A New Variational Association Process for the Verification of Geometrical Specifications

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When this new association process of a datum is performed to verify a geometrical specification, measured points are considered as perturbations which generate modifications of the nominal geometry by variation of its location, orientation, and intrinsic dimensional characteristics, without requiring rotation and translation variables as the traditional methods usually do (Bourdet, et al., 1996, Advanced Mathematical Tools in Metrology II, World Scientific) with torsors or matrices. This new association process (Choley, 2005, Ph.D. thesis, Ecole Central, Paris; Choley, et al., 2006, Advanced Mathematical and Computational Tools in Metrology, VII, World Scientific) is based on both a reduced modeling of the geometry, taken out of the computer aided design system database, and a variational distance function. The whole measured points set influence is taken into account as an optimization criterion is applied (Bourdet and Clement, 1988, Ann. CIRP **37***(1), p. 503; Srinivasan, DIMACS Workshop on Computer Aided Design and Manufacturing, Rutgers University, NJ, October 7–9). Thus, the least squares optimization is achieved using the pseudo-inverse matrix, whereas the minimax optimization is treated with an algorithm developed by the Physikalisch-Technische Bundesanstalt and adapted for this purpose. In this paper, it is explained how this association process may be applied to planes and cylinders, used as single datum, datum systems, or common datum, with the least squares and minimax criteria.* [DOI: 10.1115/1.2432900]

Keywords: variational association, geometrical specification, datum, verification, minimax criterion, least squares criterion, pseudo-inverse matrix

1 Introduction

Computer aided design (CAD) systems allow us to define the nominal geometry as well as to specify the geometrical tolerancing for each constituting part (see Fig. 1) of a product. Once a reading of the geometrical specifications in the CAD system database is completed, a metrological analysis procedure checks to determine if the machined parts are in accordance with the designer's requirements. To this end, specified datum are associated with respect to the current ISO-GPS (Geometrical Product Specifications) standards [1].

2 The Modeling of the Datum

Instead of a Cartesian [6] or a parametric modeling [7], the variational association method [2,3] is based on a reduced modeling of the geometry, such as points, vectors, and intrinsic parameters to characterize the minimal reference geometric element (MRGE) of a technologically and topologically associated surfaces (TTRS) [8] or a geometrical product specifications (GPS) situation feature as proposed in ISO 17450 standard [9]. This reduced modeling is used:

- To locate the geometry with points;
- To orientate the geometry with vectors; and
- To dimension the geometry with intrinsic characteristics.

Some examples follow:

1. The modeling of a single datum (see Fig. 2):
   - The plane is characterized by its normal vector (orientation) and a point of the surface (position);
   - The cylinder is characterized by its axis vector (orientation), a point of the axis (position), and its radius (size);
   - The cone is characterized by its axis vector (orientation), an angle (intrinsic characteristic), and the apex (or a radius and the corresponding point of the axis) in order to define the location; and
   - The sphere is characterized by its center (location) and its radius (intrinsic characteristic of size).
2. The modeling of a datum system and a common datum:
   - In Fig. 3 the whole datum (common or system), made up of three orthogonal planes, belongs to the complex surfaces class [8]. It is characterized by the point defined as the intersection of the three planes, and the three normal vectors.

     This reduced modeling of a datum is not a minimal one as it may be remarked in this case: choosing one point and three vectors means 12 parameters which are not independents. Thus, some constraints must be taken into account, such as:
   - Relations between vectors (e.g., perpendicularity, parallelism); and
   - Relations on the norms of the vectors.
   - In Fig. 4 the whole datum (common or system), made up of two coaxial cylinders and one orthogonal plane, belongs to the revolute surfaces class [8]. It is characterized by a point, intersection of the plane and the axis, the common axis vector of the cylinders, which is also the normal vector of the plane, and the two radii.

