribution of the society and technological progress. This is also the first paper to forecast the impact of population ageing on economic growth of United States.

3 The Model

3.1 The Modeling Environment

In this section, I present R&D based demographic economic growth model which is built on Jones (1995). The model assumes infinitely lived representative households maximizing their lifetime utility. The population is composed of cohorts of one-year age groups. People face age-dependent risk of mortality and can live at most until age T. Productivity of individuals changes as they get older and the way productivity changes also depends on the sector. Hansen (1993) computes relative efficiency units for various age-gender subgroups in United States by comparing average hourly earnings of each subgroup to average hourly earning over all subgroups. The data in Hansen (1993) has been used in the literature and I use this data (by averaging across genders) for age-dependent relative productivity in final-goods producing sector, $w_Y(a)$. I use average number of journal articles (co)authored by faculty members and researchers with respect to their age, reported by National Science Founda-

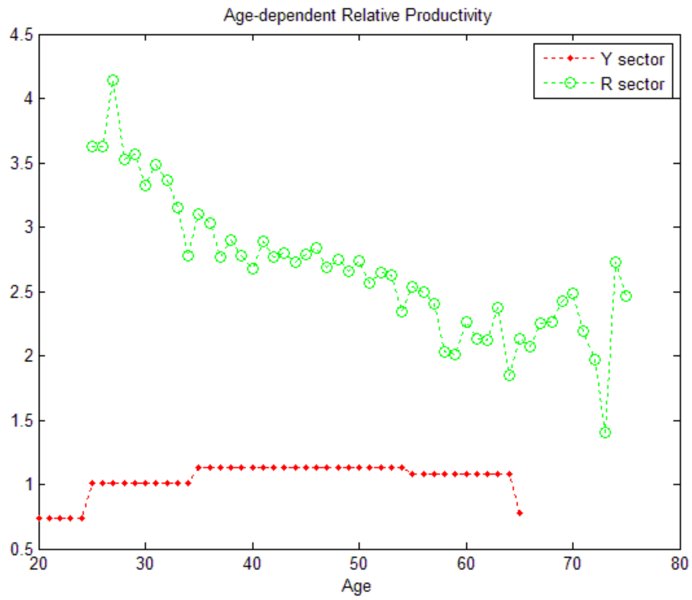

Fig. 1. Relative individual productivity in the research and economy sector with respect to age

tion 2003, as an indicator of age-dependent productivity in the research sector $w_R(a)$. Figure 1 shows relative productivity of individuals with respect to their age and occupation, as given by these data.

The organization of the economy is decentralized as the interaction between firms and sectors are through market forces. Individuals decide which sector to work considering expected lifetime wage earnings. Note that although the government subsidies scientific activities and venture financing, the real economy is still decentralized because it is research institutions, universities and private firms that carry on research and development.

In order to identify the impact of individual variables, first I isolate the social security and education expenditures at the beginning so that I can observe the effect of demographic dynamics on the allocation of labor force. Additionally, in the first stage I assume a closed economy model where the country produces technology and goods on its own; but the model can easily be extended to open economy by incorporating immigrants and international capital flows.

3.2 Sectors in the Economy

There are three sectors in the economy: A competitive final-goods production sector, a monopolistic intermediate-good production sector and the monopolistically competitive R&D sector. The final-goods sector uses labor L_{Yt} and a collection of intermediate inputs $x_{i,t}$ to produce the consumption good.

$$Y_t = {H_{Yt}}^{1-\alpha} \int_0^{A_t} x_{i,t}^{\alpha} di, \qquad H_{Yt} = \sum_{a=w_{y(\min)}}^{w_{y(\max)}} w_Y(a) L_{Yt}(a) \tag{1}$$

where $L_{Yt}(a)$ is the number of workers at age a and H_{Yt} is the human capital in final-good production sector. In the competitive equilibrium labor and intermediate inputs are paid their marginal productivity, i.e.,

$$wage_{Y,t}(a) = (1-\alpha)\frac{Y_t}{H_{Yt}} w_Y(a), \qquad a = a_{y(\min)}...a_{y(\max)} \tag{2}$$

$$p_{i,t} = \alpha \left(\frac{H_{Yt}}{x_{i,t}}\right)^{(1-\alpha)} \qquad \text{for } \forall i$$

The intermediate good producing sector is composed of an infinite number of firms on the interval $[0, A_t]$ that have purchased a design from the R&D sector and are monopolists in the production of their particular variety of intermediate good. The only factor of production is capital which is rented at a rate of r_t each period and remains without change or depreciation. A firm that has purchased a design can transform one unit of capital into one unit of intermediate input. Every intermediate firm then solves the following profit maximization problem each period:

$$\max p(x_{it}) x_{i,t} - r_t x_{i,t}$$

Every intermediate firm acting as a monopolist sets the same price, sells the same quantity of intermediate good and gets the same profit:

$$p_{it} = \overline{p_t} = \frac{r_t}{\alpha}, \ \forall i \tag{3}$$

$$x_{it} = \overline{x_t} = H_{Yt}\left(\frac{\alpha}{\overline{p_t}}\right)^{1/(1-\alpha)} = H_{Yt}\left(\frac{\alpha^2}{r_t}\right)^{1/(1-\alpha)}, \ \forall i \tag{4}$$

$$\pi_{it} = \overline{\pi_t} = (1-\alpha)\overline{p}_t \overline{x}_t = \alpha(1-\alpha)\frac{Y_t}{A_t}, \ \forall i \tag{5}$$

The last equation uses the fact that in equilibrium the output is given by $Y_t = A_t {H_{Yt}}^{1-\alpha} \overline{x_t}^{\alpha} = A_t H_{Yt} \left(\frac{\alpha^2}{r_t}\right)^{\alpha/(1-\alpha)}$. The resource constraint in the economy requires the capital stock be equal to the total stock of producer durables. This yields the following capital and interest rate values

$$K_t = A_t \overline{x_t} = A_t H_{Yt} \left(\frac{\alpha^2}{r_t} \right)^{1/(1-\alpha)} \tag{6}$$

$$r_t = \alpha \overline{p_t} = \alpha^2 \frac{Y_t}{K_t} \tag{7}$$

Observe that capital is underpaid relative to the competitive case in order to compensate the R&D expenditures. The research sector creates new designs and innovations with the human capital $H_{At} = \sum_{a=a_{r(\min)}}^{a_{r(\max)}} w_R(a) L_{At}(a)$ engaged in R&D and current stock of technology A_t. Observe that the level of technology is defined as the variety of intermediate goods A_t. The amount of new designs or equivalently the progress in the technology is given by $\Delta A_t = (\nu h_{At}^{\lambda-1} A_t^{\phi}) H_{At}$ where $h_{At} = H_{At}$ in equilibrium. L_{At} denotes the number of researchers and $(\nu h_{At}^{\lambda-1} A_t^{\phi})$ term defines the rate at which R&D labor force generates ideas. Here h_{At} captures the negative externalities caused by the inefficiency, failure or duplications in the research process. Then the knowledge stock evolves according to

$$A_{t+1} = A_t + \nu H_{At}^{\lambda} A_t^{\phi}, \qquad \lambda, \phi \in (0,1) \tag{8}$$

Any individual can enter the research sector to search for new designs so R&D labor also receives its marginal productivity:

$$wage_{R,t}(a) = \nu P_{A,t} H_{At}^{\lambda-1} A_t^{\phi} w_R(a), \qquad a = a_{r(\min)} ... a_{r(\max)} \tag{9}$$

where $P_{A,t}$ is the price of the patent that the research firm sells to the intermediate firm in each period for production of particular durable.

