

## **ON THE SPECTRAL DISTRIBUTION OF PHOTONS BETWEEN PLANAR INTERFACES**

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Using a phenomenological approach to field quantization, an expression for the Keldysh function of photons between two planar interfaces (Casimir geometry) is found for any stationary quantum state of the two bodies. The case of one interface sliding against the other is considered in detail.

**Key words:** Casimir effect, planar interlace nonequilibrium, photon distribution

### **1. Introduction**

The simplest geometry in the electrodynamics of nanosystems is presumably the Casimir system: two half-spaces of different materials separated by a vacuum gap with two parallel planar interfaces. The gap has a width in the nano-range and provides the stage for propagating and evanescent waves emitted by the materials, thus shaping a rich plethora of Casimir physics: Casimir pressure, quantum friction, near-field radiative heat transfer, etc. All of these phenomena share a purely electrodynamic origin, and the basic theoretical quantity are the different components of Maxwell's stress tensor. This tensor mirrors the quantum states of the electromagnetic field in the gap, and the latter is determined by macroscopic properties of the boundaries like their temperature distribution, current density and the relative motion of the interfaces.

In the case of stationary macroscopic conditions, a general formula for the distribution function of photons (Keldysh function) in the gap was recently found in Ref.[1], based on tools from nonequilibrium quantum field theory. This function provides a rather general description of the system and contains quantities (called fluctuation sources) which are determined by the above-mentioned macroscopic conditions of the system boundaries. In Ref.[1], the fluctuation sources were expressed by Keldysh functions evaluated on the interfaces which are quite difficult to handle. In this paper we find an alternative form for the sources that links them to the number of excitations of the compound system electromagnetic field+bulk medium, using the phenomenological theory of quantization of the electromagnetic field in a dissipative medium [2]. We also give explicit expressions for the Keldysh functions of a single interface system for further applications.

To avoid overloading of the paper with formulas, our notation and definitions follow those of Ref.[1].

## 2. Preliminaries

We consider two half-spaces of different media with parallel and homogeneous boundaries (what we know about boundaries are their reflection coefficients) located at  $z = \pm a/2$  ( $v = \pm$ ). The boundaries are in stationary conditions: their temperatures are constant in time, and their relative motion is uniform and parallel to each other. We can then assume that the electromagnetic field (EMF) in the cavity  $[-a/2 \leq z \leq +a/2]$  is stationary in time and statistically homogenous in the  $xy$ -plane. As a consequence, all relevant fields and correlation functions can be expanded in Fourier integrals with respect to frequency  $\omega$  and wave vectors  $\mathbf{q} = (q_x, q_y)$  along the interfaces. We use in the following the shorthand  $\Omega = (\omega, \mathbf{q})$ . In the Dzyaloshinskii gauge where  $\varphi = 0$ , the transversality condition for the electric field can be used to eliminate the normal components of the vector potential in favor of the tangential ones (see [1] for details) which are conveniently expressed in the basis of s- and p-polarized fields of the lower interface (index  $\lambda = s, p$ ). The EMF in the gap is not a closed system because of dissipativity of the enclosing boundaries, and the straightforward procedure of field quantization in vacuum cannot be applied. So, for a quantum description of the EMF in the gap, we have to resort to the theories of quantization in dissipative media. There are many such theories [2-5], but we shall take the simplest version called phenomenological quantization [2] which recently found its justification in the frame of the canonical quantization rules of quantum field theory in the dissipative medium [6]. We begin with the splitting of the operator of vector potential  $\hat{\mathbf{A}}$  into positive and negative parts in Fourier space:

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \int \frac{d\mathbf{q}}{(2\pi)^2} \int_0^{+\infty} \frac{d\omega}{2\pi} [\hat{\mathbf{A}}(\Omega, z) e^{i(\mathbf{q}\mathbf{r} - \omega t)} + h. c.] \quad (1)$$

and then we postulate for the positive-frequency parts of the vector potential a representation in terms of current sources

$$\hat{\mathbf{A}}_\lambda(\Omega, z) = \sum_{\nu \lambda_1} D_{\lambda \lambda_1}^R(\Omega; z, \nu) \hat{\mathbf{I}}_{\lambda_1}(\Omega, \nu) \quad (2)$$

at any point  $z$  of the gap. The surface currents  $\hat{\mathbf{I}}_\lambda(\Omega, \nu)$  here ‘live’ on the two interfaces  $\nu = \pm$ .

