Field Determination of the Hydraulic Properties of Leaky Multiple Aquifer Systems

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Abstract. A new field method is proposed for determining the hydraulic properties of aquifers and aquitards in leaky systems. Conventional methods of analyzing leaky aquifers usually rely on drawdown data from the pumped aquifer alone. Such an approach is not sufficient to characterize a leaky system; our new method requires observation wells to be placed not only in the aquifer being pumped but also in the confining layers (aquitards) above and/or below. The ratio of the drawdown in the aquitard to that measured in the aquifer at the same time and the same radial distance from the pumping well can be used to evaluate the hydraulic properties of the aquitard. The new method is supported by theory and has been applied to the coastal groundwater basin of Oxnard, California. The field results are in good agreement with laboratory measurements.

Traditionally, groundwater hydrologists have tended to focus their attention on the more permeable aquifer layers of a groundwater basin in developing water supplies. However, sedimentary groundwater basins usually consist of a series of aquifers separated by confining layers of relatively low permeability, which may act as conduits for the vertical migration of water from one aquifer to another. Since fine-grained sediments often tend to be much more compressible than associated coarse-grained aquifer materials, they also can release large quantities of water from storage and thereby increase the supply available to the aquifer. The combined effects of these phenomena are known as leakage.

Usually, when the effects of leakage can be detected by observing drawdown in the aquifer being pumped, the confining beds are called 'aquitards,' and the aquifer is referred to as being 'leaky.' When such effects cannot be easily detected in the aquifer, the confining beds are called 'aquicludes,' and the aquifer is termed 'slightly leaky' [*Neuman and Witherspoon*, 1968].

Aquitards play an important role in the hydrology of multiple aquifer systems, and we shall mention here only a few examples. Although groundwater recharge is often believed to occur in areas of aquifer outcrops, *Gill* [1969] has recently reported that substantial amounts of water produced from the Potomac-Raritan-Magothy aquifer system are coming through the aquitards. Earlier, *Walton* [1965] had shown how the Maquoketa formation in Illinois, which is essentially a shale bed, serves as an effective transmitter of water between aquifers. Land subsidence in the San Joaquin Valley and other areas in California has been shown to be associated with water withdrawal from multiple aquifer systems and is generally attributed to the resulting compaction of fine-grained aquitard sediments [*Poland and Davis*, 1969]. Similar situations exist in Venice, Japan, and other parts of the world.

For the past 20 years, aquifers at depths below 500 feet have been used for storing natural gas in the United States and Europe. Where the properties of the aquitards were not properly investigated, the gas industry has on occasion witnessed the spectacular and dangerous effects of gas leakage. The storage of other fluids, as well as the disposal of waste products underground, requires the role of aquitards to be thoroughly understood if the degradation of groundwater supplies and the pollution of the surface environment are to be avoided. The role of aquitards may also be important in determining the rate at which the seawater from a degraded aquifer may migrate vertically to an uninvaded zone. An interesting situation in which the effectiveness of aquitards in preventing seawater intrusion is largely unknown occurs where the construction of shallow harbors and marinas requires the removal of a part of the aquitard that normally provides a natural barrier between the ocean and the freshwater aquifer beneath [*California Department of Water Resources,* 1971, p. 10].

Although the importance of aquitards is being recognized more and more, there is no reliable method for their investigation, and very little is known about their hydraulic properties. This report describes an improved field method for evaluating the hydraulic properties of aquifers and aquitards in leaky multiple aquifer systems. The new approach is simple to use and applicable to a wide range of hydrogeological situations. We shall describe in detail one particular investigation performed in the coastal groundwater basin of Oxnard, California.

PROBLEMS IN ANALYZING PUMPING TESTS WITH CURRENT METHODS

In analyzing results of water pumping tests the well-known *Theis* [1935] solution is often used to determine the permeability and the specific storage of the aquifer under investigation. As long as the aquitards do not leak significant amounts of water into the aquifer, this method of analysis produces reliable results.

However, groundwater hydrologists noted many years ago that deviations from the aquifer behavior, as predicted by the Theis solution, are not uncommon. These deviations are often caused by water leaking out of the confining beds, and this led to the 'leaky aquifer' theory of *Hantush and Jacob* [1955]. This theory and its later modifications [*Hantush,* 1960] relied only on an examination of aquifer behavior and attempted to relate such behavior to the properties of the adjacent aquitards.

Unfortunately, this approach has not been entirely satisfactory. As has recently been pointed out by *Neuman and Witherspoon* [1969*b*], field methods based on the leaky aquifer theory of *Hantush and Jacob* [1955] may often lead to significant errors. These errors are such that one tends to overestimate the permeability of the aquifer and underestimate the permeability of the confining beds. Under some circumstances, one may also get the false impression that the aquifer is inhomogeneous. Furthermore, the method does not provide a means of distinguishing whether the leaking beds lie above or below the aquifer being pumped.

A new theory of flow in multiple aquifer systems has recently been developed by *Neuman and Witherspoon* [1969*a*; *California Department of Water Resources,* 1971, pp. 24–38]. This theory shows that the behavior of drawdown in each layer is a function of several dimensionless parameters β_{ij} and r/B_{ij}, which depend on the hydraulic characteristics of the aquitards as well as those of the aquifers. The new theory clearly indicates that the observation of drawdown in the pumped aquifer alone is not always sufficient to determine uniquely the values of β and r/B. For example, *Hantush's* [1960] modified theory of leaky aquifers provides an analytical solution in terms of β that we know is applicable at sufficiently small values of time. Nevertheless, since this solution relates only to drawdown in the aquifer being pumped, its usefulness in determining uniquely the properties of each aquitard or even in determining a unique value of β is very limited [*California Department of Water Resources,* 1971, p. 327; *Riley and McClelland,* 1970]. Our theory indicates that one should be able to develop improved methods of analysis by installing observation wells not only in the aquifer being pumped but also in the confining layers enclosing it. Indeed, as will be shown later, a series of observation wells in more than one layer is a prerequisite for any reliable evaluation of aquitard characteristics.

The idea of placing observation wells in a low permeability layer (aquiclude) overlying a slightly leaky aquifer was originally proposed by *Witherspoon et al.* [1962] in connection with the underground storage of natural gas in aquifers. Their purpose was to determine how effective a given aquiclude would be in preventing gas leakage from the intended underground storage reservoir. Using results obtained from a finite difference simulation model, Witherspoon et al. were able to suggest a method for evaluating the hydraulic diffusivity of an aquiclude by means of a pumping test.

Later, a theoretical analysis of flow in aquicludes adjacent to slightly leaky aquifers was developed by *Neuman and Witherspoon* [1968]. This theory led to an improved method for determining the hydraulic diffusivity of aquicludes under slightly leaky conditions [*Witherspoon and Neuman*, 1967; *Witherspoon et al.*, 1967, pp. 72–92]. Since the method relies on the ratio between drawdown in the aquiclude and drawdown in the pumped aquifer, it will henceforth be referred to as the 'ratio method.'