3.3 Households

The representative household makes the consumption decision in order to maximize life-time utility subject to the budget constraints. Namely,

$$\max_{C_t} \sum_{t=0}^{\infty} \beta^t (1+n_t)^t (\frac{c_t^{1-\psi}}{1-\psi}) \quad \textit{subject to}$$

$$K_{t+1} = (1-\delta) K_t + I_t \tag{10}$$

$$C_t+I_t = r_tK_t+\sum_{a=a_{r(\min)}}^{a_{r(\max)}} wage_{R,t}(a)L_{At}(a)+\sum_{a=a_{y(\min)}}^{a_{y(\max)}} wage_{Y,t}(a)L_{Yt}(a)+[A_t\overline{\pi_t} - P_{A,t}(A_t - A_{t-1})] \tag{11}$$

where $c_t = C_t/L_t$ per capita consumption, n_t population growth rate between period $(t+1)$ and t, and β subjective discount rate. Households' problem requires the flow of C_t and $P_{A,t}$ to be

$$\left(\frac{C_{t+1}}{C_t}\right)^{\psi} = \beta(1+n_t)^{\psi}\left[r_{t+1} + (1-\delta)\right] \tag{12}$$

$$P_{A,t+1} = \left[r_{t+1} + (1-\delta)\right] P_{A,t} - \overline{\pi}_{t+1} \tag{13}$$

In this decentralized economy labor is engaged only in the R&D and the final-good producing sector. At this point I assume that individuals decide which sector to work at certain age $\overline{a}_{\text{det}}$ in their lifetime, and then undergo a training or education process for each sector with no cost. They become productive after finishing their sector-specific education. Once the individual gives the sector decision , (s)he cannot change the sector later. [1] The individual considers expected discounted wage income throughout his/her lifetime while giving sector decision. In equilibrium, since labor is immobile across sectors, expected discounted wage earnings in both sectors must be equal to each other; otherwise there would be an arbitrage opportunity. Formally,

$$\sum_{a=a_{r(\min)}}^{a_{r(\max)}} \tau_{(t,t+a-\overline{a}_{\text{det}})}.wage_{R,t+a-\overline{a}_{\text{det}}}(a) = \sum_{a=a_{y(\min)}}^{a_{y(\max)}} \tau_{(t,t+a-\overline{a}_{\text{det}})}.wage_{Y,t+a-\overline{a}_{\text{det}}}(a) \quad \text{for } \forall t \tag{14}$$

$$\begin{aligned}\Rightarrow &\sum_{a=a_{r(\min)}}^{a_{r(\max)}} \tau_{(t,t+a-\overline{a}_{\text{det}})}.\nu.P_{A,t+a-\overline{a}_{\text{det}}} H^{\lambda-1}_{A,t+a-\overline{a}_{\text{det}}} A^{\phi}_{t+a-\overline{a}_{\text{det}}} w_R(a) = \\ &\sum_{a=a_{y(\min)}}^{a_{y(\max)}} \tau_{(t,t+a-\overline{a}_{\text{det}})}.(1-\alpha)\frac{Y_{t+a-\overline{a}_{\text{det}}}}{H_{Y,t+a-\overline{a}_{\text{det}}}} w_Y(a) \quad \text{for } \forall t\end{aligned} \tag{15}$$

[1] I assumed free entry condition when deriving wage equations in final-good production sector and the research sectors. Although labor is immobile across sectors, individuals are still free to choose the sector at the beginning of their career so that wage equations in the text apply.

$$\tau_{(t,t+a-\overline{a}_{\text{det}})} = \prod_{s=\overline{a}_{\text{det}}}^{a-1} \left(\frac{1-q_s}{1+r_{t+s-\overline{a}_{\text{det}}}} \right) \tag{16}$$

Thus at any time t, the fraction of the cohort at age $\overline{a}_{\text{det}}$ that chooses each sector is determined by lifetime earnings equality condition. $\tau_{(t,t+a-\overline{a}_{\text{det}})}$ is the individual discount rate between time period t and $t+a-\overline{a}_{\text{det}}$ i.e, between period at which s/he is at age $\overline{a}_{\text{det}}$ and period s/he is at age a. The discount rate between consecutive periods t and $t+1$, (equivalently between age a and $a+1$) is $\tau_{(t,t+1)} = \frac{1-q_a}{1+r_t}$ which is a function of interest rate r_t and probability of dying q_a at age a. Aggregating one-period discount rates until time $t+a-\overline{a}_{\text{det}}$ yields:

$$\tau_{(t,t+a-\overline{a}_{\text{det}})} = \prod_{s=\overline{a}_{\text{det}}}^{a-1} \left(\frac{1-q_s}{1+r_{t+s-\overline{a}_{\text{det}}}} \right) \tag{17}$$

Finally, labor engaged in two sector for any age cohort should sum up to total employment in that cohort:

$$L_{Yt}(a) + L_{At}(a) = Work_t(a), \qquad a = \overline{a_{\text{det}}}, ..., \max\{a_{r(\max)}, a_{y(\max)}\} \tag{18}$$

4 Steady-State of the Model

The steady-state analysis allows us to learn the behavior of the system in the long-run. In this section first I find the steady-state of the population pyramid and then solve for steady-state value and growth rate of variables in the economy.

4.1 Population Pyramid

Population pyramid shows the age-distribution in the society. Let $G_t(a)$ denote the percentage of age-group a in the population. If we assume that net population growth rate or total fertility rate [2] are constant and in addition age-dependent mortality rate is constant over time, then the population pyramid of the society defined by $G_t(a)$ eventually becomes stable. Let $L_t(a)$ denote

[2] Total fertility rate (TFR) is the total number of children that a woman would have on the average during her lifetime if she experiences age-dependent fertility rate in the society in that period.

the actual number of people at age a at time t, L_t total population at time t, n_{t+1} net population growth rate between time t and t+1, q_a the probability of death at the age of a and T the maximum survival age ($q_T = 1$) The evolution of demographic variables then can be expressed as,

$$L_{t+1} = (1 + n_t)L_t \tag{19}$$

$$L_{t+1}(0) = n_t L_t + \sum_{a=0}^{T} q_a L_t(a) \tag{20}$$

$$L_{t+1}(a+1) = (1 - q_a)L_t(a), \qquad a = 1, ..., T \tag{21}$$

$$G_t(a) = \frac{L_t(a)}{L_t} \tag{22}$$

The second equation gives the number of new born since increase in the population size is equal to new born less of total deaths. In the steady-state $G_t(a) = \widetilde{G}(a)$, $n_t = n$, and $L_t = (1+n)^t \widetilde{L}_0$ That is population grows at the rate of n and let $\widetilde{L}_t = L_t/(1+n)^t = \widetilde{L}_0$ denote the normalized value of population size. Steady-state age-distribution $\widetilde{G}(a)$ is then

$$\widetilde{G}(0) = \left[1 + \sum_{a=0}^{T} \frac{\left(\prod_{i=0}^{a-1} (1 - q_i) \right)}{(1+n)^a} \right]^{-1} \tag{23}$$

$$\widetilde{G}(a) = \left(\prod_{i=0}^{a-1} (1 - q_i) \right) \frac{\widetilde{G}(0)}{(1+n)^a}, \qquad a = 1, ..., T \tag{24}$$

$$\widetilde{L_t}(a) = \widetilde{G}(a)\widetilde{L}_t = \widetilde{G}(a)\widetilde{L}_0 \tag{25}$$

4.2 The Economy

In the steady-state balanced growth, variables grow at constant rate in time. If g_X is the growth rate of variable X_t, then $X_t = (1+g_X)^t.\widetilde{X_t}$.Here $\widetilde{X_t} = X_t/(1+g_X)^t$ is the normalized variable and stationary over time. To find the steady-state values of the model, we need to characterize $L_{Yt}(a)$, $L_{At}(a)$ and

$Work_t(a)$ for different age groups. I assume labor force participation rate, total unemployment rate U_t and sector-specific age-dependent unemployment rates $u_A(a), u_Y(a)$ to be stable over time. Because labor is immobile across sectors,

$$\frac{L_{A,t+1}(a+1)}{1-u_A(a+1)} = \frac{L_{A,t}(a)}{1-u_A(a)}, \qquad a \geq a_{r(\min)} \tag{26}$$

$$\frac{L_{Y,t+1}(a+1)}{1-u_Y(a+1)} = \frac{L_{Y,t}(a)}{1-u_Y(a)}, \qquad a \geq a_{y(\min)} \tag{27}$$

To simplify the solution, I further assume that unemployment rates are equal in research and final-good producing sector for all age groups at which both sector employees are working, i.e., $u_A(a) = u_Y(a) = u(a)$, for $\max\{a_{r(\min)}, a_{y(\min)}\} \leq a \leq \min\{a_{r(\max)}, a_{y(\max)}\}$. This assumption implies

$$\frac{L_{A,t+1}(a+1)}{L_{Y,t+1}(a+1)} = \frac{L_{A,t}(a)}{L_{Y,t}(a)} = \frac{L_{A,t-a+a_{\det}}(\overline{a}_{\det})}{L_{Y,t-a+a_{\det}}(\overline{a}_{\det})}, \qquad a \geq \max\{a_{r(\min)}, a_{y(\min)}\} \tag{28}$$

$$\frac{L_{A,t+1}(a+1)}{Work_{t+1}(a+1)} = \frac{L_{A,t}(a)}{Work_t(a)} = \frac{L_{A,t-a+a_{\det}}(\overline{a}_{\det})}{L_{t-a+a_{\det}}(\overline{a}_{\det})}, \qquad a \geq a_{r(\min)} \tag{29}$$

$$\frac{L_{Y,t+1}(a+1)}{Work_{t+1}(a+1)} = \frac{L_{Y,t}(a)}{Work_t(a)} = \frac{L_{Y,t-a+a_{\det}}(\overline{a}_{\det})}{L_{t-a+a_{\det}}(\overline{a}_{\det})}, \qquad a \geq a_{y(\min)} \tag{30}$$

The share of the R&D employment in total employment stays the same over time for the same cohort and equal to the share of individuals who chose R&D sector when they were at the age $\overline{a}_{\det}$. Similar result holds for final-good production sector.

