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Proceedings of the ASME 2009 International Design Engineering Technical Conferences &
Computers and Information in Engineering Conference
IDETC/CIE 2009
August 30 - September 2, 2009, San Diego, California, USA

DETC2009-86158

TRACKING CONTROL OF UNCERTAIN NONHOLONOMIC MOBILE ROBOTS: SMOOTH SWITCHING APPROACH

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ABSTRACT
Backstepping based adaptive tracking control of non-holonomic mobile robots in the presence of both kinematic and dynamic parametric uncertainty is presented. The major challenge is the possible singularity phenomenon due to the approach of zero of the estimated input vector field entering the denominator of the control input, a common drawback of adaptive linearization-based schemes. A hybrid control approach, which switches between an adaptive and a robust control schemes, is developed for solving such a problem. It retains the advantage of an adaptive control approach to a greatest extent while avoiding the possible blowup of the torque inputs simultaneously. A case study on a specific Type (2; 0) mobile robot is provided in the final to verify the usefulness of the proposed design.

1. INTRODUCTION
Tracking control of mobile robots with nonholonomic constraints, due to its great potential in wide varieties of applications, has received a lot of attentions recently [1], [3], [14]. The design process is generally divided into the kinematic and the dynamic stages.The former generally starts from a driftless model describing motions on the nonholonomic manifold, which is next converted into the chained canonical form to facilitate the control design. Based on such a model, numerous schemes, falling into the span of the discontinuous control, the hybrid control, and the backstepping designs, have been proposed to attain the control objectives (see a review in [3], [9]). Among them, the so-called integrator backstepping, due to its amenity to the nested-coupling structure of the chained form systems, has become the major tool for this application [17].
Assuming perfect knowledge of the system dynamics, a backstepping computed torque scheme is proposed to solve various tracking tasks in [8]. The design in [16] ensures the exponential tracking stability on a mild persistent excitation (PE) condition for a specific type of mobile robot systems. The time-varying polynomial type stabilizer in [7] ensures the global asymptotic stability for multi-input chained form systems. Based on such an approach, various schemes, such as the stabilization design for chained-form systems with parametric uncertainty [11], [9], tracking control for nonholonomic dynamic systems with inertia parametric uncertainty [6], and the simultaneous tracking of motion and force of a general mobile robot [10], have been proposed. An integrated controller, which ensures the semiglobal asymptotic stabilization and tracking stability of mobile robot systems with both kinematic and dynamic
parametric uncertainties, was proposed in [12]. Robust adaptive control design for a specific mobile robot with parametric and nonparametric uncertainty is given in [5]. However, control designs for a general mobile robot with parametric uncertainty in the input matrix are still in demand.
The major challenge is that the so-called singularity phenomenon, due to the approach of zero of the denominator of the estimated input vector field in a common adaptive linearizing approach, may occur to saturate the actuators. A Lyapunov-based smooth switching control algorithm, modified from the design in [19], is therefore developed here for alleviating such a difficulty. Such an approach is appealing to practical applications in that it retains the advantage of the adaptive control scheme to a greatest extent without igniting the singularity phenomenon and the switching-induced chattering behavior [18]. A case study on a unicycle-like mobile robot is undertaken next to demonstrate its validity.
The remainder of the paper is organized as follows: The kinematic and dynamical model of a mobile robot with nonholonomic constraint is introduced in Section 2. The essential properties and existing designs related to our work are also described. The central part of this paper, namely, the control design, is detailed in Section 3. To demonstrate its usefulness, a case study of a mobile robot with two driving wheels is given in Section 4. Concluding remarks are finally made in Section 5.

2. SYSTEM MODEL
Let $q \in R^n$ denote the generalized coordinate vector of a nonholonomic mobile robot. By definition, the corresponding velocity vector satisfies the nonintegrable linear velocity constraint

$$J(q)\dot{q} = 0 \tag{1}$$

where $J(q) \in R^{m\times n}$ is a full-rank matrix. Next, via the so called Euler-Lagrange formulation, the system dynamics can be obtained as

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$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q) = B(q)\tau+J^T(q)\lambda \quad (2)$$

where $M(q)\in\mathbb{R}^{m\times n}$ is the symmetric, positive definite inertia matrix; $\tau\in\mathbb{R}^{m\times n}$ is the available motor torque vector and B(q) is an $n\times r$ full-rank matrix; $\lambda\in\mathbb{R}^m$ represents the constraint force vector; $C(q,\dot{q})\dot{q}$ is the centripetal and Coriolis torque vector while G(q) is the gravitational torque vector. For simplicity and without loss of generality, we assume that the robot is moving on a flat surface and therefore the gravitational torque vector can be set to zero.

The following properties known to hold for a general robot are summarized here for the ease of reference.