A method for evaluating the hydraulic diffusivity of an aquitard under arbitrary conditions of leakage, which also uses observation wells completed in the confining layer itself, was recently described by *Wolff* [1970]. In his analysis Wolff assumed that, at any given radial distance from the pumping well and at a sufficiently large value of time, one can represent drawdown in the pumped aquifer by a step function. Assuming also that drawdown in the unpumped aquifer remains 0, Wolff arrived at a set of type curves that he recommended for aquitard evaluation.

Although this method gave satisfactory results for the particular site investigated by Wolff, we think that the step function approach may lead to difficulties when it is applied to arbitrary multiple aquifer systems. Fundamentally, drawdown in the pumped aquifer cannot be reliably represented by a single step function unless a quasi-steady state is reached within a sufficiently short period of time. The quasi-steady state will be reached only if the transmissibility of the aquifer is large and if the observation wells are situated at relatively small radial distances from the pumping well. To minimize the effect of early drawdowns, Wolff's method further requires that the duration of the pumping test be sufficiently long and that the vertical distance between the pumped aquifer and the aquitard observation wells not be too small.

From our new theory of flow in multiple aquifer systems, we now know that at large values of time the results in the aquitard may be affected significantly by the influence of an adjacent unpumped aquifer, especially where the aquitard observation well has been perforated close to such an aquifer. Thus, although the single step function approach renders the method inapplicable at small values of time, the assumption of zero drawdown in the unpumped aquifer introduces an additional restriction at large values of time.

In the special case where the thickness of the aquitard is known, one can determine its diffusivity directly from the step function type curves without the need for graphical curve matching. Quite often, however, the effective thickness of the aquitard is unknown. For example, the aquitard may contain unidentified or poorly defined layers of highly permeable material that act as a buffer to the pressure transient and also as a source of leakage. Another possibility is that the aquitard is situated below the pumped aquifer and that its lower limit has never been adequately defined. Then the step function approach requires the graphical matching of aquitard drawdown data with *Wolff's* [1970] type curves.

However, the intermediate parts of these type curves are essentially parallel, and therefore they cannot be matched uniquely with field results. On the other hand, neither the early nor the late parts of the type curves can be used with confidence. Thus there may be a significant element of uncertainty when *Wolff's* [1970] method is applied to real field situations.

Since the currently available direct field methods appear to be limited in their application, there is an obvious need for a new approach that would enable one to determine the characteristics of multiple aquifer systems under a wide variety of field conditions. We shall attempt to demonstrate that a rational basis for such an approach is provided by our new theory of flow in multiple aquifer systems [*Neuman and Witherspoon*, 1969*a*]. We will start by showing that the ratio method, which we originally thought was limited in application only to aquicludes under slightly leaky conditions, can in fact also be used to evaluate the properties of aquitards under very leaky conditions.

APPLICABILITY OF THE RATIO METHOD TO LEAKY CONDITIONS

To develop a method for determining the hydraulic properties of aquitards, we shall first consider a two-aquifer system (Figure 1). A complete solution for the distribution of drawdown in such a system has been developed by *Neuman and Witherspoon* [1969*a*]. In each aquifer the solutions depend on five dimensionless parameters β_{11}, r/B_{11}, β_{21}, r/B_{21}, and t_{D_1}. In the aquitard the solution involves one additional parameter z/b_1'. This large number of dimensionless parameters makes it practically impossible to construct a sufficient number of type curves to cover the entire range of values necessary for field application. For a set of type curves to be useful, they are normally expressed in terms of not more than two independent dimensionless parameters.

One way to significantly reduce the number of parameters is to restrict the analysis of field data to small values of time. In particular, we want to focus our attention on those early effects that occur prior to the time when a discernible pressure transient reaches the unpumped aquifer. At such early times the unpumped aquifer does not exert any influence on the rest of the system, and therefore drawdowns are independent of the parameters β_{21} and r/B_{21}. Furthermore, the aquitard behaves as if its thickness were infinite, which simply means that the parameters r/B_{11} and z/b_1' also have no influence on the drawdown. Thus the resulting equation will depend only on β_{11}, t_{D_1}, and an additional parameter t_{D_1}'.

In the pumped aquifer, drawdown is then given by *Hantush*'s [1960] asymptotic equation [*Neuman and Witherspoon*, 1969*a*].

$$s_1(r, t) = \frac{Q_1}{4\pi t_1} \int_{1/4t_{D_1}}^{\infty} \frac{e^{-y}}{y} \operatorname{erfc}\left(\frac{\beta_{11}}{[y(4t_{D_1}y - 1)]^{1/2}}\right) dy \qquad (1)$$

In the aquitard the solution is

$$s_1'(r, z, t) = \frac{Q_1}{4\pi T_1} \int_{1/4t_{D_1}}^{\infty} \frac{e^{-y}}{y} \cdot \operatorname{erfc}\left(\frac{\beta_{11} + y(t_{D_1}/t_{D_1}')^{1/2}}{[y(4t_{D_1}y - 1)]^{1/2}}\right) dy \qquad (2)$$

Theoretically, (1) and (2) are limited to those small values of time that satisfy the criterion

$$t_{D_1} \leq 1.6\beta_{11}^{\ 2}/(r/B_{11})^4 \qquad (3)$$

In terms of real time this criterion may also be

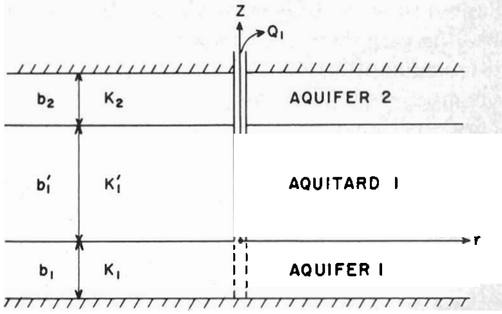


Fig. 1. A schematic diagram of a two-aquifer system.

expressed by

$$t \leq 0.1 S_{s_1}' b_1'^2 / K_1' \qquad (4)$$

indicating that the limiting value of time is independent of the radial distance from the pumping well.

From a practical standpoint the criterion given by (3) or (4) is overly conservative. For example, Figures 2–8 in *Neuman and Witherspoon* [1969*a*] reveal that the effect of the unpumped aquifer is felt in the rest of the system at times that are always greater than those predicted by (3). Note further that in these figures the effects of β_{21} and r/B_{21} are negligible as long as the log-log curve of drawdown versus time for the unpumped aquifer does not depart from its initial steep slope.

This effect of the unpumped aquifer provides a useful criterion for determining the time limit beyond which the asymptotic solutions may no longer be applicable. If an observation well can be provided in the unpumped aquifer, a log-log plot of drawdown versus time should enable the hydrologist to identify this time limit.