At the steady-state, $L_t, L_{Yt}, L_{At}, H_{At}, H_{At}, Work_t$ grow at constant rate n. If we normalize $L_t(a)$, $L_{Yt}(a)$, $L_{At}(a)$ with their growth rates, we get stable variables $\widetilde{L}(a)$, $\widetilde{L}_Y(a)$, $\widetilde{L}_A(a)$. The ratios in equations (29) and (30) now become independent of time and thus,

$$\frac{\widetilde{L}_{A,t}(a)}{\widetilde{Work}_t(a)} = \frac{L_{A,t}(a)}{Work_t(a)} = \frac{L_{A,t-a+a_{\det}}(\overline{a}_{\det})}{L_{t-a+a_{\det}}(\overline{a}_{\det})} = m_A, \qquad a \geq a_{r(\min)} \tag{31}$$

$$\frac{\widetilde{L}_{Y,t}(a)}{\widetilde{Work}_t(a)} = \frac{L_{Y,t}(a)}{Work_t(a)} = \frac{L_{Y,t-a+a_{\det}}(\overline{a}_{\det})}{L_{t-a+a_{\det}}(\overline{a}_{\det})} = m_Y = 1 - m_A, \qquad a \geq a_{y(\min)} \tag{32}$$

$$\widetilde{L}_{A,t}(\overline{a}_{\text{det}}) + \widetilde{L}_{Y,t}(\overline{a}_{\text{det}}) = \widetilde{L}_t(\overline{a}_{\text{det}}) \tag{33}$$

This means that the share of employees in R&D sector inside the workers of that age is the same over all age groups and equal to m_A. Likewise the share of employment in final-good sector in any age group is $m_Y = 1 - m_A$. Given

population pyramid $\widetilde{Work}(a)$ and m_A, one can calculate steady-state R&D employees in particular age group a.

$$\frac{\widetilde{L}_A(a)}{\widetilde{Work}(a)} = \frac{\widetilde{L}_A(a_{r(\min)})}{\widetilde{L}(a_{r(\min)})} = m_A, \qquad a_{r(\min)} \leq a \leq a_{r(\max)} \tag{34}$$

$$\frac{\widetilde{L}_Y(a)}{\widetilde{Work}(a)} = \frac{\widetilde{L}_Y(a_{y(\min)})}{\widetilde{L}(a_{y(\min)})} = m_Y, \qquad a_{y(\min)} \leq a \leq a_{y(\max)} \tag{35}$$

$$\widetilde{L}_A(a) + \widetilde{L}_Y(a) = \widetilde{Work}(a) \tag{36}$$

At steady-state balanced growth, interest rate is constant over time, $r_t = \overline{r}$. To find growth rate of variables consider equations (6), (7), (10) and (11),

$$g_Y = g_C = g_I = g_K = g \tag{37}$$

$$(1 + g_Y) = (1 + g_A)(1 + n) \tag{38}$$

$$(1 + g_A) = (1 + n)^{\lambda}(1 + g_A)^{\phi} \Rightarrow g_A = (1 + n)^{\frac{\lambda}{(1-\phi)}} \tag{39}$$

$$g_{P_A} = g_{\overline{\pi}} = n \tag{40}$$

$$g_{wage_R} = g_{wage_Y} = g_A \tag{41}$$

Scaling variables in model equations and using normalized variables, I obtain the model in terms of stationary variables as

$$\widetilde{Y}_t = \widetilde{A}_t \widetilde{H}_{Yt} \left(\frac{\alpha^2}{r_t}\right)^{\alpha/(1-\alpha)} \tag{42}$$

$$\widetilde{K}_t = \widetilde{A}_t \widetilde{H}_{Yt} \left(\frac{\alpha^2}{r_t} \right)^{1/(1-\alpha)} \tag{43}$$

$$(1+g_A)\widetilde{A}_{t+1} = \widetilde{A}_t + \nu \widetilde{H}_{At}^{\lambda} \widetilde{A}_t^{\phi}, \qquad \lambda, \phi \in (0,1) \tag{44}$$

$$\widetilde{H}_{Yt} = \sum_{a=a_{y(\min)}}^{a_{y(\max)}} w_Y(a) \widetilde{L}_{Yt}(a) \tag{45}$$

$$\widetilde{H}_{At} = \sum_{a=a_{r(\min)}}^{a_{r(\max)}} w_R(a) \widetilde{L}_{At}(a) \tag{46}$$

$$\sum_{a=a_{r(\min)}}^{a_{r(\max)}} \tau_{(t,t+a-\overline{a}_{\mathrm{det}})} . \nu . \widetilde{P}_{A,t+a-\overline{a}_{\mathrm{det}}} \widetilde{H}_{A,t+a-\overline{a}_{\mathrm{det}}}^{\lambda-1} \widetilde{A}_{t+a-\overline{a}_{\mathrm{det}}}^{\phi} w_R(a) =$$

$$\sum_{a=a_{y(\min)}}^{a_{y(\max)}} \tau_{(t,t+a-\overline{a}_{\mathrm{det}})} . (1-\alpha) \frac{\widetilde{Y}_{t+a-\overline{a}_{\mathrm{det}}}}{\widetilde{H}_{Y,t+a-\overline{a}_{\mathrm{det}}}} w_Y(a) \tag{47}$$

$$\widetilde{L}_{Yt}(a) + \widetilde{L}_{At}(a) = \widetilde{W}ork_t(a), \qquad a = \overline{a}_{\mathrm{det}}, ..., \max\{a_{r(\max)}, a_{y(\max)}\} \tag{48}$$

$$(1+g)\widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \widetilde{I}_t \tag{49}$$

$$\widetilde{C}_t + \widetilde{I}_t = r_t \widetilde{K}_t + \sum_{a=a_{r(\min)}}^{a_{r(\max)}} \widetilde{w}age_{R,t}(a) \widetilde{L}_{At}(a) + \sum_{a=a_{y(\min)}}^{a_{y(\max)}} \widetilde{w}age_{Y,t}(a) \widetilde{L}_{Yt}(a) + \left[\widetilde{A}_t \widetilde{\overline{\pi}}_t - \widetilde{P}_{A,t}(\widetilde{A}_t - \frac{\widetilde{A}_{t-1}}{1+g_A}) \right] \tag{50}$$

$$\widetilde{\overline{\pi}}_t = \alpha(1-\alpha) \frac{\widetilde{Y}_t}{\widetilde{A}_t}, \ \forall i$$

$$r_t = \alpha^2 \frac{\widetilde{Y}_t}{\widetilde{K}_t} \tag{51}$$

$$(1+g_{P_A})\widetilde{P}_{A,t+1} = \left[r_{t+1} + (1-\delta) \right] \widetilde{P}_{A,t} - \widetilde{\overline{\pi}}_{t+1} \tag{52}$$

$$(1+g_C)^{\psi}\left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t}\right)^{\psi}=\beta(1+n)^{\psi}\left[r_{t+1}+(1-\delta)\right] \tag{53}$$

At the steady-state, $\widetilde{X}_{t+1}=\widetilde{X}_t=\widetilde{X}_{ss}=\widetilde{X}, \forall t$ so given model parameters and the data for labor force participation rate and age-dependent unemployment rate, I compute the decentralized steady-state values with the above formulae.