• P1): The left-hand side of (2) is linear in the physical parameters (masses, moment of inertia, etc.) and therefore can be written in a compact form of

$$M(q)\dot{v}+C(q,\dot{q})v= H(q,\dot{q},v,\dot{v})\beta \quad (3)$$

where $\beta\in\mathbb{R}^p$ denotes the lumped parameter vector while $H\in\mathbb{R}^{n\times p}$ is the regression matrix depending on $q,\dot{q},v$ and $\dot{v}$.

• P2): The selection of the matrix $C(q,\dot{q})$ is not unique, and in particular, it can always be selected so that the matrix $(\dot{M}-2C)$ is skew symmetric.

Given a bounded reference state vector $(q_d,\dot{q}_d)$ fulfilling the constraint (1), the control objective is, under the condition of the parameters β and those in the input matrix B being unknown, to determine a control law for $\tau(t)$ such that $(q,\dot{q})\to(q_d,\dot{q}_d$ as $t\to\infty$.

The first step is to obtain the dynamics on the reduced constraint manifold, which is $(n-m)$ dimensional and free from constraint forces. The assumption of J(q) being full-rank implies the existence of a smooth distribution, denoted by $J^\perp$, which totally annihilates the row vectors of J(q) for all $q\in\mathbb{R}^n$ [2]. Let $S(q)\in\mathbb{R}^{n\times(n-m)}$ be the matrix whose column vectors span $J^\perp(q)$, i.e.,

$$S^T(q)\,J^T(q) = 0 \quad (4)$$

Clearly, $\dot{q}$ must lie in $J^\perp$. More formally, there exists a set of linearly independent vector field $v(q)\in\mathbb{R}^{n-m}$ such that

$$\dot{q} = S(q)v(q), \quad (5)$$

Taking time derivatives of (5) results in

$$\ddot{q} = S\dot{v}+\dot{S}v \quad (6)$$

By substituting (5) and (6) into (2) and then multiplying both sides by $S^T(q)$, it yields

$$M_1(q)\,\dot{v}+C_1(q,\dot{q})v= B_1(q)\,\tau \quad (7)$$

where $M_1(q) = S^T M(q)\,S(q)$, $C_1(q,\dot{q}) = S^T M(q)\,\dot{S}(q) + S^T C(q,\dot{q})S(q)$, and $B_1(q) = S^T B(q)$. Equations (5) and (7) constitute a set of $2n-m$ algebraic-differential equations describing the dynamics on the constraint manifold. They are underactuated in the sense that the actual control inputs appear only in the last $n-m$ differential equations [2]. The control objective can still be attained if the coupling state v(t) in (5) provides desired virtual control signal, which in turn is realized via actuating the torque inputs properly.

To that end, it is quite common to first convert (5) into certain canonical forms to facilitate the control designs [2]. For simplicity and without loss of generality, we consider the case with $v\in\mathbb{R}^2$ in the sequel. Assume there exists a diffeomorphic coordinate transformation $y=\phi(q), u=\varphi(q)v$ with $\varphi(q)\in\mathbb{R}^{2x2}$, under which the kinematic subsystem (5) can be transformed into the one-chain single-generator chained canonical form

$$\dot{y}_1= u_1$$
$$\dot{y}_2= u_1 y_{j+1},$$
$$\dot{y}_n= u_2,\ 2\le j\le n-1 \quad (8)$$

As a consequence, the dynamic model can be rewritten as

$$M(y)\,\dot{u}+C(y,\dot{y})u = B(y)\,\tau \quad (9)$$

where

$$M(y) = \varphi(q)^{-T}M_1(q)\,\varphi(q)^{-1}\,|_{q=\varphi^{-1}(y)}$$
$$C(y,\dot{y}) = \varphi^{-T}[C_1(q,\dot{q}) - M_1(q)]\dot{\varphi}(q)^{-1}\,|_{q=\varphi^{-1}(y)}$$
$$B(y) = \varphi^{-T}B_1(q)\ \ |_{q=\varphi^{-1}(y)} \quad (10)$$

It is easy to prove that M(y) remains as a symmetric positive-definite matrix and

• P3): $M(y)\,\dot{\eta} + C(y,\dot{y})\eta = \Phi(y,\dot{y},\eta,\dot{\eta})\beta$, where Φ is some certain known regression matrix depending on q, $\dot{q}$, η and $\dot{\eta}$.

• P4): The matrix $(\dot{M}-2C)$ is skew symmetric.

The desired trajectory q_d should also comply with the same constraint such that

$$\dot{q_d} = S(q_d)v_d \quad (11)$$

which, via the similar transformation $y_d = \phi(q_d)$ and $u_d = \varphi(q_d)v_d$, can be converted into the same canonical form

$$\dot{\zeta_1} = v_{d,1,}$$
$$\dot{\zeta_j} = v_{d,1,}\zeta_{j+1},$$
$$\dot{\zeta_n} = v_{d,2,}\ 2\le j\le n-1 \quad (12)$$

The trajectory tracking task has been converted into a model following problem, i.e, under the condition of the system's parameters being unknown, the goal is to seek a robust switching adaptive controller, such that $y\to\zeta$ as $t\to\infty$.