To illustrate the previous case, the following example describes

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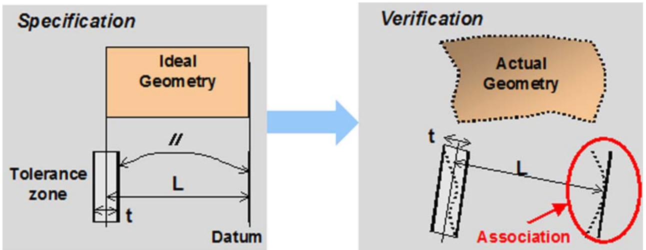

Fig. 1 The association of a datum for the verification of a GPS specification

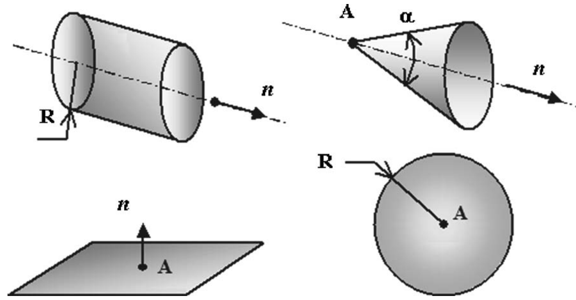

Fig. 2 The modeling of single datum

a gear unit (see Fig. 5) with a three-dimensional (3D) CAD model and a design drawing of the main casing of the unit with geometrical specifications.

In order to achieve the association for the chosen location specification, Fig. 6 described the modifications of the nominal geometry as follows:

- The normal and common axis vector $\boldsymbol{n}$ is modified with $\boldsymbol{dn}$, thus becoming $\boldsymbol{n}+\boldsymbol{dn}$;
- The common point **A** is also modified, thus becoming $\mathbf{A}+\mathbf{dA}$; and
- The two radii $\mathbf{R_a}$ and $\mathbf{R_c}$ become $\mathbf{R_a}+\mathbf{dR_a}$ and $\mathbf{R_c}+\mathbf{dR_c}$.

3 Description of the Variational Association Process

3.1 A Variational Distance Function. A distance function (see Fig. 7) is defined to associate an ideal geometry to the actual geometry with the association process:

1. (O,x,y,z) is a unique reference system for the CAD model and the CMM;
2. $G(\boldsymbol{u})$ is an ideal geometry characterized by its parameter $\boldsymbol{u}$ (position, location, and intrinsic characteristics depending of its class); and
3. $E=\{M_i, i=1..n\}$ is a set of measured points the locations of which are defined by the vectors $\boldsymbol{Mi}=OMi$.

$F(Mi,G)$ is a variational distance function delivering the scalar, smallest Euclidian orthogonal distance between Mi and $G(\boldsymbol{u})$. It inherits of the parameter $\boldsymbol{u}$. Some examples are as follows

For a plane defined by

$$\boldsymbol{u}=\begin{bmatrix}\boldsymbol{n}\\ \boldsymbol{A}\end{bmatrix}$$

the distance function may be

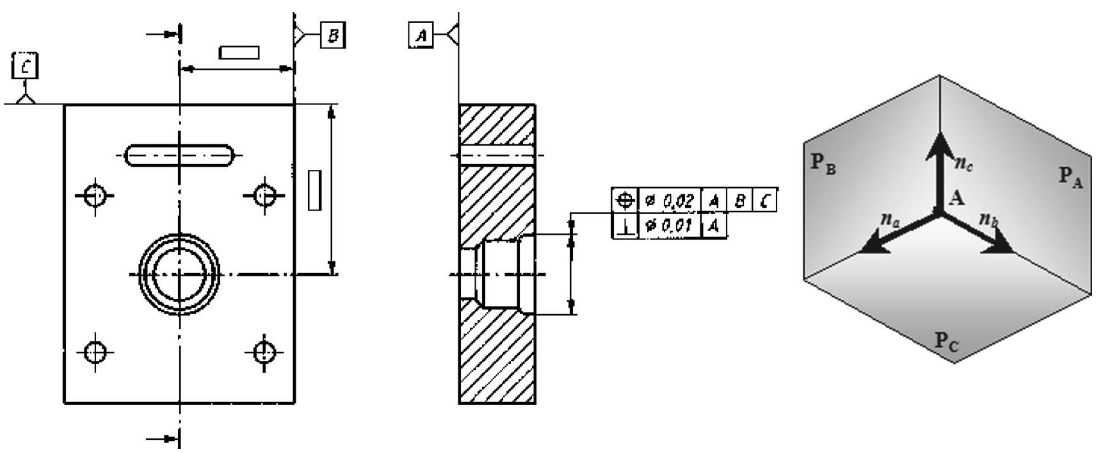

Fig. 3 The modeling of a common datum or datum system (three orthogonal planes)