5 Calibration of The Model

As an empirical application, I attempt to fit the model to the United States data and compute model parameters. I choose United States since the model assumes the country producing its own technology. The equilibrium outcome of the model heavily depends on the choice of parameters, so calibrated parameters are helpful to understand the economy and perform forecast.

5.1 Calibration Method

Based on the availability of data, the model is calibrated using 1950-2000 data, and variables are predicted for the period 2001-2050. The period is one year and the first period is 1950. There are 8 parameters to calibrate: $\alpha,\lambda,\phi,\psi,\beta,\nu,\delta,\eta$ with constraints $\alpha,\lambda,\phi,\beta,\delta\in(0,1)$ and $\psi,\nu,\eta>0$. Data for $w_Y(a)$ and $w_Y(a)$ defines productivity relative to age groups but not in absolute terms. Thus the last parameter η multiplies $w_Y(a)$ data to obtain the absolute value. Since ν multiplies $w_R(a)$ in (8) an additional parameter is not required.

I calibrate the model so that its steady-state matches the statistics of the US data. First we need to find values of exogenous variables. For $\widetilde{L}$ and n, I regress United States total population 1950-2000 assuming population growth rate is constant. I use time-average of 1960 to 2000 age-specific labor force participation rate and age-specific unemployment rate to calculate $\widetilde{Work}$ variable.

Observable endogenous variables of the model are $Y_t, C_t, I_t, r_t, L_{Yt}, L_{At}$. I choose target values of calibration as g_Y, $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{I}$, $\overline{r}$. I perform log-linear regressions on data to find the growth rate and intercepts g_Y, $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{I}$. Observe that I chose growth rate of Y_t as the growth rate g of the model. Interest rate $\overline{r}$ is the median of annual bank loan rates in 1950-2000 since interest rate has experienced large fluctuations. $\widetilde{L}_A$ or $\widetilde{L}_A/\widetilde{Work}$ ratio is not chosen a target of calibration because the share of R&D employment in total employment has not stabilized in the period.

There are 8 parameters but 5 statistics to match, thus I need to set 3 of the variables. Following the convention in the literature, I use $\alpha = 0.33$, $\beta = 0.995$ and $\delta = 0.05$.Demographic parameters are maximum age $T = 90$ and sector decision age $\overline{a}_{\text{det}} = 18$. Scientists, engineers and technology labor force are productive between age $a_{r(\min)} = 25$ and $a_{r(\max)} = 85$. Workers in final-good production sector are productive between age $a_{y(\min)} = 20$ and $a_{y(\max)} = 65$.

5.2 Calibration Results

Table 1 shows the estimated parameter values, corresponding steady-state of the model and the target statistics in the data. Fitted variables are, then computed via their growth rate and steady-state value. Figure 2 plots actual time series data together with the fitted variables from 1950 to 2005. The figure also shows the age distribution of R&D employees in NSF Integrated Database 2003. In this respect the model prediction of R&D workforce age distribution is close to the actual data. Note that according to model the share of R&D employment in total employment is %45.32, while in reality it is never above %6 (although increasing). This is due to the fact that labor force allocation has not reached its steady-state and there is an inertia in labor force.

$\widehat{\alpha}$	$\widehat{\lambda}$	$\widehat{\phi}$	$\widehat{\psi}$	$\widehat{\beta}$	$\widehat{\nu}$	$\widehat{\delta}$	$\widehat{\eta}$	n	$\widehat{L}_0$
0.33	0.648	0.74	0.58	0.995	3.981x10^{-4}	0.05	1.0x10^{-8}	0.1146	1.630x10^{8}

Table 1: Optimum parameter values for the model

6 Assessing Steady-States

Steady-state characteristics of the model depends on the parameters, so it's beneficial to see how steady-state values and growth rate of variables change as different set of parameters are used. In this section I make a comparison of steady-states to see the influence of population growth rate. I have already shown steady-state of the economy with the calibrated parameters and calibrated population growth rate $\widehat{n} = \%1.11$. Now I use different population growth rates for n and keep remaining parameters the same.

In the first case, I use the same set of coefficients as in the first one, except I choose a lower steady-state population growth rate: $n = \%0.75$. In the second case, I choose a higher steady-state population growth rate: $n = \%1.25$. Thereby, one can observe the change in steady-state values and growth rates in the models. The steady-states of the model is tabulated for different population growth rates in Table 2.

	$\widetilde{Y}$	$\widetilde{C}$	$\widetilde{I}$	$\widetilde{K}$	$\widetilde{A}$	$\widetilde{I}/\widetilde{C}$	
$n = \%0.75$	1.04x10^{13}	9.20x10^{12}	1.32x10^{12}	1.72x10^{13}	3.55x10^{13}	0.143	
$n = \%1.11$	1.33x10^{12}	1.17x10^{12}	1.82x10^{11}	2.03x10^{12}	3.14x10^{12}	0.155	
$n = \%1.25$	6.82x10^{11}	6.00x10^{11}	9.57x10^{10}	1.01x10^{12}	1.52x10^{12}	0.160	

	$\widetilde{Work}$	$\widetilde{L}_A/\widetilde{Work}$	$\widetilde{L}_Y/\widetilde{Work}$	$\widetilde{P}_A$	r	$g_Y = g$	g_A
$n = \%0.75$	7.83x10^7	0.642	0.358	7.74	%6.59	2.64x10^{-2}	1.88x10^{-2}
$n = \%1.11$	7.50x10^7	0.453	0.547	9.34	%7.13	3.95x10^{-2}	2.80x10^{-2}
$n = \%1.25$	7.37x10^7	0.407	0.593	9.33	%7.32	4.43x10^{-2}	3.14x10^{-2}

Table 2: Steady-state values and growth rates for different population growth rates. The model parameters are $\alpha = 0.33, \lambda = 0.648, \phi = 0.74, \psi = 0.58, \beta = 0.995, \nu = 3.981x10^{-4}, \delta = 0.05, \eta = 1.0\text{x}10^{-8}$. Population intercept $\widetilde{L} = 1.63\text{x}10^8$.

As seen in the table, growth rate of model variables increase as population growth rate increases. Population growth thus fosters economic growth and rate of technological progress in the steady-state. Higher population growth rate, however, results smaller steady-state value (intercept) for $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{I}$, $\widetilde{K}$, $\widetilde{A}$ because population involves more young members and ratio of labor force to total population is lower. However when population is growing faster, households prefer to invest more portion of their income because they benefit from higher output growth rate in the future.

Raising population growth decreases the share of employees in R&D sector since final-good production has non-diminishing constant returns to scale in labor while research sector has diminishing returns to labor. Therefore higher population growth rate decreases $\widetilde{A}$.

Figure 3 shows the age-distribution among the employees of the R&D sector for the three cases. The resultant distributions are similar. Note that when population growth rate is lower, the research sector is relatively older because the population consists of more old members.

7 Impulse Response

Since fertility and technological progress are inherently random, it's reasonable to define A_t and $G_t(a)$ as stochastic variables. This enables us to understand response of the demographic multisector economy against shocks. With this modification, the dynamic equations become

$$(1+g_A)\widetilde{A}_{t+1} = \widetilde{A}_t + \nu \widetilde{H}_{At}^{\lambda} \widetilde{A}_t^{\phi} + \varepsilon_A, \qquad \lambda, \phi \in (0,1) \tag{54}$$

$$\widetilde{G}_{t+1}(a) = \widetilde{G}_t(a) + \varepsilon_G(a), \qquad a = 0, ..., T \tag{55}$$

where ε_A and $\varepsilon_G(a)$ are technology and demographic shocks respectively. Here we (distort) apply one-period shock to the system at time t when it was in steady-state previously. Thereafter the population pyramid and the economic system react to shocks and eventually return to steady-state after some transitional period. We can apply one type of shock or both shocks together.

To get the impulse response, we need to find the state transition equation. I achieve this using log-linear approximation of the dynamic equations. Note that we cannot use the original deterministic system of dynamic equations because $\widetilde{A}_t$ and $\widetilde{G}_t(a)$ are now stochastic, thus equation (44) should be replaced with the new stochastic $\widetilde{A}_t$ equation (54). Furthermore, the Euler equations now turn into expectational equations, in particular expectation operator is introduced to the right-hand side of equations (52) and (53).