3. MAIN RESULTS

In this section, a backstepping based control design will be formulated for attaining the objective mentioned above. In particular, the smooth switching mechanism originated in [19] will be included for avoiding the possible singularity phenomenon arising from the denominator of the proposed adaptive controller being close to zero.

The assumptions required for the upcoming designs are summarized here first.

• A1): The reference trajectory q_d is bounded and smooth, and $\lim_{t\to\infty}\inf|u_{d,0}|>0$.

• A2): There exists a known matrix $B_s(y)\in\mathbb{R}^{r\times 2}$, such that $\boldsymbol{\lambda_0 x^T x \le x^T B B_s} x$, $\forall x\in\mathbb{R}^2$ with $\lambda_0>0$ being known *a priori*.

By subtracting (8) from (12), the kinematic tracking error dynamics can be obtained

$$\dot{e_1} = u_1 - u_{d,1}$$
$$\dot{e_j} = u_{d,1}e_{j+1} + (u_1-u_{d,1})\,y_{j+1},\ 2\le j\le n-1$$
$$\dot{e_n} = u_2 - u_{d,2} \quad (13)$$

where $e = y-\zeta$. Denote the virtual controller (the desired behavior) for the coupling state e_{j+1} in $\dot{e}_j$ by α_j. The backstepping tool aims to specify α_j recursively and in the final a suitable actual controller is sought for stabilizing the error dynamics between e_{j+1} and α_j to attain the objectives. To that end, the following set of error states are defined first

$$\dot{z}_1 = e_1,$$
$$\dot{z}_j = e_j - \alpha_{j-1},\quad 2\le j\le n$$

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$$\dot{z}_n = u_k - u_{b,k}, \quad k = 1,2 \quad (14)$$

where $\alpha \in R^n$ and $u_b \in R^2$ are the virtual controllers at disposal. By a direct differentiation and taking (13) into account, it yields

$$\dot{z}_1 = \xi_1 + u_{b,1} - u_{d,1},$$
$$\dot{z}_j = u_{d,1}(z_{j+1} + \alpha_j) + (\xi_1 + u_{b,1} - u_{d,1})y_{j+1} - \dot{\alpha}_{j-1}, \quad 2 \le j \le n-1$$
$$\dot{z}_n = \xi_2 + u_{b,2} - u_{d,2} - \dot{\alpha}_{n-1}, \quad (15)$$

On the other hand, the dynamics in (9) can be expressed in terms of the error vector ξ as follows

$$M(y)\dot{\xi} = B(y)\tau - \Phi(y, \dot{y}, u_b, \dot{u}_b)\beta - C(y, \dot{y})\xi \quad (16)$$

Denote $f^{(k)}(t) = d^k f/dt^k$, $\bar{e}_j = [e_1, \cdot \cdot \cdot, e_j]^T$, and $\bar{u}_{d,1}^{(k)} = [u_{d,1}, \cdot \cdot \cdot, u_{d,1}^{(k)}]^T$ for brevity of notation. The proposed virtual controllers are given by

$$\alpha_1 = 0,$$
$$\alpha_1(\bar{e}_2, u_{d,1}) = -k_{z,2} u_{d,1}^{\bar{n}} z_2,$$
$$\alpha_1(\bar{e}_j, \bar{u}_{d,1}^{(j-2)}) = -z_{j-1} - k_{z,j} u_{d,1 z_j}^{\bar{n}} + \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e_k} e_{k+1} + \frac{1}{u_{d,1}} \sum_{k=0}^{j-2} \frac{\partial \alpha_{j-1}}{\partial u_{d,1}^{(k)}} u_{d,1}^{(k+1)}, \; 3 \le j \le n-1$$
$$u_{b,1} = u_{d,1} + \eta,$$

$$u_{b,2} = u_{d,2} - u_{d,1} z_{n-1} - k_{z,n} z_n + u_{d,1} \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial e_k} e_{k+1} + \sum_{k=0}^{n-2} \frac{\partial \alpha_{n-1}}{\partial u_{d,1}^{(k)}} u_{d,1}^{(k+1)}, \quad (17)$$

where $\bar{n} = 2k + 1, k \ge n - 3$ and $\eta \in R$ is a dynamic variableobeying

$$\dot{\eta} = -k_0 \eta - h_1 \quad (18)$$

with

$$h_1 = z_1 + \sum_{j=2}^{n-1} (y_{j+1} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e_k} y_{k+1}) - z_n \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e_k} y_{k+1} \quad (19)$$