Note that there may be field situations in which the procedure above is not applicable. For example, when the transmissibility of the unpumped aquifer is large in comparison to that of the aquifer being pumped, drawdowns in the unpumped aquifer will be too small to measure, and one would not be able to determine the time limit as outlined above. This procedure may also fail when the water levels in the unpumped aquifer are fluctuating during the pumping test owing to some uncontrolled local or regional effect. Then a more conservative estimate of the time limit can be established from drawdown data observed in one of the aquitard wells. In general, the smaller the vertical distance between the perforated interval in the aquitard well and the boundary of the pumped aquifer is, the more conservative the time indicated by the procedure above is.

Having established a practical method for estimating the time within which (1) and (2) are valid, we can now proceed to show how these equations lead to the ratio method for evaluating aquitards. Remember that Hantush's equation does not by itself lead to a reliable method for determining a unique value of β_{11} from field results. The same can be said of (2), because it involves three independent parameters β_{11}, t_{D_1}, and t_{D_1}'. However, the usefulness of these two equations becomes immediately evident when one considers s_1'/s_1, i.e., the ratio of drawdown in the aquitard to that in the pumped aquifer at the same elapsed time and the same radial distance from the pumping well.

In the discussion that follows we shall be dealing with only one aquifer and one aquitard, and for the sake of simplicity we shall omit all subscripts. Figure 2 shows the variation of s'/s versus t_D' for a practical range of t_D and β values. Note that at $t_D = 0.2$ changing the value of β from 0.01 to 1.0 has practically no effect on the ratio s'/s. The same is true as t_D increases, and this relationship is shown by the additional results for $t_D = 10^4$.

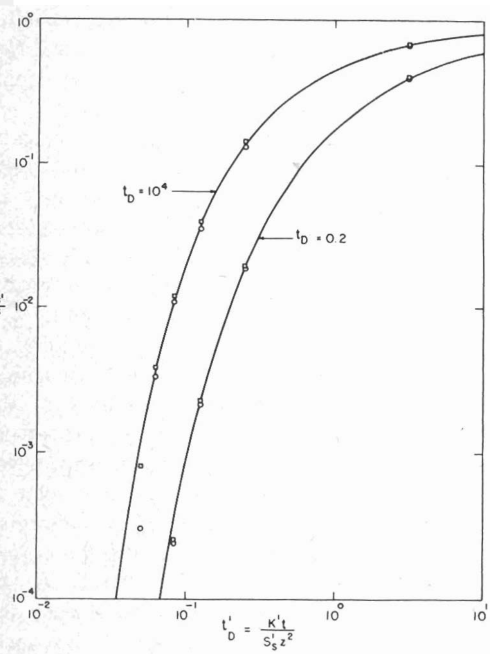


Fig. 2. The variation of s'/s with t_D' for $\beta = 0.0$ (solid lines), $\beta = 0.01$ (squares), and $\beta = 1.0$ (circles).

If we now use our theory for slightly leaky situations [*Neuman and Witherspoon,* 1968] where s' is given by

$$s'(r, z, t) = \frac{Q}{4\pi T}\frac{2}{\pi^{1/2}} \cdot \int_{1/(4t_D')^{1/2}}^{\infty} -\mathrm{Ei}\left(-\frac{t_D' y^2}{t_D(4t_D' y^2 - 1)}\right) e^{-y^2}\, dy \tag{5}$$

and s is obtained from the Theis solution, we have in effect the special case where $\beta = 0$. This is represented by the two solid lines in Figure 2.

We also examined the case where $\beta = 10.0$ and found that the values of s'/s deviate significantly from those shown in Figure 2. Thus one may conclude that for all practical values of t_D the ratio s'/s is independent of β as long as β is of order 1.0 or less. Since β is directly proportional to the radial distance from the pumping well, its magnitude can be kept within any prescribed bounds simply by placing the observation wells close enough to the pumping well. A quick calculation will show that distances of the order of a few hundred feet will be satisfactory for most field situations.

Thus we arrive at the very important conclusion that the ratio method, which we originally thought was restricted to only slightly leaky situations, can in effect be used to determine the hydraulic diffusivities of aquitards under arbitrary leaky conditions. We therefore decided to adopt the ratio method as a standard tool for evaluating the properties of aquitards.

USE OF THE RATIO METHOD IN AQUITARD EVALUATION

The ratio method can be applied to any aquifer and its adjacent aquitards, above and below, in a multiple aquifer system (see sketch in Figure 3). The method relies on a family of curves of s'/s versus t_D', each curve corresponding to a different value of t_D as obtained from (5) and the Theis equation. The curves in Figure 3 have been prepared from tables of values published previously by *Witherspoon et al.* [1967, Appendix G].

In the ratio method, one first calculates the value of s'/s at a given radial distance from the pumping well r and at a given instant of time t. The next step is to determine the magnitude of t_D for the particular values of r and t at which s'/s has been measured. When $t_D < 100$, the curves in Figure 3 are sensitive to minor changes in the magnitude of this parameter, and therefore a good estimate of t_D is desirable. When $t_D > 100$, these curves are so close to each other that they can be assumed to be practically independent of t_D. Then even a crude estimate of t_D will be sufficient for the ratio method to yield satisfactory results. A procedure for determining the value of t_D from drawdown data in the aquifer will be discussed later in connection with methods dealing with aquifer characteristics.

Having determined which one of the curves in Figure 3 should be used in a given calculation, one can now read off a value of t_D' corresponding to the computed ratio of s'/s. Finally, the diffusivity of the aquitard is determined from the simple formula

$$\alpha' = (z^2/t)t_D' \tag{6}$$

Note in Figure 3 that, when $s'/s < 0.1$, the value of t_D' obtained by the ratio method is not very sensitive to the magnitude of s'/s. As a result the value of α' calculated from (6) depends very little on the actual magnitude of the drawdown in the aquitard. Instead, the critical quantity determining the value of α' at a given elevation z is the time lag t between the start of the test and the time when the aquitard observation well begins to respond. The time lag is very important because in using the ratio method one need not worry about having extremely sensitive measurements of drawdown in the aquitard observation wells. A conventional piezometer with a standing water column will usually give sufficiently accurate information for most field situations. The time lag between a change in pressure and the corresponding change in water level in the column is usually so small in comparison to the time lag between the start of the test and this change in pressure that its influence can be safely ignored.