In Eq.(2),  $\hat{D}^R$  is the retarded Green function (RGF) of the EMF defined as ( $\hbar = 1$ )

$$D_{\lambda \lambda'}^R(\mathbf{r}, t; \mathbf{r}', t') = -i\vartheta(t - t') \langle [\hat{A}_\lambda(\mathbf{r}, t), \hat{A}_{\lambda'}(\mathbf{r}', t')] \rangle. \quad (3)$$

The latter is a solution of an inhomogeneous wave equation with boundary conditions. The intensity of the radiation is distributed according to the Keldysh-Green function (KGF) which is a quantum expectation value of a symmetrized field correlation

$$D_{\lambda \lambda'}^K(\mathbf{r}, t; \mathbf{r}', t') = -i \langle \{ \hat{A}_\lambda(\mathbf{r}, t), \hat{A}_{\lambda'}(\mathbf{r}', t') \} \rangle. \quad (4)$$

To find RGF and KGF in Eqs.(3, 4), we need averages of the commutator and anticommutator of  $\hat{A}_\lambda$ [Eq.(2)] with its Hermitian conjugate  $\hat{A}_\lambda^+$ , and then we come to compute averages of the commutator and anticommutator of the surface currents  $\hat{I}_\lambda, \hat{I}_\lambda^+$  introduced in Eq.(2). Our key assumption at this point is that we claim locality of the latter, writing

$$\langle [\hat{I}_\lambda(\Omega, \nu), \hat{I}_{\lambda'}^+(\Omega', \nu')] \rangle = \delta(\Omega - \Omega') \delta_{\nu\nu'} c_{\lambda\lambda'}(\Omega, \nu), \quad (5.a)$$

$$\langle \{ \hat{I}_\lambda(\Omega, \nu), \hat{I}_{\lambda'}^+(\Omega', \nu') \} \rangle = \delta(\Omega - \Omega') \delta_{\nu\nu'} a_{\lambda\lambda'}(\Omega, \nu), \quad (5.b)$$

where we use the short-hand notation  $\delta(\Omega) \equiv (2\pi)^3 \delta(\mathbf{q}) \delta(\omega)$ . Using Eqs.(5.a, 2) and the definition (3) of RGF, we come to an expression for the RGF that has the same structure as Eq.(D9) of Ref.[1]. Comparison of these two expressions gives the relation

$$\vartheta(\omega) \hat{c}(\Omega, \nu) - \vartheta(-\omega) \hat{c}^T(-\Omega, \nu) = i \hat{\Gamma}^\nu(\Omega) \quad (6)$$

between the unknown matrix  $\hat{c}$  in the r.h.s. of Eq.(5.a) and the matrix  $\hat{\Gamma}^\nu$  of Eq.(3.26) in Ref.[1]. Taking into account the properties of  $\hat{\Gamma}^\nu(\Omega)$  (a symmetric matrix and an odd function of  $\Omega$ ), we find for  $\hat{c}$  in Eq.(6)

$$\hat{c}(\Omega, \nu) = i \hat{\Gamma}^\nu(\Omega) \quad (7)$$

With respect to the KGF (4), using Eqs.(1, 2), we can express it in the form of Eq.(4.21) of Ref.[1], where the photon sources  $\hat{P}^\nu(\Omega, \nu)$  are expressed by surface current anticommutators (5.b) according to

$$i \hat{P}(\Omega, \nu) = \vartheta(\omega) \hat{a}(\Omega, \nu) + \vartheta(-\omega) \hat{a}^T(-\Omega, \nu), \quad (8)$$

where  $-\Omega = (-\omega, \mathbf{q})$ . Using a quantization procedure, we will show in the coming two sections that the photon sources  $\hat{P}^\nu(\Omega, \nu)$  are proportional to  $\hat{\Gamma}^\nu(\Omega)$  matrices as well:

$$\hat{P}(\Omega, \nu) = \hat{\mathcal{N}}_\nu^f(\Omega) \hat{\Gamma}^\nu(\Omega), \quad (9)$$

and the coefficients  $\hat{\mathcal{N}}_\nu^f$  ( $\nu = \pm$ ) of proportionality are defined by excitation (occupation) numbers of the compound system electromagnetic field+body below the interface at  $z = \nu a/2$ .