By substituting (17) into (15), the resulting closed-loop kinematics becomes

$$\dot{z}_1 = \xi_1 + \eta$$
$$\dot{z}_2 = u_{d,1} z_3 - k_{z,2} u_{d,1}^{\bar{n}+1} z_2 + (\xi_1 + \eta) y_3$$
$$\dot{z}_j = u_{d,1} z_{j+1} - u_{d,1} z_{j-1} - k_{z,j} u_{d,1}^{\bar{n}+1} + (\xi_1 + \eta) (y_{j+1} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e_k} y_{k+1}), \; 3 \le j \le n-1$$
$$\dot{z}_n = \xi_2 - u_{d,1} z_{n-1} - k_{z,n} z_n - (\xi_1 + \eta) \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial e_k} y_{k+1} \quad (20)$$

This type of kinematic controllers, originated from [7], is extensively adopted in many existing schemes [10], [6]. Next, it will be backstepped into the dynamic model (16) where the inertia parameters and those in the input matrix B are unknown. We aim to develop a torque controller at this stage to attain the objectives mentioned above. The following switching function, which plays a key role in the upcoming design, is introduced here first [19]

$$\rho(x) \triangleq 1 - \exp[-(x/\omega)^2)], \; x \in R \quad (21)$$

where $w > 0$ is the corresponding transition width at disposal. It possesses the following useful properties

- P7): $0 \le \rho(x) \le 1, \; \forall x \in R$.
- P8): $\frac{\rho(x)}{x} \to 0$, as $x \to 0$.
- P9): $\rho(x) \to 1$, as $x \to \infty$.
- P10): The value $\max_{x \ne 0} \rho(x)/x$ is bounded.

The first one is a general criterion of a switching function. In the later derivation, x represents the estimated input vector fields entering the denominator of the controller. Therefore, a small x signifies the approach of singularity and a switch from the adaptive controller to its deputy should be initiated. The second property can then be used to guarantee a safe switch in that case, while the third one is to totally recover the adaptive control. Finally, the last property guarantees the finiteness of the torque input.

Assume that n= 2 for simplicity and without loss of generality. Also, for brevity of notation, the argument of a function will be omitted in the sequel when no ambiguity arises. The torque input is specified as

$$\tau = \tau_a + \tau_r,$$
$$\tau_{a,i} = \begin{cases} 0, & \xi^T \hat{B}_i = 0 \\ \frac{p_i \xi_i}{\xi^T \hat{B}_i} (-k_a \xi_i - h_i + (\Phi\hat{\beta})_i), & \text{else,} \end{cases} \quad i = 1,2$$
$$\tau_r = \sum_{i=1}^{2} (1 - p_i) \left[k_1 + \frac{k_2 \|h - \Phi\hat{\beta}\|}{\|\xi\|} \right] B_s \xi, \quad (22)$$

where $k_a, k_j > 0, j = 1,2$ are gain constants, B_i is the i'th column vector of the matrix B, $\widehat{(\cdot)}$ is the estimation of $(\cdot)$ and $\widetilde{(\cdot)} = \widehat{(\cdot)} - (\cdot)$, $p_i = p(\xi^T \hat{B}_i)$ and h = $[h_1, z_n]^T$.

The corresponding update algorithms for $\hat{\beta}$ and $\hat{B}$ are given by

$$\dot{\hat{\beta}} = -\Gamma_a \Phi^T \xi$$
$$\dot{\hat{B}} = \Gamma_b p_i \frac{-\xi_i [k_a \xi_i + h_i - (\Phi\hat{\beta}_i]}{\xi^T \hat{B}_i} \xi, \quad i = 1,2 \quad (23)$$

where $\Gamma_a, \Gamma_b > 0$ are the diagonal gain matrices.

We're now in a position to conclude that

Theorem 1: Consider the error dynamics in (14) and (16), with the control in (17) and (22), the update algorithm in (23). If the control gains are selected to fulfill

$$k_1 \ge k_a/\lambda_0 \quad k_2 \ge 1/\lambda_0 \quad (24)$$

then

•T1): all the signals in the closed-loop system remain bounded $\forall t \ge 0$;

•T2): the tracking error e(t)→ 0 as t→ ∞.

Proof:

Select the Lyapunov function as follows

$$V(t) = \frac{1}{2}(z^T z + \eta^2 + \xi^T M \xi + \tilde{\beta}^T \Gamma_a^{-1} \tilde{\beta} + \sum_{i=1}^{2} \tilde{B}_i^T \Gamma_b^{-1} \tilde{B}_i \quad (25)$$

where. z=$[z_1, z_2, \cdots, z_n]^T \in R^n$. The time derivative of V (t), taking (20) and (22)-(23) into account, can be calculated as follows