To evaluate the permeability and specific storage of an aquitard from its hydraulic diffusivity, one of these quantities must first be determined by means other than the ratio

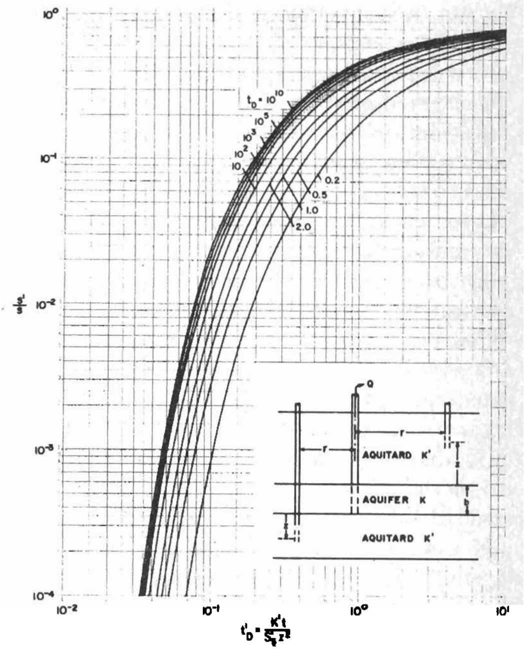


Fig. 3. The variation of s'/s with t_D' for a semi-infinite aquitard.

method. Experience indicates that permeability may vary by several orders of magnitude from one aquitard to another and even from one elevation to another in the same aquitard. A much more stable range of values is usually encountered when one is dealing with specific storage.

Recent field measurements in areas of land subsidence (F. S. Riley, personal communication, 1971) have shown that the specific storage of fine-grained sediments depends on the relationship between the load generated by pumping and the past history of loading. When this relationship is such that the sediments react elastically, the value of S_s' is relatively small. When the sediments are undergoing irreversible consolidation, the value of S_s' may be larger by 1 or 2 orders of magnitude. Presently, the most reliable measurements of S_s' are performed in the field by using borehole extensometers. Another way to determine approximate values of S_s' is to perform standard consolidation tests on core samples in the laboratory. In the total absence of field and laboratory measurements, S_s' can be estimated by correlating published results on similar sediments. Once the value of

S_s' has been determined, K' is easily calculated from $K' = \alpha' S_s'$.

We also studied the effects of aquitard heterogeneity and anisotropy on the value of K' obtained by the ratio method at a given elevation a. In our investigation we used the finite element method to examine the behavior of a two-aquifer system when: (1) the aquitard was a homogeneous anisotropic layer with a horizontal permeability as much as 250 times greater than the vertical and (2) the aquitard consisted of three different layers, each of which was homogeneous and anisotropic. The results of this study indicated that for homogeneous anisotropic aquitards the ratio method will always give a value of K' that corresponds to the vertical permeability of the aquitard. For a heterogeneous aquitard, K' is simply the weighted average vertical permeability over the thickness z. If there are N layers of thickness b^n and vertical permeability K_v^n inside this interval, K' represents the average value

$$K' = z \bigg/ \left(\sum_{n=1}^{N} \frac{b^n}{K_v^n} \right) \tag{7}$$

Boulton [1963] and *Neuman* [1972] have shown that, at early values of time, drawdown in an unconfined aquifer can safely be approximated by the Theis solution. At later values of time, drawdown is affected by the delayed response of the water table, and the effect is similar to that of leakage in a confined aquifer. Thus, if the ratio method is applicable to aquitards adjacent to confined leaky aquifers, it should also be applicable to situations in which the pumped aquifer is unconfined. This conclusion is further supported by the fact that the ratio method depends less on the actual values of drawdown in the aquifer than on the time lag observed in the aquitard. To test this applicability of the ratio method to an unconfined aquifer, we took data from *Wolff* [1970] for a pumping test in which observation wells were placed in a confining layer underneath a water table aquifer. We analyzed these data by using the ratio method, and the results are in excellent agreement with those obtained by Wolff.

When we showed that our slightly leaky theory was applicable to the so-called leaky aquifer, our previous discussion was restricted to a two-aquifer system. By now, however, the reader will recognize that such a restriction is not necessary and that the ratio method is actually applicable to arbitrary multiple aquifer systems. The only requirement is that the sum of the β_{ij} values with respect to the overlying and underlying aquitards be of order 1 or less.

In summary, note once again the following features of the ratio method.

1. The method applies to arbitrary, leaky multiple aquifer situations.
2. The pumped aquifer can be either confined or unconfined.
3. The confining layers can be heterogeneous and anisotropic. Then the ratio method gives the average vertical permeability over the thickness z of the aquitard being tested.
4. The method relies only on early drawdown data, and therefore the pumping test can be of relatively short duration.
5. The drawdown data in the unpumped aquifer or in the aquitard provide an in situ indication of the time limit at which the ratio method ceases to give reliable results.
6. Since the method is more sensitive to time lag than to the actual magnitude of s'/s, the accuracy with which drawdowns are measured in the aquitard is not overly critical.
7. The method does not require prior knowledge of the aquitard thickness.
8. The ratio method is simple to use and does not involve any graphical curve-matching procedures. This lack of curve-matching procedures is an advantage because curve matching is often prone to errors due to individual judgment and because a more reliable result can be obtained by taking the arithmetic average of results from several values of the ratio s'/s.

METHOD FOR EVALUATING AQUIFERS

When the pumped aquifer is slightly leaky, one can evaluate its transmissibility and storage coefficient by the usual procedures based on the Theis equation. When leakage is appreciable, these procedures will not always yield satisfactory results. Alternative methods for analyzing the results of pumping tests in leaky aquifers were proposed by *Jacob* [1946] and *Hantush* [1956, 1960]. Still another method based on the r/B solution has recently been proposed by *Narasimhan* [1968]. All these methods rely on drawdown data from the pumped aquifer alone.

Their purpose is to determine not only the properties of the aquifer but also the so-called 'leakage factors' r/B and β that depend on the characteristics of the confining layers as well as on those of the aquifer. We have shown earlier that these methods have a limited application and that they can often lead to erroneous results.

Since we have introduced the ratio method as a means for evaluating aquitards, the only remaining unknowns to be determined are the aquifer transmissibility T and the storage coefficient S. When the aquifer is leaky, the use of methods based on the Theis solution will lead to errors whose magnitudes are a function of β and r/B. A look at *Neuman and Witherspoon* [1969*a*] will reveal that the smaller the values of β and r/B are, the less the drawdowns in the pumped aquifer deviate from the Theis solution, and therefore the smaller the errors introduced by such methods are. At this point we must recognize that β and r/B do not necessarily reflect the amount of water that leaks into the aquifer. In fact, both these parameters are directly proportional to r, which simply means that their magnitude in a given aquifer varies from nearly 0 at the pumping well to relatively large values further away from this well. Thus the extent to which leakage can affect the behavior of the drawdown in any given aquifer is a function of the radial distance from the pumping well. Thus the closer one is to this well, the smaller the deviations of drawdown from the Theis curve are. On the other hand, the rate of leakage is obviously greatest near the pumping well where the vertical gradients in the aquitard are largest and diminishes as the radial distance from this well increases. Therefore, in a given system, β and r/B increase with radial distance, whereas the actual rate of leakage decreases.