Note that the number of people choosing R&D sector and final-good production sector at each period of time are implicitly given by the equation (47) stating equality of lifetime discounted wage earnings among sectors. In order to perform log-linear approximation of model equations, I find explicit formula for number of people choosing each sector at decision age cohort $\widetilde{L}_{A,t}(\overline{a}_{\text{det}})$ and $\widetilde{L}_{Y,t}(\overline{a}_{\text{det}})$ in terms of other model variables. For this I approximate the policy function $\widetilde{L}_{A,t}(\overline{a}_{\text{det}})$ with Chebsyhev polynomials and determine its optimal parametrization by equality of lifetime wage earnings condition. Details of deriving policy function and parametrization are explained in the Appendix.

Having explicit form of policy functions, I obtain the following state-transition equation by log-linear approximation:

$$A.\widetilde{x}_{t+1} = B.\widetilde{x}_t + C.\vartheta_{t+1} + D.\varepsilon_{t+1} \tag{56}$$

where A and B are state matrices and $\widetilde{x}_t$ is the state vector including variables $\widetilde{y}_t, \widetilde{c}_t, \widetilde{i}_t, \widetilde{k}_t, \widetilde{a}_t, \widetilde{l}_t, \widetilde{h}_{At}, \widetilde{h}_{Yt}, \widetilde{work}_t, \widetilde{p}_{At}, \widetilde{\pi}_t, \widetilde{r}_t, \widetilde{l}_{At}(a)$, $a = \overline{a}_{\det}...a_{r(\max)}$ and $\widetilde{l}_{Yt}(a)$, $a = \overline{a}_{\det}...a_{y(\max)}$. Note that the state vector consists of logged deviations of normalized variables from their steady-state. ϑ_{t+1} is the vector of expectational errors and ε_{t+1} is the composite vector that carries structural shocks and time-dependent values of exogenously determined variables $\widetilde{Work}_t$ and $\widetilde{L}_t$. The structural shock ε_A to $\widetilde{A}_t$ is now in terms of standard deviation of $\widetilde{A}_t$ (since we use logged deviation of $\widetilde{A}_t$ from steady-state in the state vector) while demographic shock $\varepsilon_G(a)$ is still in its absolute value. Using Sim's solution method, we get the evolution of state as

$$\widetilde{x}_{t+1} = F.\widetilde{x}_t + H.\epsilon_{t+1}, \qquad t \geq 0 \tag{57}$$

where F is the state transition matrix and ϵ_{t+1} is as defined before. In the first experiment, I choose the original set of parameters in calibration and apply only a technology shock of $\varepsilon_A = 0.05$, that is 5 percent standard deviation shock. In the second experiment I use the same set of parameters but apply a 10 percent demographic shock to the cohort at age $\overline{a}_{\det}$ to see the impact more clearly. That is I apply $\varepsilon_G(\overline{a}_{\det}) = 0.10\ \widetilde{G}(\overline{a}_{\det})$ and then scale $\widetilde{G}_t(a)$ properly (so that it adds up to 1). The impulse response of economy are shown in figure 4 and 5 in the Appendix C. Shocks are applied at $t = 0$ when the system is at steady-state[3].

The first observation is that even if a relatively small amount of $\varepsilon_G(\overline{a}_{\det})$ shock to single cohort has been applied, it takes more than hundred years for population pyramid to stabilize. If the demographic shock $\varepsilon_G(a)$ was applied to more than one cohort, the population pyramid would stabilize in longer time period. This also causes other variables to return to steady-state in a long period of time. Therefore demographic economic models have very long time-horizon, and in practice they never reach their steady-state because technology shocks are frequent and population growth rate is not constant in the long-run. Other factors like war or disease may further alter the age distribution in the society.

[3] Because the space is limited, I can only show several samples of $\widetilde{G}_t$ response. The interested reader can see the interative response of all variables and experiment with different parameters using the Matlab code provided.

The impulse response to the technology shock is relatively smooth whereas impulse response to the demographic shock has fluctuations and irregularities. As far as the amplitude is concerned, variables show greater response to the technology shock ε_A compared to the demographic shock, as the demographic shock is applied to single cohort. The impact of the demographic shock however is more durable and the variables show variations above and below their steady-state. The exception is that it takes longer for $\widetilde{A}_t$, $\widetilde{L}_{A,t}(\overline{a}_{\text{det}})$ and $\widetilde{L}_{A,t}(\overline{a}_{\text{det}})$ to return to their steady-state in reaction to the technology shock.

8 Forecasting Economic Growth

After calibrating the model and finding evolution of the economy, I make predictions for TFP growth of United States to observe the impact of population ageing. I forecast the model from year 2001 to 2050. I use the set of parameters obtained in calibration. I take L_t, $G_t(a), Work_t$ and n_t as exogenous. Since there is no data for annual employment forecast up to 2050 for United States, I use year 2000 data for age-specific labor force participation rate and unemployment rate.

At each time period t, given present values of state variables $\widehat{K_t}, \widehat{A}_t, \widehat{P}_{At}$, $\widehat{H}_{At}$, $\widehat{H}_{Yt}$, one can compute the choice variables $\widehat{C}_t$, $\widehat{I}_t$, $\widehat{L}_{At}(\overline{a}_{\text{det}}), \widehat{L}_{Yt}(\overline{a}_{\text{det}})$ using intratemporal optimization conditions. Number of people at the age of $\overline{a}_{\text{det}}$ at time t choosing R&D sector $\widehat{L}_{At}(\overline{a}_{\text{det}})$ is computed using Chebshyev function derived in the previous section. Calculation of the Chebshyev function is explained in the Appendix. The other sector decision then becomes $\widehat{L}_{Yt}(\overline{a}_{\text{det}}) = Work_t(\overline{a}_{\text{det}}) - \widehat{L}_{At}(\overline{a}_{\text{det}})$. Having time t choice variables, next period state variables $\widehat{K_{t+1}}, \widehat{A}_{t+1}, \widehat{P}_{A,t+1}$, can be estimated using state transition equations. Knowing $\widehat{L}_{At}(a)$, $a = \overline{a}_{\text{det}}...a_{r(\max)}$ and $\widehat{L}_{Yt}(a)$, $a = \overline{a}_{\text{det}}...a_{y(\max)}$ I compute human capital in the next period $\widehat{H}_{A,t+1}$, $\widehat{H}_{Y,t+1}$. Forecasting procedure then continues with the next period and so on. Note that capital and knowledge stock in the first period of forecast (year 2001) are evaluated using fitted variables $\widehat{K}_{2000}$, $\widehat{A}_{2000}$ and year 2000 investment data. I obtain initial age-distribution in the R&D sector from National Science Foundation Integrated Database 2003. I evaluate share of age groups among those employees who are in the research and development category in the database.

Figure 6 in Appendix C shows actual data from 1950 to 2000 with the forecasted variables from 2001 to 2050. Logged values are plotted to see the change in growth rates. Note that actual A_t and K_t are unobservable, so fitted values $\widehat{A}_t$, $\widehat{K_t}$, $t = 1950$ to 2000 are plotted. According to forecasts, share of employees in R&D sector steadily increases until 2050 and reaches %45 level, which is the steady-state value of the model. Level of technology $\widehat{A}_t$, will continue to grow with slight decrease in the early 2000s.

Observe that there are discontinuities in $\widehat{Y}_t$ and $\widehat{I}_t$ in year 2000. One explanation for the discontinuity between the data and the forecasts is that the economy has not reached its steady-state, especially share of employment in science, technology and engineering sectors. Decreasing population growth rate and variations in population pyramid over time are also reason for the discontinuity. Besides one may argue that the actual consumption rate is greater than its steady-state value. Note that the model predicts lower $\widehat{H}_{Yt}$ value than the actual data because approximately % 95 of employees work in economy-wide sector in 2000 while this ratio for the model is %54.7. So forecasted human capital $\widehat{H}_{Yt}$ decreases over time which causes interest rate also to decrease. Because interest rate is decreasing, the model then predicts consumption growth to slow down and then total consumption to decrease.

Figure 7 depicts the age-distribution forecasts in the R&D sector for 2025 and 2050. Because ratio of people choosing science sector $\widehat{L}_{At}(\overline{a}_{\text{det}})/\widehat{L}_t(\overline{a}_{\text{det}})$, is increasing, labor force in research sector will be relatively younger in 2025 compared to 2003. Age-distribution will be closer to its steady-state distribution in 2050.