$$\dot{V}(t) = z^T \dot{n} + \eta\dot{\eta} + \xi^T M \dot{\xi} + \frac{1}{2} \xi^T \dot{M} \xi + \tilde{\beta}^T \Gamma_a^{-1} \dot{\hat{\beta}} + \sum_{i=1}^{2} \tilde{B}_i^T \Gamma_b^{-1} \dot{\hat{B}}_i$$
$$= (\xi_1 + \eta) z_1 + z_2 [u_{d,1} z_3 - k_{z,2} u_{d,1}^{\bar{n}+1} z_2 + (\xi_1 + \eta) y_3]$$
$$+ \sum_{j=3}^{n-3} z_j [u_{d,1} z_{j-1} - k_{z,j} u_{d,1}^{\bar{n}+1} z_j + (\xi_1 + \eta) (y_{j+1} - \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial e_k} y_{k+1})] + z_n [\xi_2 - u_{d,1} z_{n-1} - k_{z,n} z_n$$
$$-(\xi_1 + \eta) \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial e_k} y_{k+1}] - k_0 \eta^2 - \eta h_1$$
$$+ \xi^T [B(y)\tau - \Phi\hat{\beta} + \Phi\tilde{\beta}] + \tilde{\beta}^T \Gamma_a^{-1} \dot{\hat{\beta}}$$
$$+ \xi^T \left[\frac{1}{2} \dot{M} - C(y, \dot{y}) \right] \xi + \sum_{i=1}^{2} \tilde{B}_i^T \Gamma_b^{-1} \dot{\hat{B}}_i \quad (26)$$

By taking advantage of P4) and using (23), it leads to

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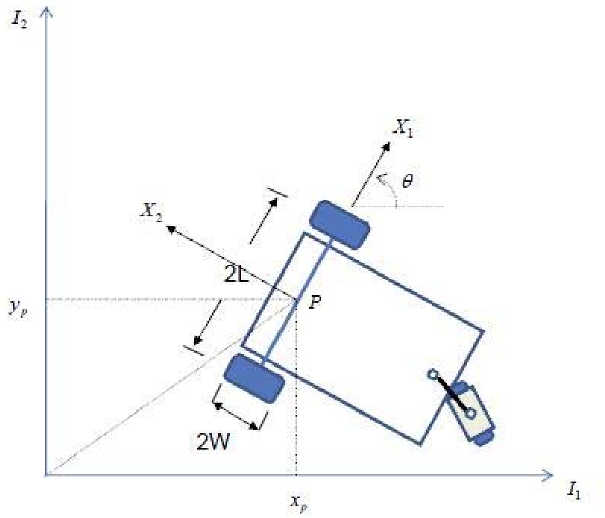

Fig. 1 Unicycle-like mobile robot

$$
\begin{aligned}
\dot{V}(t) &= -\Big(u_{d,1}^{\bar{n}+1}\sum_{j=2}^{n-1} k_{z,j}z_j^2 + k_{z,n}z_n^2\Big) - k_0\eta^2 + \xi^T h \\
&\quad + \xi^T\big[B_\tau - \Phi\hat{\beta}\big] + \textstyle\sum_{i=1}^{2}\tilde{B}_i^T\Gamma_b^{-1}\dot{\hat{B}}_i \\
&= -\Big(u_{d,1}^{\bar{n}+1}\sum_{j=2}^{n-1} k_{z,j}z_j^2 + k_{z,n}z_n^2\Big) - k_0\eta^2 + \xi^T h \\
&\quad + \textstyle\sum_{i=1}^{2}\big[\xi^T B_i\tau_{a,i} - p_i\xi_i(\Phi\hat{\beta})_i\big] \\
&\quad + \xi^T B\tau_a - \textstyle\sum_{i=1}^{2}(1-p_i)\,\xi_i(\Phi\hat{\beta})_i + \sum_{i=1}^{2}\tilde{B}_i^T\Gamma_b^{-1}\dot{\hat{B}}_i \\
&= -\left(u_{d,1}^{\bar{n}+1}\sum_{j=2}^{n-1} k_{z,j}z_j^2 + k_{z,n}z_n^2\right) - k_0\eta^2 - k_a\xi^T\xi \\
&\quad + \textstyle\sum_{i=1}^{2}\tilde{B}_i^T\{\Gamma_b^{-1}\dot{\hat{B}}_i + \frac{-p_i\xi_i[k_a\xi_i + h_i - (\Phi\hat{\beta})_i]}{\xi^T\hat{B}_i}\xi\} \\
&\quad + \textstyle\sum_{i=1}^{2}(1-p_i)\{[k_1 + k_2\frac{\|h-\Phi\hat{\beta}\|}{\|\xi\|}]\xi^T BB_s\xi \\
&\quad - \xi_i[k_a\xi_i + h_i - (\Phi\hat{\beta})_i]\}
\end{aligned}
\tag{27}
$$