At first glance, we seem to be faced with a paradox: The greater the leakage is, the less the deviations from the nonleaky Theis solution are. However, a closer examination of the flow system will show that there is a simple physical explanation for this phenomenon. The reader will recognize that, although vertical gradients in the aquitard do not vary appreciably with radial distance from the pumping well, the same cannot be said about drawdown in a pumped aquifer. As a result the rate of leakage per unit area relative to this drawdown is negligibly small in the immediate vicinity of the pumping well but becomes increasingly important at larger values of r. In addition, the water that leaks into the aquifer at smaller values of r tends to act as a buffer to the pressure transient. This transient cannot propagate as fast as it otherwise might have had there been no increase in aquifer storage. The effect is to reduce further the drawdown at points farther away from the pumping well. The net result is a situation in which larger values of r are associated with less leakage but also with greater deviations from the Theis curve.

Thus we arrive at the important conclusion that one can evaluate the transmissibility and storage coefficient of a leaky aquifer by using conventional methods of analysis based on the Theis solution. The errors introduced by these methods will be small if the data are collected close to the pumping well, but they may become significant when the observation well is placed too far away. Therefore a distance drawdown analysis based on the Theis curve is not generally applicable to leaky aquifers and should be avoided whenever possible.

Ideally, the values of T and S should be evaluated by using drawdown or buildup data from the pumping well itself because here the effect of leakage is always the smallest. We recommend this approach whenever the effective radius of the pumping well is known (e.g., wells in hard rock formations). However, when a well derives its water from unconsolidated materials, its effective radius usually remains unknown owing to the presence of a gravel pack. In these situations the approach above can still be used to evaluate T but cannot be used to determine S.

As a general rule, early drawdown data are affected by leakage to a lesser degree than data taken at a later time are. Therefore we feel that in performing the analysis most of the weight should be given to the earliest data available, if, of course, there is confidence in their reliability.

Once S and T have been determined, one can calculate the dimensionless time at any given radial distance from the pumping well by

$$t_D = Tt/Sr^2 \tag{8}$$

Equation 8 can then be used with the ratio method as we discussed earlier.

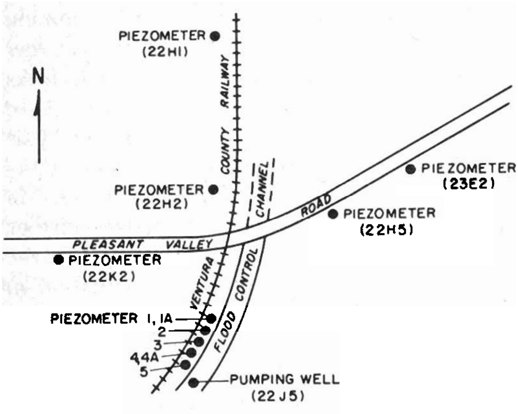


Fig. 4. The locations of the piezometers used in field pumping tests.

FIELD PUMPING TESTS IN THE OXNARD, CALIFORNIA, BASIN

The California Department of Water Resources had previously investigated the Oxnard basin in connection with seawater intrusion problems and constructed several wells at various locations in the basin. For our field studies we selected a particular location in the city of Oxnard where a large capacity pumping well (Figure 4, 22J5) was available to produce water from the Oxnard aquifer. Four additional piezometers (22H2, 22H5, 22K2, and 23E2) were available to monitor water levels in the Oxnard aquifer at radial distances of 502–1060 feet.

In addition, seven new piezometers were installed at various elevations relative to that of the Oxnard aquifer. Table 1 summarizes the vertical distances above or below the Oxnard for each piezometer and also gives the radial distances from pumping well 22J5. Ideally, the seven piezometers should have been arranged along a circular arc with its center at the pumping well so that responses would be given at various elevations but at only one unique value of r. However, this arrangement was not possible under the local conditions, and we therefore had to design the well field according to the scheme shown in Figure 4. For details of the construction, the completion, and the development methods, the reader is referred to *California Department of Water Resources* [1971, pp. 63–68].

The following is a brief description of the lithology in the vicinity of the test area. The semiperched zone is composed of fine- to medium-grained sand with interbedded silty clay lenses. The upper aquitard is made up of predominantly silty and sandy clays, mainly montmorillonite. The Oxnard aquifer, which is the most important water producer in the Oxnard basin, is composed of fine- to coarse-grained sand and gravel. Silty clay with some interbedded sandy clay lenses makes up the lower aquitard. The material that forms the Mugu aquifer is fine- to coarse-grained sand and gravel with some interbedded silty clay. Figure 5 shows an electric log through this series of sediments.

ANALYSIS OF PUMPING TEST RESULTS

Two pumping tests were performed in the field. Their purpose was to determine the hydraulic characteristics of the Oxnard aquifer and the confining layers above and below it and to confirm our theoretical concepts [*Neuman and Witherspoon*, 1969*a*] regarding the response of multiple aquifer systems to pumping.

The first pumping test lasted 31 days. Figure 6 shows the response in the Oxnard aquifer at

TABLE 1. Location of Piezometers

Piezometer	Distance from 22J5, feet	Depth, feet	Vertical Distance*, feet	Layer
1	100	120	...	Oxnard aquifer
1A	100	239	...	Mugu aquifer
2	91	225	−26	lower aquitard
3	81	205	−6	lower aquitard
4	72	95	+11	upper aquitard
4A	72	58.5	+50	semiperched aquifer
5	62	84	+22	upper aquitard

* The vertical distance is the distance above the top of the Oxnard aquifer at a depth of 105 feet or below the bottom at a depth of 198 feet.

† Failed to operate satisfactorily.

various radial distances from the pumping well. Piezometer 1, which is nearest to the pumping well, demonstrated an anomalous behavior during the first 6 min of pumping. This was apparently due to a surging effect in the pumping well. At about 6000 min the entire basin started experiencing a general drop in water levels probably due to the beginning of intermittent pumping for irrigation at this time of the year. Table 2 gives the values of T and S as calculated from these data by using *Jacob*'s [1950] semilogarithmic approach.

Table 2 shows that in general the values of T become progressively larger as r increases. This relationship can be explained as follows. Since the Oxnard aquifer is obviously leaky, the actual drawdown curve at any given well will lie below the Theis solution, as is shown diagrammatically in Figure 7. To demonstrate this positioning, we shall choose a particular point on the data curve that corresponds to some given value of s and t. If we could match the data to the true type curve where β and r/B are not 0, we would obtain the true value of s_{D_1} for the point chosen. However, such type curves were not available for this investigation, and we used a method that is essentially equivalent to matching the field data to the Theis curve. Therefore the field data are being shifted upward from their true position, and our chosen point will now indicate an apparent value of $s_{D_2} > s_{D_1}$.