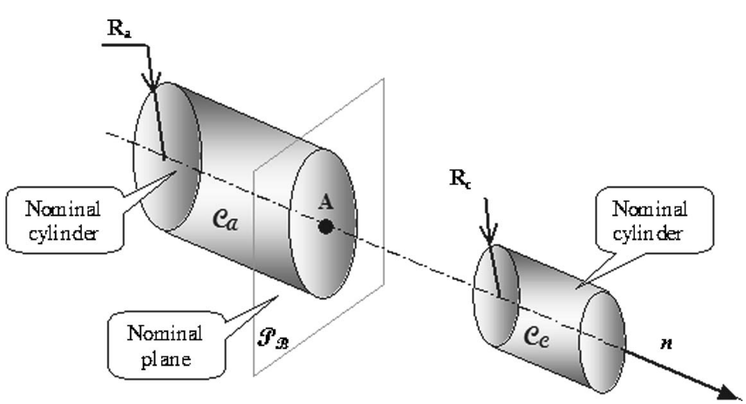

Fig. 4 The modeling of a common datum or a datum system (a plane and two cylinders)

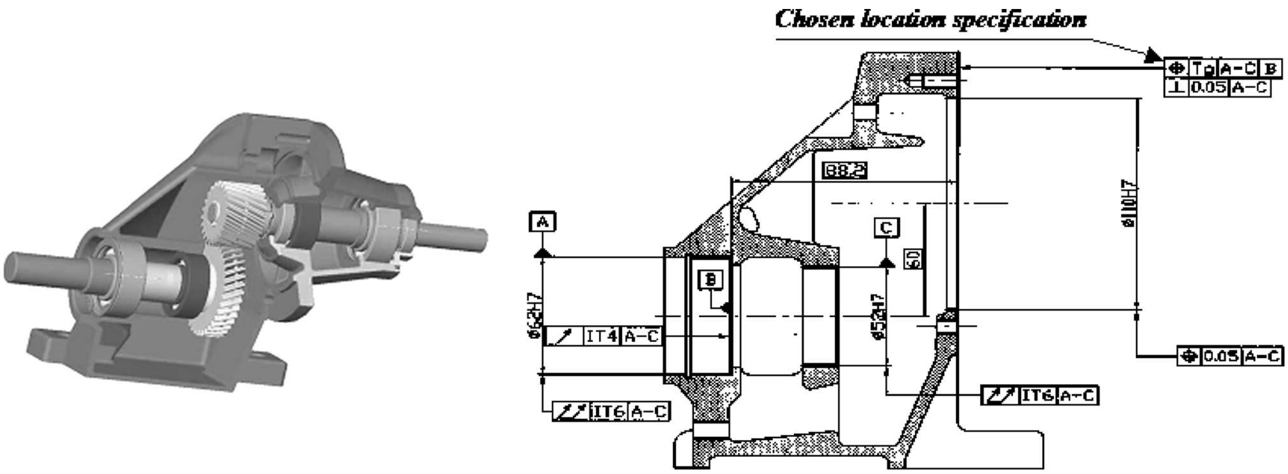

Fig. 5 CAD model and main casing design drawing of a gear unit

$$F(M,G) = AM \cdot \frac{\boldsymbol{n}}{|\boldsymbol{n}|} \tag{1}$$

For a cylinder defined by

$$\boldsymbol{u} = \begin{bmatrix} \boldsymbol{n} \\ A \\ R \end{bmatrix}$$

the distance function may be

$$F(M,G) = |(AM - (AM \cdot \boldsymbol{n})\boldsymbol{n})| - R \tag{2}$$

3.2 The Association Process. This association process relies on the variational distance function and is described by Fig. 8.