Based on A2) and using (23) again, it can be further simplified as follows

$$
\begin{aligned}
\dot{V}(t) &\le -\left(u_{d,1}^{\bar{n}+1}\sum_{j=2}^{n-1} k_{z,j}z_j^2 + k_{z,n}z_n^2\right) - k_0\eta^2 - k_a\xi^T\xi \\
&\quad - \textstyle\sum_{i=1}^{2}(1-p_i)\{[k_1 + k_2\frac{\|h-\Phi\hat{\beta}\|}{\|\xi\|}]\xi^T BB_s\xi \\
&\quad - \|\xi\|(k_a\|\xi\| + \|h-\Phi\hat{\beta}\|)\} \\
&\le -\left(u_{d,1}^{\bar{n}+1}\sum_{j=2}^{n-1} k_{z,j}z_j^2 + k_{z,n}z_n^2\right) - k_0\eta^2 - k_a\xi^T\xi \\
&\quad - \textstyle\sum_{i=1}^{2}(1-p_i)\{[k_1\lambda_0 - k_a + (k_2\lambda_0 - 1)\frac{\|h-\Phi\hat{\beta}\|}{\|\xi\|}]\xi^T\xi \\
&\le -\big(u_{d,1}^{\bar{n}+1}\textstyle\sum_{j=2}^{n-1} k_{z,j}z_j^2 + k_{z,n}z_n^2\big) - k_0\eta^2 - k_a\xi^T\xi
\end{aligned}
\tag{28}
$$

The Lyapunov function V (t) is thus nonincreasing and therefore V (∞) is well defined since V (t) is lower-bounded. It follows immediately that η,z, and ξ are bounded, which in turn implies the boundedness of e by the definition of z in (14) and α in (17).

Therefore, y = e+ξ and $\xi_k + u_{b,k}, k = 1,2$ are bounded.In view of (18)-(20), (16) and (22), one obtains immediately that $\dot{\eta}, \dot{z}$ and $\dot{\xi}$ are uniformly bounded. Next, by integrating (28) from 0 to ∞, it can be seen that the signals η, $u_{d,1}^{\bar{n}+1}z_j, z_n$,

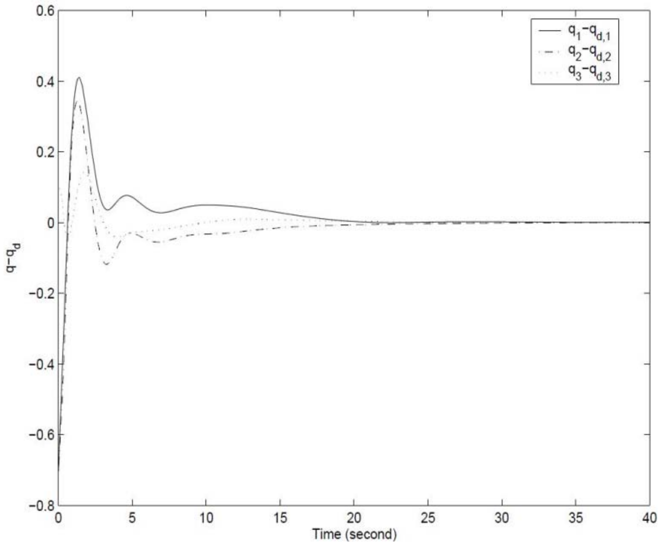

Fig. 2 Trajectories of tracking errors

$2 \le j \le n-1$, and ξ belong to L_2 functions, which, together with the fact of their derivatives being uniformly bounded, imply their convergence to zero as t → ∞, from Barbalat's lemma. By the assumption A1), it can be further concluded that $z_j, 2 \le j \le n$ tend to zero.

To prove that z_1 also tends to zero, we differentiate the vanishing signal $u_{d,1}^{\bar{n}+1}\eta$ to yield

$$
\begin{aligned}
\frac{d}{dt}[u_{d,1}^{\bar{n}+1}\eta] &= -u_{d,1}^{\bar{n}+1}z_1 + (\bar{n}+1)u_{d,1}^{\bar{n}}\dot{u}_{d,1}\eta \\
&\quad - u_{d,1}^{\bar{n}+1}(k_0\eta + h_1 - z_1)
\end{aligned}
\tag{29}
$$

From the definition of h1 in (19) and the vanishing nature of the signals of η and$z_j, 2 \le j \le n-1$ mentioned above, it can be easily checked that the second and the third terms on the right-hand side of (29) tend to zero eventually. This fact, together with the uniform continuity of the first term, imply the convergence of $(d/dt)(u_{d,1}^{\bar{n}+1}\eta)$ to zero and therefore $u_{d,1}^{\bar{n}+1}z_1 \to 0$ as $t \to \infty$ by Lemma 2 in [10]. Sustained A1), it follows immediately that $z_1 \to 0$ as $t \to \infty$. Finally, to establish the asymptotic tracking stability, i.e., $e(t) \to 0$ as $t \to \infty$, it is first noted that $e_1, \dot{e}_1, e_2$ and $\dot{e}_2$ all tend to zero, by (13)-(14) and (17). From the smoothness and boundedness of $u_{d,1}$, one obtains that α_2 and $\dot{\alpha}_2$ go to zero, which imply that $e_3, \dot{e}_3$ and $\dot{\alpha}_3 \to 0$ as t → ∞.The convergence of α_3 and $\dot{\alpha}_3$ to zero leads to the vanishing of α_4 and $\dot{\alpha}_4$ to zero eventually. By similar reasoning, it can be easily concluded that$\lim_{t\to\infty} e_j = \lim_{t\to\infty}\dot{e}_j = 0, 5 \le j \le n$.