9 Discussion and Further Research

Demographic economy models are powerful to study the relationship between society, total factor productivity, economic output and sectors as the models employ population pyramid to estimate labor and population variables in detail. One can extend the model by endogeneouzing population growth, for instance Becker and Lewis (1973) model can be used to determine the fertility choice. In addition social security expenditures and (possibly sector-dependent [4]) educational cost can be added. The social planner in this case also considers training costs while allocating the labor force between sectors. Similarly individuals in the economy can consider training costs while choosing their sector. Introducing social security expenditures makes the model more realistic and promising since health expenditures and needs of old people will become significant as population ages. Note that although promising, the educational and social security expenditures will make the model more complicated and more parametric. Furthermore, data for these variables are available only for recent years. Another extension is open-economy model where migration and international capital flows are allowed. But since immigrants bring their own human capital, the model environment should be modified to adapt immigrant profiles. Finally, demographic profile of OECD countries, China, Japan and global economic equilibrium, similar to Fehr, Jokisch and Kotlikoff (2005) is a potential long-term project.

[4] I thank David Dejong for this suggestion.

10 Conclusion

In this paper I have founded the basis of demographic economic growth models and thus established a framework in which the impact of age-distribution of the society on economic growth can be examined. The way demographic structure influences economy is through age-dependent productivity and endogenous allocation of individuals between sectors. In the model, there are three sectors in the economy, wages and market forces determine the allocation. Higher population growth provides greater economic growth and faster technological progress. As an empirical application I fit the model to United States data for 1950 to 2000, and perform 50 year forecast to see the impact of population ageing. Technological progress seems to be sustainable despite population ageing. The size and the share of employment in the R&D sectors will continue to rise. Aside from the model presented in this paper, there are other forces such as social security or education expenditures by which population ageing may enhance or suspend economic growth. These extensions may be the topic of subsequent papers.

11 Data and Sources

United States 1929 to 2007 annual real consumption expenditures (nondurables and services) and investment data are all in year 2000 US dollars and taken from Federal Reserve Bank of St. Louis online database. Data for United States total population, population growth rate and population pyramid between 1950 and 2050 are from United Nations Population Database (2006 revision). Population pyramid data is available every five year from 1950 to 2050 and total population forecast is every five year from 2005 to 2010 so I perform linear approximation to calculate the annual data. In the data for population pyramid, ages groups are 0-4, 5-10,...,95-100 and 100+. I assume the same portion for all five ages inside the same age group. Annual total employment is obtained from United States Bureau of Labor Statistics. OECD online statistics database supplies 1960-2007 annual labor force participation and employment rate for different age groups. Number of employees in science, technology and engineering sectors are obtained from United States Census data between 1950 and 2000. Hansen (1993) gives relative individual productivity with respect to age in the whole economy and this data is used to form $w_Y(a)$. National Science Foundation conducted survey of doctorate recipients in 2003 and I use the average number of papers (co)authored since 1990 among faculty members and R&D employees, in this survey as a relative measure of age-dependent scientific productivity $w_R(a)$. National Science Foundation Integrated Database provides number of employees for each age subgroup working in research and development category and I use 2003 data to calculate age-distribution inside R&D sector with interpolation.

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12 Appendix.

12.1 Appendix A: Nonlinear Approximation of Policy Function

The policy function I seek for sector decision variable is of 2^{nd} order (r=2) complete Chebsyhev polynomial form:

$$
\begin{aligned}
\widetilde{L}_{A,t}(\overline{a}_{\text{det}}) = {} & \chi_1 + \chi_2 T_1(\widetilde{s}_{A_t}) + \chi_3 T_1(\widetilde{s}_{P_{At}}) + \chi_4 T_1(\widetilde{s}_{K_t}) + \chi_5 T_1(\widetilde{s}_{H_{At}}) \\
& + \chi_6 T_2(\widetilde{s}_{A_t}) + \chi_7 T_2(\widetilde{s}_{P_{At}}) + \chi_8 T_2(\widetilde{s}_{K_t}) + \chi_9 T_2(\widetilde{s}_{H_{At}}) \\
& + \chi_{10} T_1(\widetilde{s}_{A_t}) T_1(\widetilde{s}_{P_{At}}) + \chi_{11} T_1(\widetilde{s}_{A_t}) T_1(\widetilde{s}_{K_t}) + \chi_{12} T_1(\widetilde{s}_{A_t}) T_1(\widetilde{s}_{H_{At}}) + \chi_{13} T_1(\widetilde{s}_{P_{At}}) T_1(\widetilde{s}_{K_t}) \\
& + \chi_{14} T_1(\widetilde{s}_{P_{At}}) T_1(\widetilde{s}_{H_{At}}) + \chi_{15} T_1(\widetilde{s}_{K_t}) T_1(\widetilde{s}_{H_{At}}) + \chi_{16} T_1(\widetilde{s}_{A_t}) T_1(\widetilde{s}_{P_{At}}) T_1(\widetilde{s}_{H_{At}})
\end{aligned}
\tag{58}
$$

where $\chi_1 ... \chi_{16}$ are parameters to estimate and

$$
T_j(\widetilde{s}_{X_t}) = \cos(j. \cos^{-1}(\widetilde{s}_{X_t})), \qquad \widetilde{s}_{X_t} = \frac{\widetilde{X}_t - \widetilde{X}_{ss}}{\omega_{\widetilde{X}}}
$$

$\widetilde{s}_{X_t}$ is a measure of deviation of $\widetilde{X}_t$ variable from its steady-state with respect to a range $\omega_{\widetilde{X}}$. If $\widetilde{X}_t$ varies $\omega_{\widetilde{X}}/2$ units above or below its steady-state then $\widetilde{s}_{X_t}$ is a transformation of $\widetilde{X}_t$ to [-1, 1] scale. Here I set $\omega_{\widetilde{X}} = 4\sigma_{\widetilde{X}}$ i.e., I assume $\widetilde{X}_t$ lies in its 4 standard deviation range.

With this equation for $\widetilde{L}_{A,t}(\overline{a}_{\text{det}})$ and given candidate χ parameters, the dynamic system is ready to solve. For this I define the state vector $\widetilde{x}_t$ composed of variables $\widetilde{y}_t$, $\widetilde{c}_t$, $\widetilde{i}_t$, $\widetilde{k}_t$, $\widetilde{a}_t$, $\widetilde{l}_t$, $\widetilde{h}_{At}$, $\widetilde{h}_{Yt}$, $\widetilde{work}_t$, $\widetilde{p}_{At}$, $\widetilde{\pi}_t$, $\widetilde{r}_t$, $\widetilde{l}_{At}(a)$, $a = \overline{a}_{\text{det}} ... a_{r(\max)}$ and $\widetilde{l}_{Yt}(a)$, $a = \overline{a}_{\text{det}} ... a_{y(\max)}$. Note that the state vector consists of logged deviations of normalized variables from their steady-state. Log-linear approximation of the state vector around its steady-state yields the state transition equation:

$$
A_{(\chi)}.\widetilde{x}_{t+1} = B_{(\chi)}.\widetilde{x}_t + C.\vartheta_{t+1} + D.\varepsilon_{t+1} \tag{59}
$$

where $A_{(\chi)}$ and $B_{(\chi)}$ are state matrices (depending on the choice of χ parameters). ϑ_{t+1} is the vector of expectational errors and ε_{t+1} is the composite vector that carries structural shocks and time-dependent values of exogenously determined variables $\widetilde{Work}_t$ and $\widetilde{L}_t$. The structural shock ε_A to $\widetilde{A}_t$ is in terms of standard deviation of $\widetilde{A}_t$ (since we use logged deviation of $\widetilde{A}_t$ from steady-state in the state vector) while demographic shock $\varepsilon_G(a)$ is still in its absolute value. I solve this dynamic stochastic linear system using Sim's solution method and get the evolution of state as

$$
\widetilde{x}_{t+1} = F_{(\chi)}.\widetilde{x}_t + H_{(\chi)}.\epsilon_{t+1}, \qquad t \geq 0 \tag{60}
$$

where $F_{(\chi)}$ is the state transition matrix. Thereafter I determine the future trajectory of the state and evaluate discounted lifetime wage earnings in both sectors. Assuming individuals' expectations are rational, $\chi_1 ... \chi_{16}$ parameters should be chosen so that the equality of discounted lifetime earnings among sectors, mentioned in the text, holds: Thus $\chi_1 ... \chi_{16}$ parameters should be chosen so as to In the first experiment, I choose the original set of

$$\sum_{a=a_{r(\min)}}^{a_{r(\max)}} \tau_{(t,t+a-\overline{a}_{\text{det}})}.\nu.\widetilde{P}_{A,t+a-\overline{a}_{\text{det}}} \widetilde{H}^{\lambda-1}_{A,t+a-\overline{a}_{\text{det}}} \widetilde{A}^{\phi}_{t+a-\overline{a}_{\text{det}}} w_R(a) =$$

$$\sum_{a=a_{y(\min)}}^{a_{y(\max)}} \tau_{(t,t+a-\overline{a}_{\text{det}})}.(1-\alpha)\frac{\widetilde{Y}_{t+a-\overline{a}_{\text{det}}}}{\widetilde{H}_{Y,t+a-\overline{a}_{\text{det}}}} w_Y(a) \tag{61}$$