### 3. Quantization on the interface in rest

There are two interfaces in the problem: the lower one is in at rest ( $\nu = -$ ) and the other one ( $\nu = +$ ) is in a state of parallel uniform motion in the x-direction, say. We begin our consideration with the lower interface which is taken as a reference frame of the problem.

We introduce the annihilation operator  $\hat{f}_{-\lambda}$  of a Bose field which is associated with the elementary excitations of the composed system EMF+lower medium in the following way:

$$\hat{I}_\lambda(\Omega, -) = \alpha_{\lambda\lambda_1}(\Omega, -)\hat{f}_{-\lambda_1}(\Omega). \quad (10)$$

In Eq.(10),  $\alpha$  is an as yet unknown matrix, and we suppose the canonical commutation rules for  $\hat{f}_{-\lambda}$

$$[\hat{f}_{-\lambda}(\Omega), \hat{f}_{-\lambda'}^+(\Omega')] = \delta_{\lambda\lambda'}\delta(\Omega-\Omega') \quad (11.a)$$

$$[\hat{f}_{-\lambda}(\Omega), \hat{f}_{-\lambda'}(\Omega')] = [\hat{f}_{-\lambda}^+(\Omega), \hat{f}_{-\lambda'}^+(\Omega')] = 0 \quad (11.b)$$

We restrict our consideration considering a quantum state for the lower surface which is diagonal in the chosen basis and characterized by the excitation number  $N_{-\lambda}^f(\Omega)$ :

$$\langle \hat{f}_{-\lambda}^+(\Omega)\hat{f}_{-\lambda'}(\Omega') \rangle = \delta_{\lambda\lambda'}\delta(\Omega-\Omega')N_{-\lambda}^f(\Omega) \quad (12)$$

Then, for the nonzero averages of commutator and anticommutator (5) of the surface currents on the lower interface, we have correspondingly

$$\hat{c}(\Omega, -) = \hat{\alpha}(\Omega, -)\hat{\alpha}^+(\Omega, -), \quad (13.a)$$

$$\hat{a}(\Omega, -) = \hat{\alpha}(\Omega, -)(\hat{I} + 2\hat{N}_-^f)\hat{\alpha}^+(\Omega, -), \quad (13.b)$$

where  $\hat{I}$  is a  $2 \times 2$  unit matrix and we introduced the diagonal matrix

$$\hat{N}_-^f \equiv \begin{pmatrix} N_{-,s}^f & 0 \\ 0 & N_{-,p}^f \end{pmatrix}. \quad (14)$$

Taking into account that  $\hat{\Gamma}^-$  in Eq.(7) is diagonal, we also suggest the diagonality of  $\hat{\alpha}(\Omega, -)$  in (13.a). This provides the anticommutator (13.b) and using Eq.(12), we find

$$\hat{a}(\Omega, -) = i[\hat{I} + 2\hat{N}_-^f(\Omega)]\hat{\Gamma}^-(\Omega). \quad (15)$$

Insertion of Eqs.(15, 13.b) into Eq.(8) gives us expression (9) for  $v = -$  where  $\hat{N}_-^f$  is defined in the same way as the photon number in free space [see Eq.(C4) of Ref.[1]]:

$$\hat{N}_-^f(\Omega) = \text{sign}\omega\hat{I} + 2[\vartheta(\omega)\hat{N}_-^f(\Omega) - \vartheta(-\omega)\hat{N}_-^f(-\Omega)]. \quad (16)$$