From the definition of s_D it is clear that since s remains unchanged the value of T is increased. The greater the radial distance r, the larger β and r/B become, and therefore the larger the difference between the true type curve and the Theis curve is. In other words, as r increases, the magnitude of T should become more and more exaggerated, which is clearly evident in Table 2.

With regard to errors in S, the shifting of field data as indicated on Figure 7 may be either to the left or to the right. Thus the effect on the calculated values of S is not predictable (Table 2). With this unpredictability in mind, we decided to select the results from piezometer 1 of $T = 130{,}600$ gpd/ft and $S = 1.12 \times 10^{-4}$ as being most representative of the Oxnard aquifer, at least in the area of the pumping test.

Having estimated the properties of the pumped aquifer, we shall now consider the results from other parts of this three-aquifer

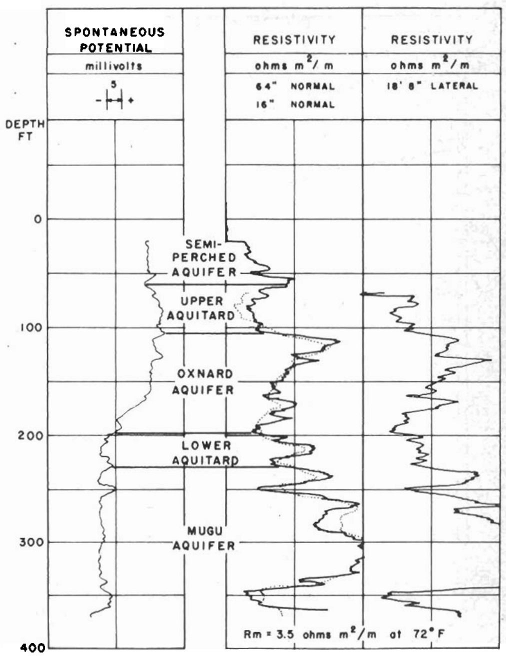


Fig. 5. The electric log from the first exploratory hole.

subsystem. Figure 8 shows the response at one particular point in the lower aquitard (well 3) as well as the responses in the Oxnard above (well 1) and the Mugu below (well 1A). Figure 9 shows the response at two different elevations in the upper aquitard (wells 4 and 5) as well as the response in the overlying semiperched aquifer (well 4A). Since piezometer 1 is located farthest from the pumping well, we do not have the response in the pumped aquifer directly below the piezometers where drawdowns in the upper aquitard were measured. However, from distance–drawdown curves in the Oxnard aquifer and from the behavior of piezometer 4, we concluded that the aquifer response was approximately as shown by the dashed curve in Figure 9. Remember that the ratio method for evaluating aquitards is more sensitive to the time lag than to the actual magnitude of drawdown in the aquifer. Therefore the dashed curve in Figure 9 can be considered sufficiently accurate for our purposes. Note that the shapes of the curves in Figures 8 and 9 are quite similar to those of our theoretical curves [*Neuman and Witherspoon*, 1969*a*].

To evaluate the lower aquitard, we shall determine the ratio s'/s at two early values of

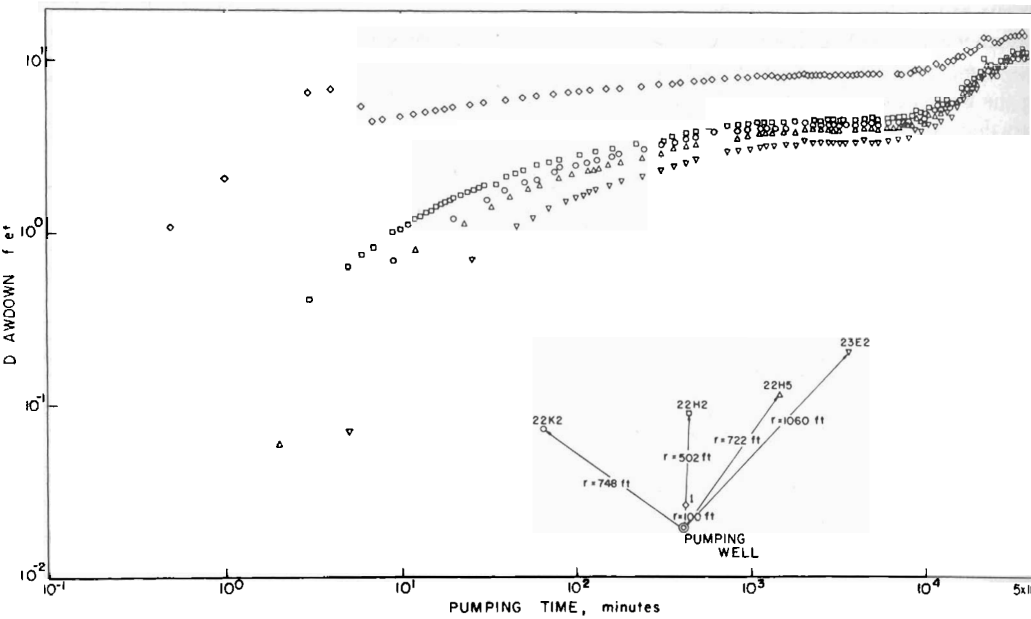


Fig. 6. The fluid levels in the Oxnard piezometers during the first pumping test. The diamonds represent well 1, the squares represent well 22H2, the triangles represent well 22H5, the circles represent well 22K2, and the inverted triangles represent well 23E2.

time, $t = 80$ min and $t = 200$ min. At $t = 80$ min, one can read on Figure 8 that $s' = 0.078$ and $s = 6.6$ feet. The ratio is simply $s'/s = 0.078/6.6 = 1.18 \times 10^{-2}$. To obtain t_D, one can use the equation

$$t_D = 9.28 \times 10^{-5} Tt/r^2S \quad (9)$$

where T is in gallons per day per foot, t is in minutes, and r is in feet. Then, using the known values of T and S and noting from Table 1 that, at piezometer 3, $r = 81$ feet, one can calculate

$$t_D = \frac{(9.28 \times 10^{-5})(130{,}600)(80)}{(81)^2(1.12 \times 10^{-4})} = 1.32 \times 10^3$$

TABLE 2. Results of Oxnard Aquifer Using Jacob's Semilog Method

Well	r, feet	T, gpd/ft	S
1	100	130,600	1.12×10^{-4}
22H2	502	139,000	3.22×10^{-4}
22H5	722	142,600	3.08×10^{-4}
22K2	748	136,700	2.48×10^{-4}
23E2	1060	157,000	2.53×10^{-4}