For a given measured point M_i, Eq. (3) describes the process, which can be linearized as shown in Eqs. (4) and (5)

$$F[M_i, G(\boldsymbol{u})] = F[M_i, G(\boldsymbol{u_0} + \boldsymbol{du})] \tag{3}$$

$$F[M_i, G(\boldsymbol{u})] = F[M_i, G(\boldsymbol{u_0})] + dF[M_i, G(\boldsymbol{u_0})] \tag{4}$$

with

$$dF[M_i, G(\boldsymbol{u_0})] = \sum_{j=1}^{j=m} \frac{\partial F[M_i, G(\boldsymbol{u_0})]}{\partial u_j} du_j \tag{5}$$

Equation (6) permits us to formulate the process as a system of equations that takes into account all the measured points of the actual geometry. J is the Jacobian matrix of the variational distance function for parameter $\boldsymbol{u_0}$

$$\boldsymbol{F} = \boldsymbol{F_0} + J\,\boldsymbol{du} \tag{6}$$

4 Optimization of the Association

In order to define the associated geometry, it is necessary to determine $\boldsymbol{du}$ while minimizing the norm of the optimized dis-

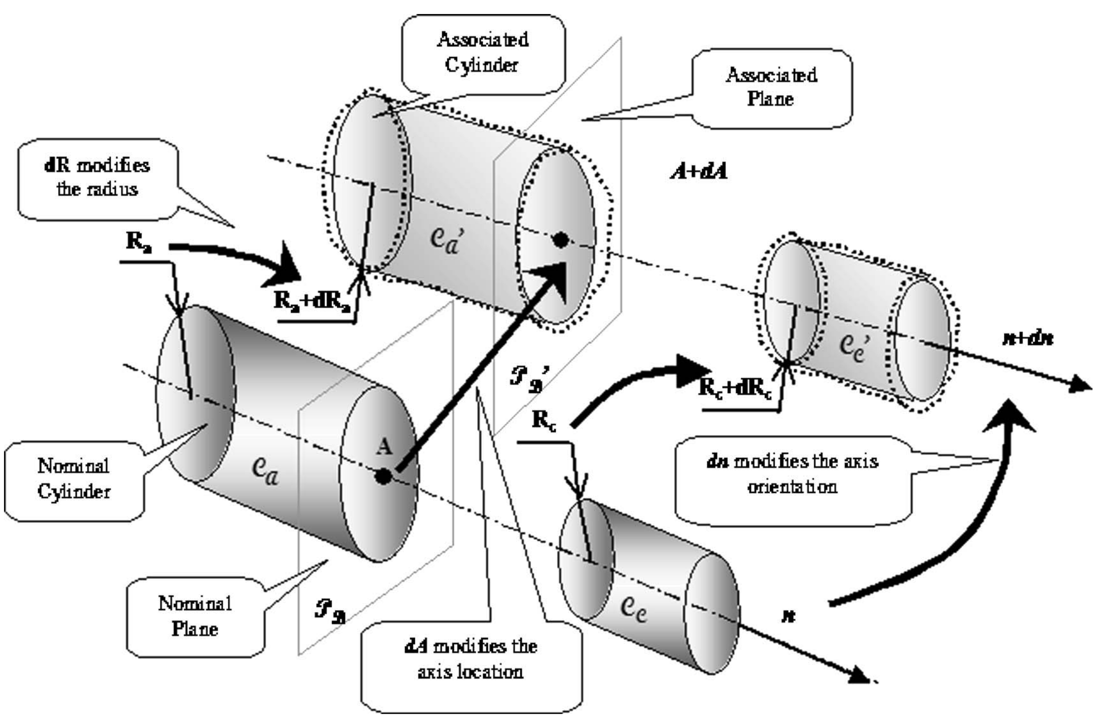

Fig. 6 The variational association process for the chosen location specification

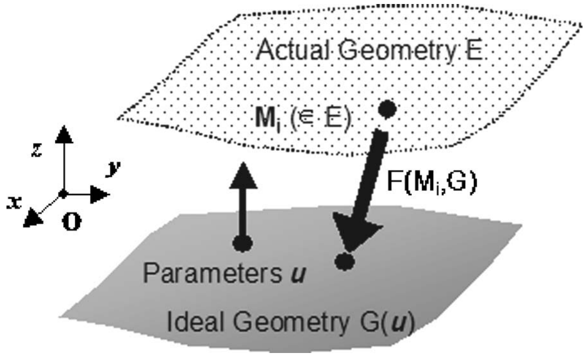

Fig. 7 Distance function between the actual geometry and the ideal geometry

tances $\boldsymbol{F_0}+J\,\boldsymbol{du}$. Thus, the least squares optimization is achieved using the pseudo-inverse matrix, whereas the minimax optimization is treated with an algorithm developed by the Physikalisch-Technische Bundesanstalt (PTB) [6] and adapted for this purpose.

4.1 Optimization With the Least Squares Criterion. The optimization problem is expressed by Eq. (7), Eq. (8) gives its formal solution, with J^+ being the pseudo-inverse matrix that permits to solve the problem while minimizing the $L2$ norm [4,5] of $\boldsymbol{F_0}+J\,\boldsymbol{du}$

$$J\,\boldsymbol{du} = -\boldsymbol{F_0} \tag{7}$$

$$\boldsymbol{du} = -J^+\boldsymbol{F_0} \tag{8}$$

If an additional constraint—Eq. (9)—needs to be taken into account, the use of a Lagrangian multiplier—Eq. (10)—is recommended. The optimized constrained solution is then obtained from the unconstrained one as described in Eq. (11)