4. SIMULATION

To demonstrate the validity of the proposed design, simulation on a unicycle-like wheeled mobile robot moving on a horizontal plane as shown in Fig. 1 (see [4] for more details), is undertaken in this section.

The nonsliding assumption on the mobile robot restricts its linear velocity perpendicular to the symmetric axis being zero, i.e.

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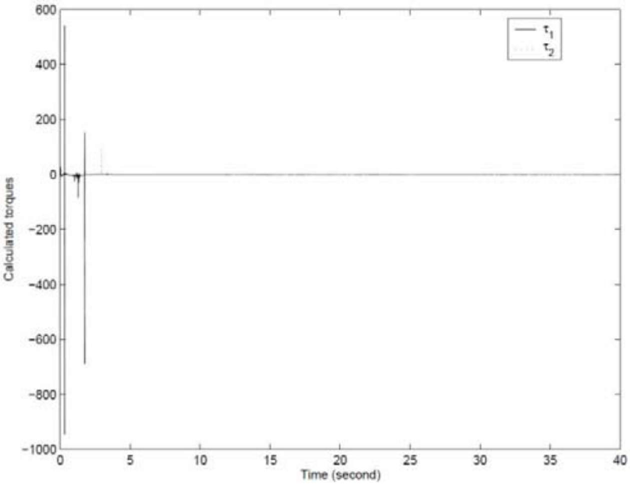

Fig. 3 Torques of purely adaptive linearizing control

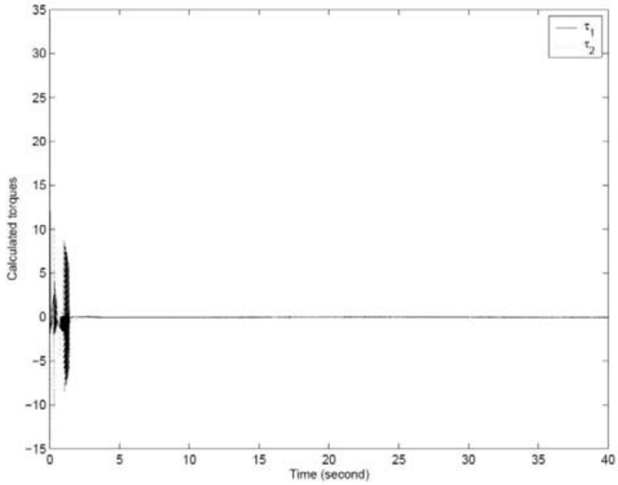

Fig. 4 Torques of the switching adaptive control

$$J(q)\dot{q} \triangleq [\cos\theta, \sin\theta, 0]\begin{bmatrix}\dot{x}_p\\ \dot{y}_p\\ \dot{\theta}\end{bmatrix} = 0 \qquad (30)$$

where (x_p, y_p) is the coordinate of the centre of mass P with respect to the inertial frame I_1I_2 while θ is the orientation of the reference frame XY with respect to the inertial frame. The null space of $J(q)^T$, as can be easily verified, is spanned by the column vector field of the matrix S(q) defined by

$$S(q) = \begin{bmatrix}-\sin\theta & 0\\ \cos\theta & 0\\ 0 & 1\end{bmatrix} \qquad (31)$$

The constraint kinematics and dynamics can be respectively written as follows

$$\dot{q} = \begin{bmatrix}-\sin\theta\\ \cos\theta\\ 0\end{bmatrix} v_1(q) + \begin{bmatrix}0\\0\\1\end{bmatrix} v_2(q), \qquad (32)$$

$$\begin{bmatrix}m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I_0\end{bmatrix}\begin{bmatrix}\ddot{x}_p\\ \ddot{y}_p\\ \ddot{\theta}\end{bmatrix} = \frac{1}{W}\begin{bmatrix}-\sin\theta & -\sin\theta\\ \cos\theta & \cos\theta\\ L & -L\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} + \lambda\begin{bmatrix}\cos\theta\\ \sin\theta\\ 0\end{bmatrix} \qquad (33)$$

where m is the mass of the robot, I_0 is the moment of inertia of the z-axis, W is the radius of the wheel, 2L is the distance between the centers of the two rear wheels, τ_1 and τ_2 are the torques from the two motors mounted on the rear wheels, $v_1(q)$ is the translational velocity along the x_2 axis and $v_2(q)$ represents the angular velocity of the mobile robot with respect to the vertical axis. Note that the gravitational torque is mull in this case for the motion is on a non-elevated surface. The coordinate and input transformations for converting the kinematic subsystem (32) into a canonical chained form are defined by [13]

$$\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} = \phi(q) \equiv \begin{bmatrix}x_p\\ y_p\\ -\cot\theta\end{bmatrix} \qquad (34)$$

$$\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \varphi(q) \triangleq \begin{bmatrix}-\sin\theta & 0\\ 0 & \csc^2\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix} \qquad (35)$$