#### 4. Quantization on the sliding interface

Using the transformation law for the surface currents and the properties of Fourier transforms under the Lorentz transformation of the space-time coordinates, we find the positive frequency part of the surface current operator in the laboratory frame  $\hat{I}_\lambda(\Omega, +)$ . It is expressed via corresponding operators in the reference frame of the moving interface [see Eq.(D2) of Ref.[1]]

$$\hat{I}_\lambda(\Omega, +) = O_{\lambda\lambda'}(\Omega)[\vartheta(\omega')\hat{I}'_\lambda(\Omega', +) + \vartheta(-\omega')\hat{I}'_\lambda^+(-\Omega', +)]; \quad \omega \geq 0, \quad (17)$$

where  $\Omega' = (\omega', q'_x, q'_y)$  is related to the reference frame  $K'$  co-moving with the upper interface; it is connected with  $\Omega = (\omega, q_x, q_y)$  via the Lorentz transformation. The transformation matrix  $\widehat{O}(\Omega)$  in Eq.(17) is given in Eq.(D4) of Ref.[1].

Introducing a Bose field (with annihilation operator  $\hat{f}'_{+\lambda}$ ) of excitations in the rest frame of the upper body analogous to Eqs.(10, 11)

$$\hat{I}'_{\lambda}(\Omega'; +) = \alpha'_{\lambda\lambda_1}(\Omega', +)\hat{f}'_{+\lambda_1}(\Omega'), \quad (18.a)$$

we find for the surface current anticommutator in  $K'$

$$\langle \{\hat{I}'_{\lambda_1}(\Omega'_1, +), \hat{I}'_{\lambda_2}(\Omega'_2, +)\} \rangle = i\delta(\Omega'_1 - \Omega'_2) [\hat{I} + 2\widehat{N}'_+{}^f(\Omega'_1)] \hat{\Gamma}'^+(\Omega'_1). \quad (18.b)$$

And finally, using expression (17) for the positive-frequency part of the current operator in  $K$ , we find the anticommutator in the laboratory frame for  $\omega \geq 0$

$$\hat{a}(\Omega, +) = i\widehat{O}\widehat{N}'_+{}^f(\Omega')\hat{\Gamma}'^+(\Omega')\widehat{O}^T, \quad (19.a)$$

where

$$\widehat{N}'_+{}^f(\Omega') = \text{sign}\omega' \hat{I} + 2[\vartheta(\omega')\widehat{N}'_+{}^f(\Omega') - \vartheta(-\omega')\widehat{N}'_+{}^f(-\Omega')]. \quad (19.b)$$

Using Eq.(8) for  $v = +$  we find for the photon sources  $\widehat{P}(\Omega, +)$  on the moving interface the expression

$$\widehat{P}(\Omega, +) = \widehat{O}\widehat{N}'_+{}^f(\Omega')\hat{\Gamma}'^+(\Omega')\widehat{O}^T. \quad (20)$$

Taking into account the transformation law of  $\hat{\Gamma}^+$  [Eq.(D11) of Ref.[1]], we come to the expression (9) for  $v = +$  where the excitation number in the laboratory frame  $K$  is given by

$$\widehat{N}_+{}^f(\Omega) = \widehat{O}\widehat{N}'_+{}^f(\Omega')\widehat{O}^{-1}. \quad (21)$$

Because of the invariance of the trace of the matrix under the similarity transformation in Eq.(21), the number of excitations in  $K$  and  $K'$  are the same, and the relative motion only changes their polarization.

Obviously Eq.(20) recovers the result (7.3) of Ref.[1] in the case of thermal equilibrium in  $K'$ .

## 5. Examples of Keldysh Green functions

In this paragraph we give expressions for the KGF of Casimir system and for the single interface system for further applications.

The expression for KGF of the Casimir system with planar parallel boundaries is already written in Ref.[1], Eqs.(6.7–6.11). This is cumbersome enough not to repeat it here. We only give explicitly the matrices  $\hat{\gamma}_\nu$  defined in Eq.(6.11) of Ref.[1] which characterize the fluctuation sources in KGF of the Casimir system.