$$\boldsymbol{c}^T\boldsymbol{du} = k \tag{9}$$

$$\lambda = \frac{k - \boldsymbol{c}^T\boldsymbol{du}}{1 + \boldsymbol{c}^T(J^TJ)^{-1}\boldsymbol{c}} \tag{10}$$

$$\boldsymbol{du}' = \boldsymbol{du} + \lambda(J^TJ)^{-1}\boldsymbol{c} \tag{11}$$

4.2 Optimization With the Minimax Criterion. Applying the PTB combinatory method [6] implies that the "tangent outside material" and the "tangent inside material" geometries are simulated as shown in Fig. 9. The t parameters allow us to allocate and to permute the contacting points on each side of the "nonideal simulated" geometry, in order to perform the combinatory process based on essential subsets of points

$$F[M_k, G(\boldsymbol{u})] = t_k \,.\, e \tag{12}$$

$$dF[M_k, G(\boldsymbol{u_0})] - t_k \,.\, e = -F[M_k, G(\boldsymbol{u_0})] \tag{13}$$

The system of equations has to be written as follow, with $\boldsymbol{dv}$ being $\boldsymbol{du}$ complemented with the thickness parameter e, and A being the modified Jacobian matrix with an additional column for the t parameters

$$A\,\boldsymbol{dv} = -\boldsymbol{F_0} \tag{14}$$

As long as essential subsets of points [6] are processed, A is a square matrix of maximal rank, thus easily invertible. For each subset, it is then possible to find the solution that minimizes e, allowing us to carry on with the PTB algorithm (see Fig. 10).

5 The Process Applied to Datums Systems and Common Datums

The variational treatment of a common datum or a datums system implies building the matrix J (least squares criterion) and A (minimax with PTB algorithm) with different distance functions relative to each single surface included in the datum. In order to respect the ISO standards, the association process needs a "one shot" solving in the case of a common datum, while it has to remain a sequential solving in the case of a datums system.

5.1 A Common Datum With Least Squares Criterion. Figure 11 displays the structure of the matrix J for a common datum made of three orthogonal planes A, B, and C.