Referring to Figure 3, one finds that these values of s'/s and t_D correspond to $t_D' = 0.086$. From the definition of t_D', one can verify the formula

$$\alpha' = 1.077 \times 10^4 t_D' z^2/t \quad (10)$$

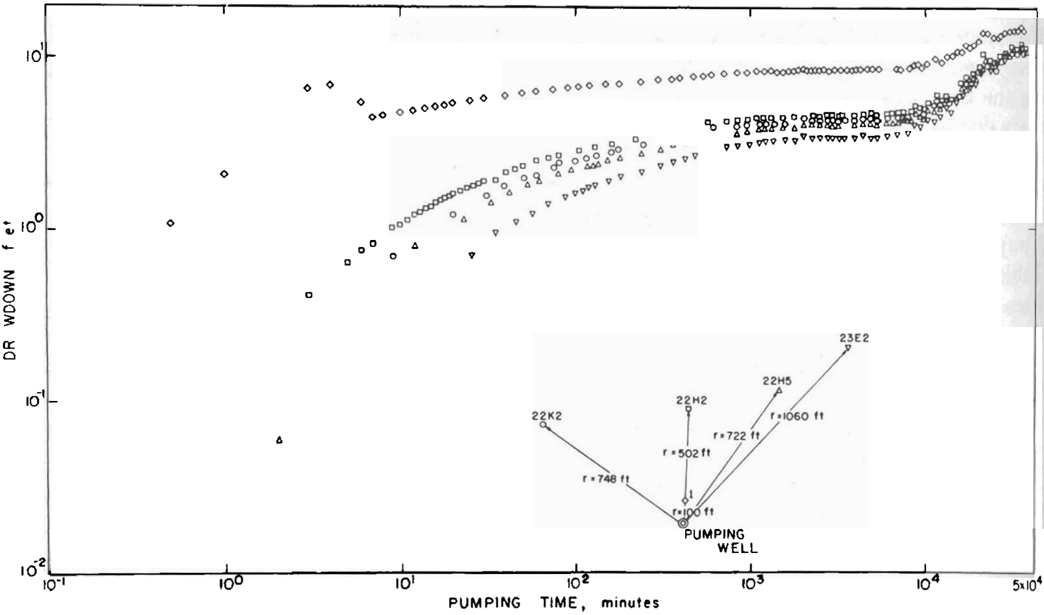
where α' is in gallons per day per foot, z is in feet, and t is in minutes. One notes from Table 1 that, for piezometer 3, $z = 6$ feet, and therefore

$$\alpha' = \frac{(1.077 \times 10^4)(0.086)(6)^2}{(80)} = 4.17 \times 10^2 \text{ gpd/ft}$$

Similarly, one finds that, at $t = 200$ min, $\alpha' = 3.39 \times 10^2$ gpd/ft. Since the method gives more reliable results when t is small, we adopted $\alpha' = 4.17 \times 10^2$ gpd/ft as the representative value for the top 6 feet of the lower aquitard. The results of similar calculations for both aquitards are summarized in Table 3. Note that the diffusivity of the Oxnard aquifer is

$$\alpha = \frac{T}{S} = \frac{130{,}600}{1.12 \times 10^{-4}} = 1.17 \times 10^9 \text{ gpd/ft}$$

which is more than 1 million times the values obtained for the aquitards.



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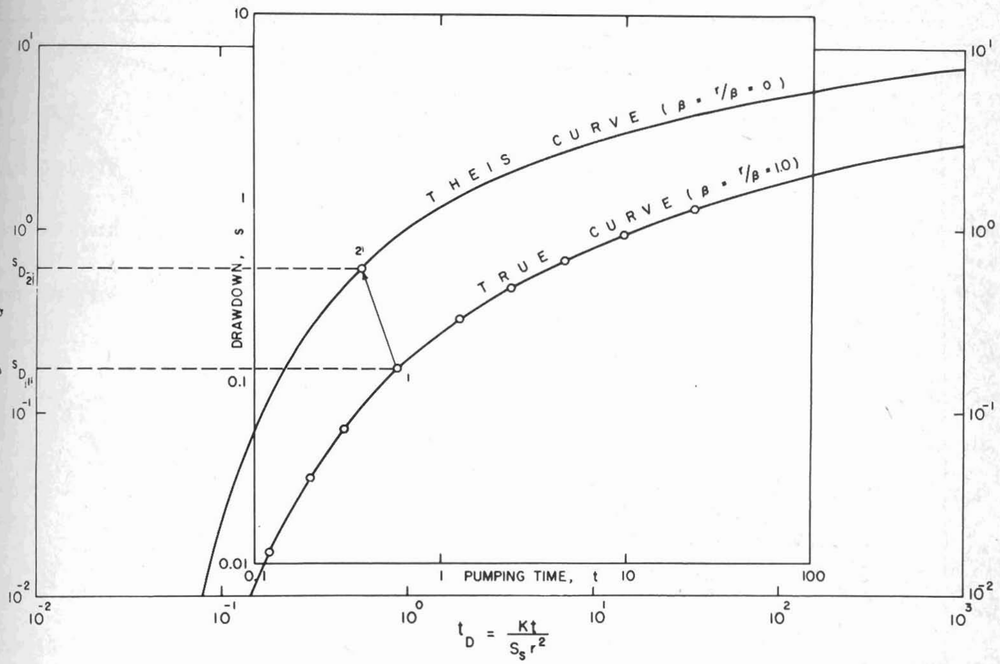

Fig. 7. A comparison of hypothetical field data with leaky and nonleaky type curves.

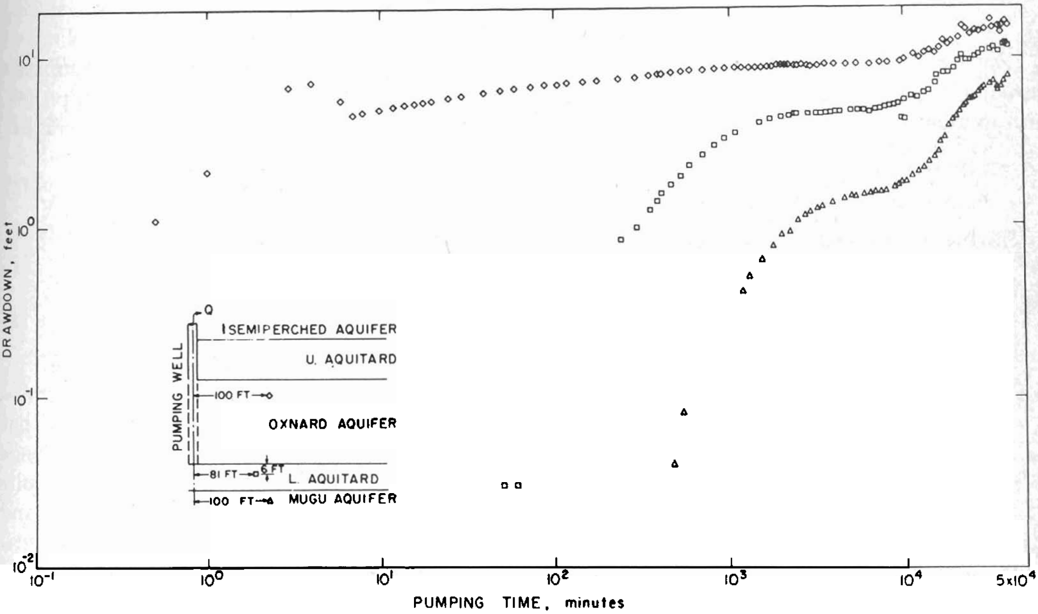

Fig. 8. The response of the piezometers in the lower aquitard (well 3, squares) to that in the Oxnard (well 1, diamonds) and Mugu (well 1A, triangles) aquifers during the first pumping test.