It is diffeomorphic for all q with $\sin\theta \neq 0$. Regarding this, we assume that the workspace is $D \triangleq \{q | q \in R^3, 0 < q_3 < \pi\}$ in the sequel. Within D, the kinematic model in (32) can be converted into the following chained form

$$\dot{y}_1 = u_1,\quad \dot{y}_2 = y_3 u_1,\quad \dot{y}_3 = u_2, \qquad (36)$$

For this application, the reduced dynamics on the constraint space in (9) can now be written explicitly as

$$M(y)\dot{u} + C(y, \dot{y})u = B(y)\tau \qquad (37)$$

where

$$\begin{aligned}M(y) &= [m(y_3^2+1), 0; 0, I_0(y_3^2+1)^{-2}]^T,\\ C(y,\dot{y}) &= [m y_3\dot{y}_3/\sqrt{y_3^2+1}, 0; 0, -2I_0 y_3\dot{y}_3/(y_3^2+1)^2]^T,\\ B(y) &= W^{-1}[-\sqrt{y_3^2+1}, -\sqrt{y_3^2+1}; L/(y_3^2+1),\\ &\quad -L/(y_3^2+1)]^T.\end{aligned} \qquad (38)$$

Define $\beta = [m, I_0]^T$. By inspecting (37), the corresponding regression matrix can be easily obtained as

$$\Phi(y, \dot{y}, u_b, \dot{u}_b) = \begin{bmatrix}(y_3^2+1)\dot{u}_{b,1} + y_3\dot{y}_3(y_3^2+1)^{-\frac{1}{2}}u_{b,1} & 0\\ 0 & \frac{\dot{u}_{b,2} - 2y_3\dot{y}_3u_{b,2}}{(y_3^2+1)^2}\end{bmatrix} \qquad (39)$$

The desired trajectory is a line segment $q_d = [3\sin 0.2t, 3\sin 0.2t, 3\pi/4]^T$ with the corresponding $\xi_d = [3\sin 0.2t, 3\sin 0.2t, 1]^T$. By definition, $u_{d,1} = \zeta_1 = 0.6\cos 0.2t$ and $u_{d,2} = \xi_2 = 0$ under such circumstances. Clearly, Assumption A1) sustains with respect to the y_d and u_d here. For finding a proper B_s, we assume the availability of the nominal values of L and W, denoted by L_0 and W_0, respectively, such that

$$c_0L_0 \leq L, \text{ and } W \leq c_1W_0 \qquad (40)$$

with c_0, c_1 being known a priori. Select

$$B_s = \begin{bmatrix}-c_1W_0/\sqrt{y_3^2+1} & \frac{c_1W_0}{c_0L_0}(y_3^2+1)\\ -c_1W_0/\sqrt{y_3^2+1} & -\frac{c_1W_0}{c_0L_0}(y_3^2+1)\end{bmatrix} \qquad (41)$$

It is not hard to obtain that

$$\begin{aligned}x^TBB_sx &= 2c_1\frac{W_0}{W}(x_1^2 + \frac{L}{c_0L_0}x_2^2)\\ &\leq 2x^Tx,\end{aligned} \qquad (42)$$

which implies $\lambda_0 = 2$.

The adopted numerical values in this simulation are $k_0 = k_2 = 1$, $k_{z,2} = k_{z,3} = 1, k_a = 10, k_1 = 15, \Gamma_a = \mathrm{diag}[0.1]$, $\Gamma_b =$

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diag[0.2] $L = 1.0, L_0 = 0.5, W = 0.2, W_0 = 0.1, c_0 = 1.0, c_1 = 2$, and $w = 0.02$. The initial state is $q(0) = [-0.7, -0.7, 2.46]^T$. The tracking errors converge to zero in about thirty seconds, as can be seen in Fig. 2. To highlight the main achievement of our design, the calculated control torques, without and the switching algorithm, are depicted in Fig. 3 and Fig. 4, respectively. As can be seen, the peaks in the former case have been effectively suppressed by our scheme.

5. CONCLUSION

We have constructed a switching adaptive controller for a general nonholonomic mobile robot with both kinematic and dynamic parametric uncertainty. Sustained the assumptions A1) and A2), it ensures the asymptotic tracking stability and suppressing the singularity phenomenon at the same time. The numerical results in Section IV further justify this assertion.
Extension of such an approach to more general cases such as systems with both parametric and nonparametric uncertainty, etc., is quite challenging and is currently under our investigation.

6. ACKNOWLEDGEMENT

The work is supported in part by National Science Council,Taiwan, with contract number NSC 96-2221-E-238-019.

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