1. With one measured point on A, m measured points on B and n measured points on C, the matrix is built with $1+m+n$ lines; and
2. Since the parameters are the components of the three normal vectors and the coordinates of the common point, there are

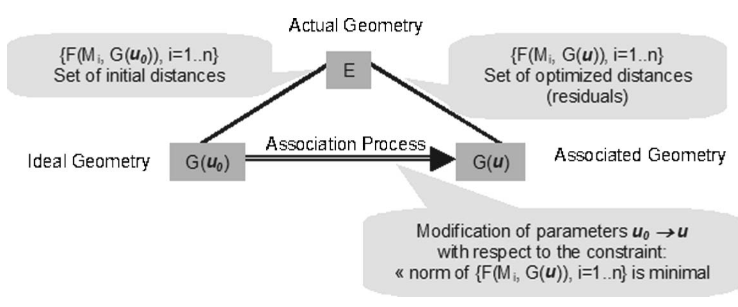

Fig. 8 The variational association process

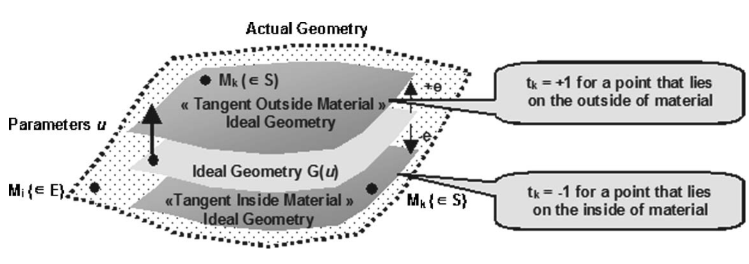

Fig. 9 The "nonideal simulated" geometry

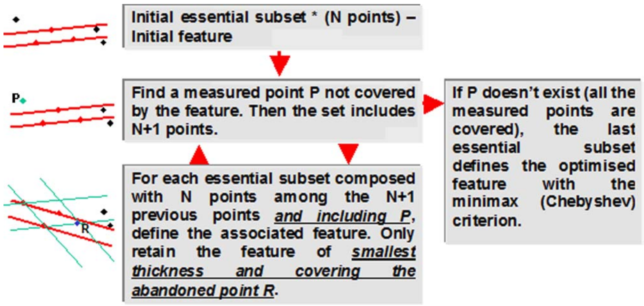

Fig. 10 The PTB combinatorial algorithm

12 columns in matrix A.

The association process with the least squares criterion implies that:

1. Three constraints are taken into account with three Lagrangian multipliers, in order to assure that the vectors remain orthogonal; and
2. Three others constraints are taken into account with three Lagrangian multipliers, in order to preserve the norm of each vector equal to 1.

Since the matrix is not directly invertible, the optimization process uses the pseudo-inverse matrix, with SVD decomposition if necessary. An offset is applied to the result in order to be "tangent outside material."

5.2 A Datums System With Minimax Criterion. Figure 12 displays the structure of matrix A in the case of a datums system (see Fig. 4) made of three orthogonal planes: A (primary datum), B (secondary datum), and C (tertiary datum). Additional geometrical constraints are added, thus allowing the matrix to be invertible. The optimization criteria is the minimax, with PTB algorithm. Since the datum is a datum system:

1. Four lines of the matrix correspond to four measured points on A. Thus, the primary datum is completely defined;
2. Three lines of the matrix correspond to three measured points on B. These define the secondary datum, along with a perpendicularity constraint between B and A; and
3. Two lines of the matrix correspond to two measured points on C. These define the tertiary datum, along with two per-

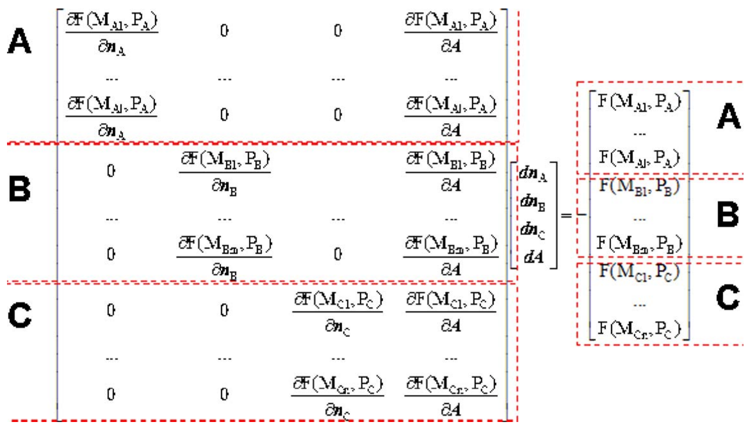

Fig. 11 Matrix *J* for a common datum, three orthogonal planes, least squares criterion