At this point, Chebyshev interpolation theorem helps us to find the desired χ parameters. According to the theorem, if the Chebyshev function is zero at the roots of rth order polynomial $T_r(\widetilde{s}_{X_t})$, then the function is close to zero on the whole $\widetilde{s}_{X_t}$ domain. The roots of the rth order polynomial $T_r(\widetilde{s})$ are

$$\widehat{s}_j = \cos\left(\frac{2j-1}{2r}\pi\right) \qquad j = 1, 2, ...r \tag{62}$$

In this problem r=2 and the roots of $T_2(\widetilde{s}_{X_t})$ polynomial are $\widehat{s}_j = \sqrt{2}/2$ and $\widehat{s}_j = -\sqrt{2}/2$ for each variable $\widetilde{X}_t = \widetilde{A}_t$, $\widetilde{P}_{At}$, $\widetilde{K}_t$, $\widetilde{H}_{At}$. With two roots of each four variable, there are 16 possible combinations of roots. The objective is to choose $\chi_1 ... \chi_{16}$ so that condition (61) is satisfied at each 16 combination of the Chebyshev roots. In the programming stage I find the χ parameters using Matlab's fminsearch routine. The optimum parameters of the Chebyshev function turns out to be:

χ_1	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
1.165x10^{6}	1.878x10^{2}	2.864x10^{-4}	8.613x10^{-10}	1.55x10^{-2}	2.51x10^{3}	4.836x10^{-8}	3.743x10^{-19}

χ_9	χ_{10}	χ_{11}	χ_{12}	χ_{13}	χ_{14}	χ_{15}	χ_{16}
9.097x10^{-5}	2.79x10^{-2}	7.091x10^{-8}	1.236	1.168x10^{-13}	2.54x10^{-6}	6.385x10^{-12}	1.957x10^{-4}

Table 4: Optimized χ values for the policy function $\widetilde{L}_{A,t}(\overline{a}_{\text{det}})$

12.2 Appendix B: Sector Decision in Forecast of the Model

Forecasts of $\widehat{L}_{At}(\overline{a}_{\text{det}})$ is computed using the 2nd order Chebshyev function derived in Appendix A. Now the Chebshyev variable becomes:

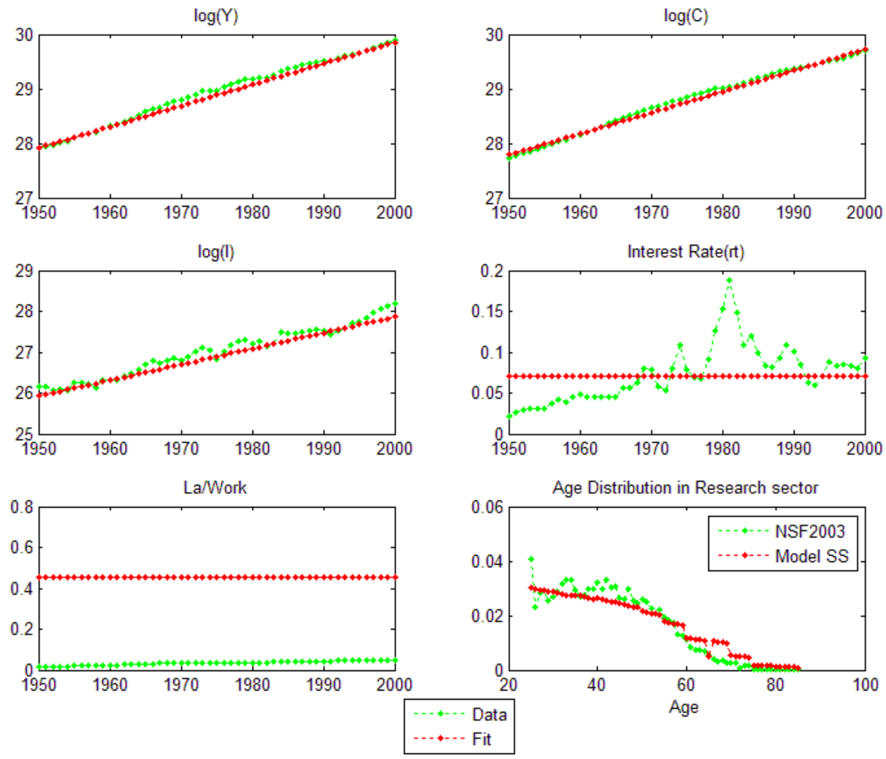

Fig. 2. Fitted variables and actual data for United States, 1950-2050

$$\tilde{s}_{\widehat{X}_t} = \frac{\widetilde{X}_t - \widetilde{X}_{ss}}{\omega_{\widetilde{X}}} \qquad \text{where } \widetilde{X}_t \; = \frac{\widehat{X}_t}{(1+g_X)^{(t-1950)}}$$

That is I normalize forecasted state variables with their respective growth rates.

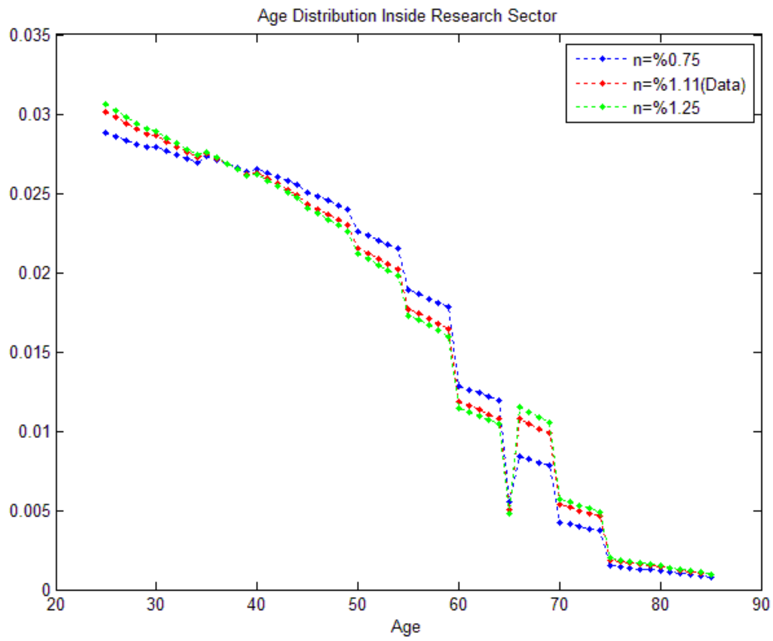

Fig. 3. Share of age groups among R&D employees in the steady-state, depicted for three different population growth rates