Insertion of Eq.(9) into the definition of  $\hat{\gamma}_\nu$ , we find after some algebra

$$\hat{\gamma}_- = e^{-\text{almq}_z} \hat{\mathcal{N}}_-^f \hat{\mathcal{R}}_- \hat{\Delta}_0 \quad (22.a)$$

for the sources in the lower interface and

$$\hat{\gamma}_+ = e^{-\text{almq}_z} (\hat{\mathbb{I}} + \hat{\mathcal{R}}_+) \hat{\Delta}_0 \hat{\mathcal{N}}_+^f \hat{\Delta}_0^{-1} (\hat{\mathbb{I}} + \hat{\mathcal{R}}_+)^{-1} \hat{\mathcal{R}}_+ \hat{\Delta}_0 \quad (22.b)$$

in the upper, moving one. Here, we have put (22.b)

$$\hat{\mathcal{R}}_\nu = \begin{cases} \hat{\mathbb{I}} - \hat{\mathcal{R}}_\nu \hat{\mathcal{R}}_\nu^* & \text{for propagating waves} \\ 2i\text{Im}\hat{\mathcal{R}}_\nu & \text{for evanescent waves,} \end{cases} \quad (23)$$

where propagating (evanescent) waves are defined by the observer-independent inequality  $q_z^2 = \omega^2 - q^2 > 0$  ( $q_z^2 < 0$ ).

We get the KGF for a single, moving body (located in  $z \geq 0$ ) by applying a limiting procedure where the two points  $z, z' \leq 0$  remain fixed, while the lower interface recedes to infinity (limit called (C) in Ref.[1]). This yields

$$\begin{aligned} \hat{D}_+^K(\Omega; z, z') = & \{(\hat{\mathbb{I}} + \hat{\mathcal{R}}_+) \hat{\Delta}_0 \hat{\mathcal{N}}_+^f \hat{\Delta}_0^{-1} (\hat{\mathbb{I}} + \hat{\mathcal{R}}_+)^{-1} \hat{\mathcal{R}}_+ e^{-i(q_z z - q_z^* z')} + \\ & + \vartheta(q_z^2) [\hat{\mathbb{I}} e^{iq_z z} + \hat{\mathcal{R}}_+ e^{-iq_z z}] \hat{\mathcal{N}}_-^f [\hat{\mathbb{I}} e^{-iq_z z'} + \hat{\mathcal{R}}_+^* e^{iq_z z'}] \} \hat{\Delta}_0, \end{aligned} \quad (24.a)$$

where  $\hat{\mathcal{N}}_-^f(\Omega)$  is now interpreted as the average number of ‘up-propagating’ photons that are incident on the moving body. If the upper interface is at rest, all matrices in (24.a) are diagonal, and we arrive at

$$\begin{aligned} \hat{D}_{0+}^K(\Omega; z, z') = & \{ \hat{\mathcal{N}}_+^f \hat{\mathcal{R}}_+ e^{-i(q_z z - q_z^* z')} + \\ & + \vartheta(q_z^2) [\hat{\mathbb{I}} e^{iq_z z} + \hat{\mathcal{R}}_+ e^{-iq_z z}] \hat{\mathcal{N}}_-^f [\hat{\mathbb{I}} e^{-iq_z z'} + \hat{\mathcal{R}}_+^* e^{iq_z z'}] \} \hat{\Delta}_0. \end{aligned} \quad (24.b)$$

Finally, letting both bodies recede to infinity, with  $z, z'$  kept finite [limit(A) of Ref.[1]], we find the KGF of the EMF in free space of the EMF in free space

$$\hat{D}_0^K(\Omega; z, z') = \vartheta(q_z^2) [\hat{\mathcal{N}}_+ e^{-iq_z(z-z')} + \hat{\mathcal{N}}_- e^{iq_z(z-z')}] \hat{\Delta}_0. \quad (24.c)$$

In the expressions (24),  $\hat{\mathcal{N}}_\pm$  are defining the number of free photons moving in opposite directions, which can be found in free space also by elementary plane wave quantization [Eq.(C4) of Ref.[1]].

## 6. Summary

We complete our considerations [1] of the photonic Keldysh function in the Casimir geometry of two plates by deriving explicit expressions for the fluctuation sources using phenomenological quantization of the electromagnetic field in a dissipative medium. We find, in particular, simple expressions for the Keldysh functions for a single interface system in an arbitrary stationary non-equilibrium state.

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