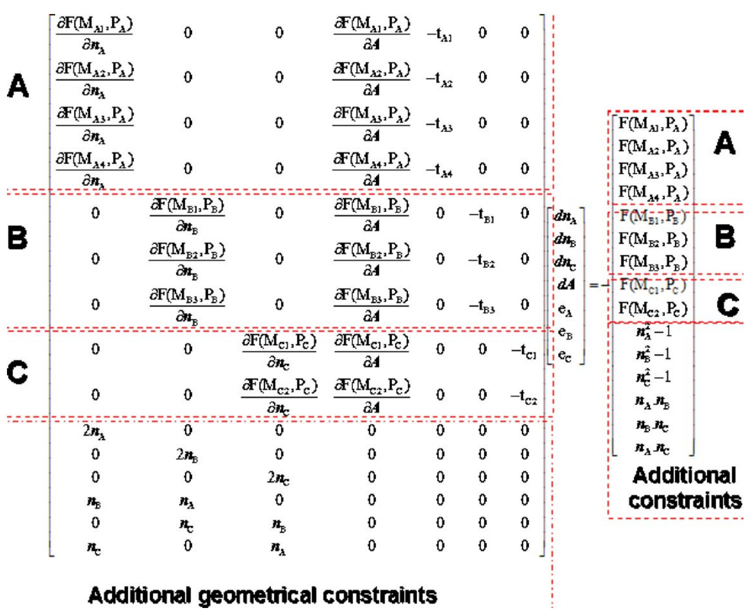

Fig. 12 Matrix *A* for a datum system, three orthogonal planes, minimax criterion

pendicularity constraints between C and A, and between C, and A.

The association process with the minimax criterion implies that:

1. Three constraints are taken into account with three more lines in the matrix, in order to assure that the vectors remain orthogonal. For the vectors $\boldsymbol{n}_{\mathbf{A}}$ and $\boldsymbol{n}_{\mathbf{B}}$, the constraint is
$$(\boldsymbol{n}_A + \boldsymbol{dn}_A) \cdot (\boldsymbol{n}_B + \boldsymbol{dn}_B) = 0 \tag{15}$$
that can be simplified (second order is neglected)
$$\boldsymbol{n}_A \,.\, \boldsymbol{dn}_B + \boldsymbol{n}_B \,.\, \boldsymbol{dn}_A = -\,\boldsymbol{n}_A \,.\, \boldsymbol{n}_B \tag{16}$$
2. Three others constraints are taken into account with three more lines in the matrix, in order to keep the norm of each vector equal to 1. For the vector $\boldsymbol{n}_A$, the constraint is
$$(\boldsymbol{n}_A + \boldsymbol{dn}_A) \,.\, (\boldsymbol{n}_A + \boldsymbol{dn}_A) = 1 \tag{17}$$
that can be simplified (second order is neglected)
$$2\boldsymbol{n}_A \,.\, \boldsymbol{dn}_A = 1 - \boldsymbol{n}_A^2 \tag{18}$$
Since the matrix is directly invertible (15 columns and 15 lines, maximal rank), the PTB algorithm can be used. An offset is applied to the result in order to be “tangent outside material.”

6 Conclusion and Perspectives

This new association process has been tested on single datum, common datum, and datum systems defined with cylinders and planes, using least squares and minimax criteria. Based on a unique common mathematical modeling and treatment, it can be applied to compare the association criteria used on different types of datum. It still needs to be extended to other geometrical features, such as cone, torus, and complex surfaces, as well as to other criteria (minimum circumscribed, maximum inscribed, both easily derived from the minimax algorithm…).

Since this association process does not use geometric transformations such as rotations and translations, it may also be applied to multiphysic systems (e.g., mechatronic systems) that conjugate geometry, mechanisms, electronics, electromagnetism, or even hydraulic, in order to evaluate the global behavior of a whole complex system (e.g., a system of systems). In this particular case, each multiphysic parameter (speed, torque, voltage, intensity, etc.) can be treated as an intrinsic parameter is treated for the modification of the geometry (radius, angle, etc.).

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