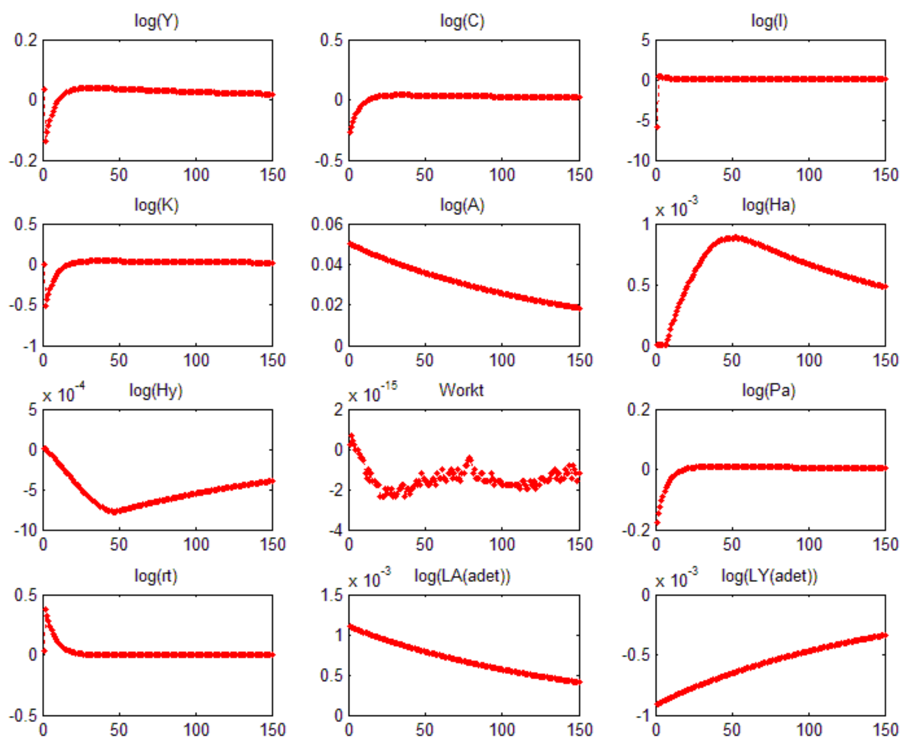

Fig. 4. Impulse response of variables against 5% A_t technology shock

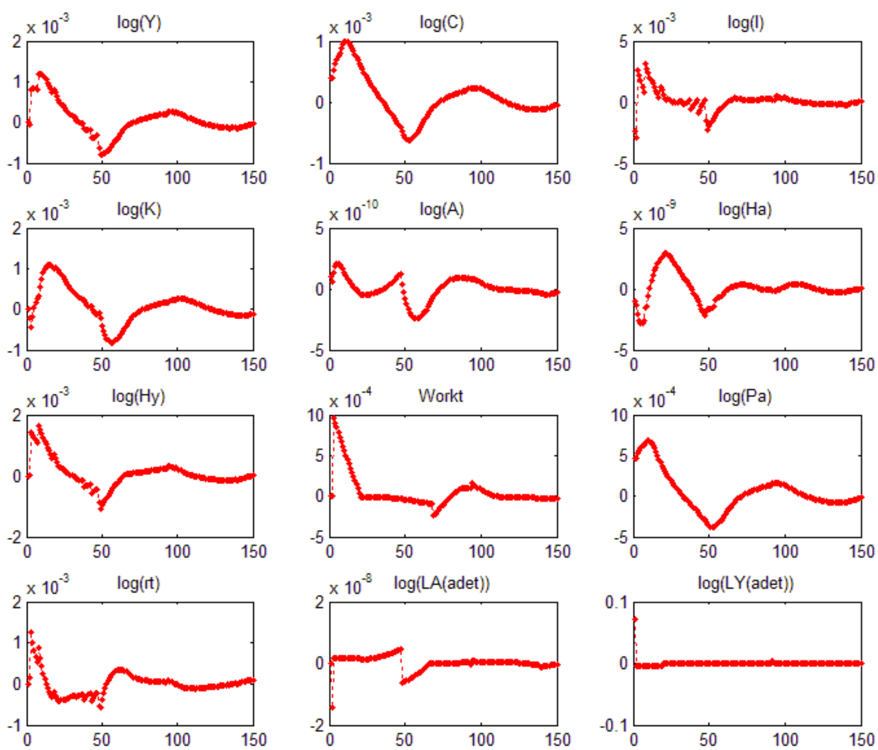

Fig. 5. Impulse response of variables against 10% $\varepsilon_G(\overline{a}_{\text{det}})$ demographic shock to the cohort at age $\overline{a}_{\text{det}}$

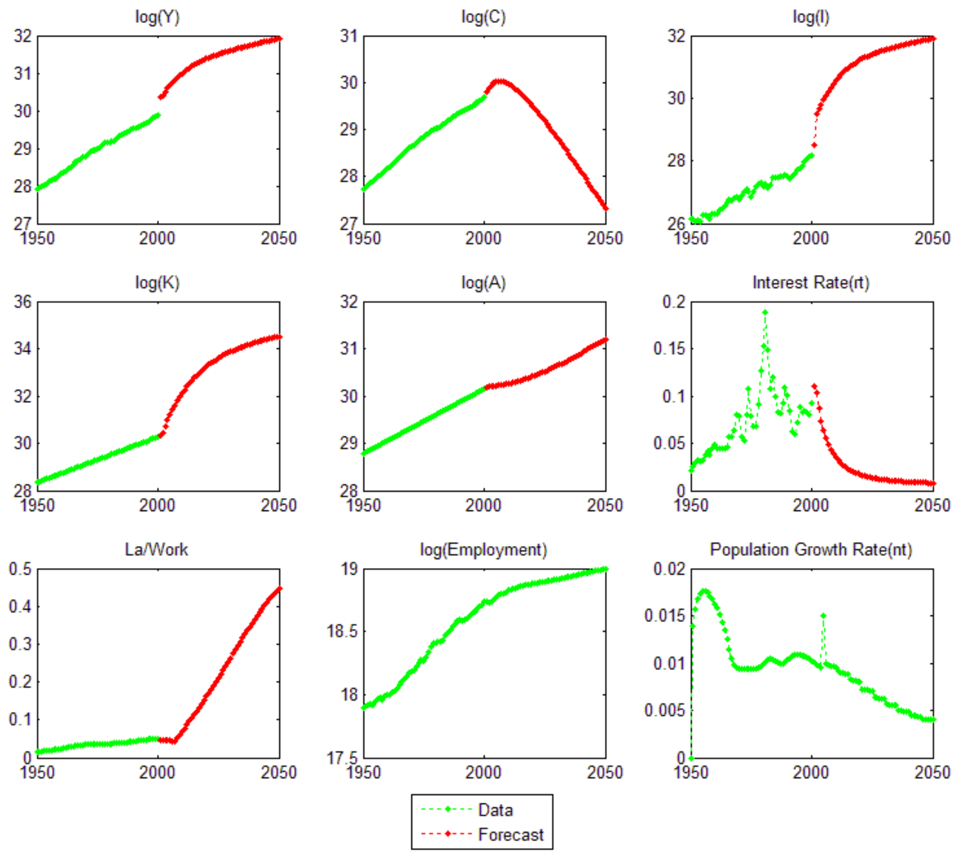

Fig. 6. Economic Forecasts for United States: 2001-2050. (Actual values are plotted for 1950-2000)

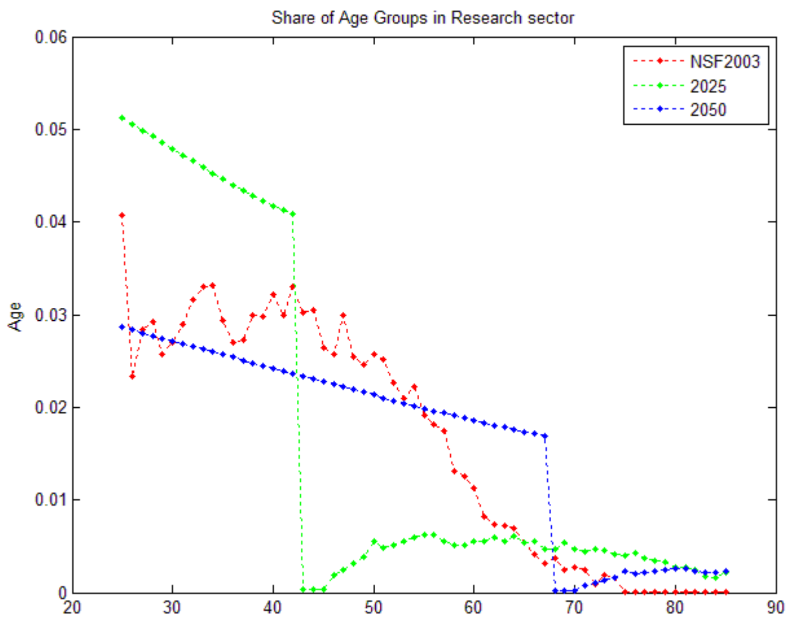

Fig. 7. Share of age groups among R&D employees: Year 2003 (actual), 2025 (forecast), 2050 (forecast)