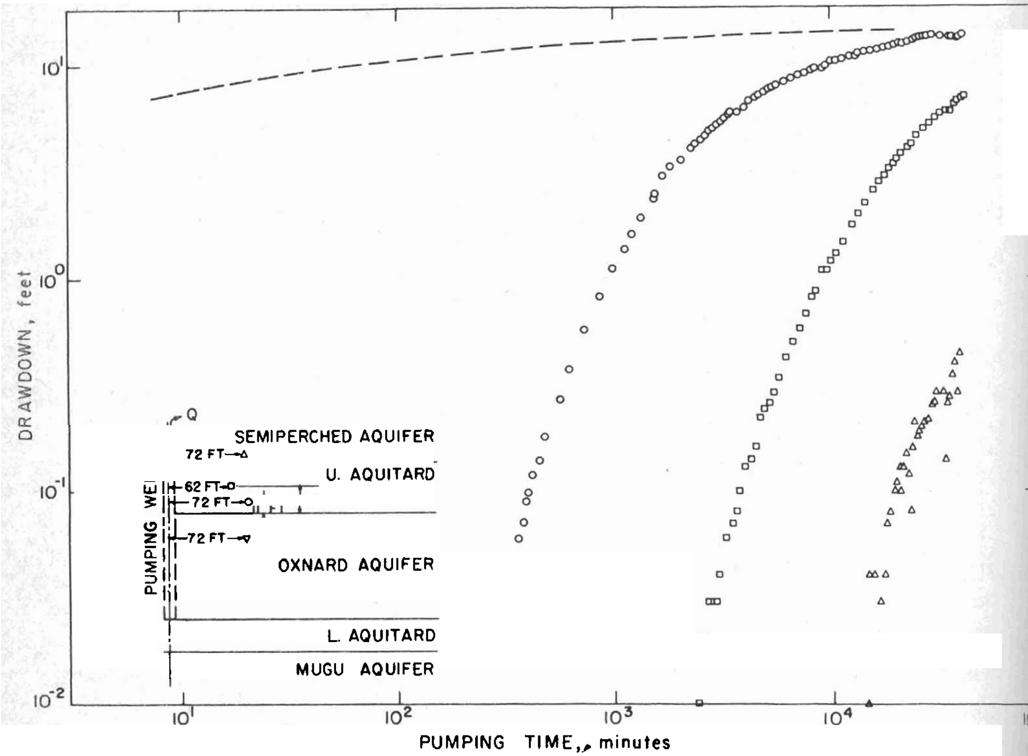

Fig. 9. The response of the piezometers in the upper aquitard (well 4, circles, and well 5, squares) and the semiperched aquifer (well 4A, triangles) during the first pumping test. The broken line indicates the probable response of the Oxnard aquifer at $r = 72$ feet.

The results of the second pumping test were essentially the same as those of the first test and will therefore not be presented here.

DETERMINATION OF AQUITARD PROPERTIES USING FIELD AND LABORATORY RESULTS

Having determined the hydraulic diffusivities, we can evaluate the permeability K' of each aquitard if the storage factor is known. The values of S_s' were calculated from consolidation tests performed in the laboratory [*California Department of Water Resources*, 1971, pp. 106-110] by using the formula

$$S_s' = a_v \gamma_w / (1 + e) \tag{11}$$

These values were then used to calculate K' according to

$$K' = \alpha' S_s'$$

and the results are summarized in Table 4.

TABLE 3. Results for Hydraulic Diffusivity of Aquitards from First Pumping Test

Layer	Section Tested	K'/S_s', gpd/ft	K'/S_s', cm²/sec
Upper aquitard	bottom 22 feet	1.02×10^2	1.47×10^{-1}
Upper aquitard	bottom 11 feet	2.44×10^2	3.51×10^{-1}
Lower aquitard	top 6 feet	4.17×10^2	5.99×10^{-1}

Direct measurements performed on undisturbed samples in the laboratory indicated that the aquitard permeabilities vary within a range of at least 3 orders of magnitude. The results in Table 4 fall on the high side of this range and thus are an indication that the average permeability in the field cannot always be reliably estimated from laboratory measurements.

It is interesting to compare the specific storage and permeability of the aquitard with those of the Oxnard aquifer. Using an aquifer thickness

TABLE 4. Hydraulic Properties of Aquitard Layers

Layer	Section Tested	Specific Storage S_s'		Permeability K'	
		cm^{-1}	ft^{-1}	cm/sec	gpd/ft^2
Upper aquitard	bottom 21 feet	7.88×10^{-6}	2.4×10^{-4}	1.11×10^{-6}	2.45×10^{-2}
Upper aquitard	bottom 11 feet	7.88×10^{-6}	2.4×10^{-4}	2.66×10^{-6}	5.85×10^{-2}
Lower aquitard	top 6 feet	3.28×10^{-6}	1.0×10^{-4}	1.89×10^{-6}	4.17×10^{-2}

of 93 feet, one has

$$K = \frac{T}{b} = \frac{130,600}{93} = 1405 \text{ gpd/ft}^2$$

and

$$S_s = \frac{S}{b} = \frac{1.12 \times 10^{-4}}{93} = 1.20 \times 10^{-6} \text{ ft}^{-1}$$

Thus the permeability of the aquifer exceeds that of the aquitards by more than 4 orders of magnitude. However, note that the specific storage of the aquifer is less than S_s' in the aquitards above and below by 2 orders of magnitude. In other words, for the same change in head a unit volume of aquitard material can contribute about 100 times more water from storage than a similar volume of the aquifer can. This statistic confirms our belief that storage in the aquitards must be considered when one deals with leaky aquifer systems.

NOTATION

a_v, coefficient of compressibility, equal to $-\Delta e/\Delta p$, LT^2M^{-1};
b_i, thickness of ith aquifer, L;
b_j', thickness of jth aquitard, L;
e, void ratio;
K_i, permeability of ith aquifer, LT^{-1};
K_j', permeability of jth aquitard, LT^{-1};
p, pressure, $ML^{-1}T^{-2}$;
Q_i, pumping rate from ith aquifer, L^3T^{-1};
r, radial distance from pumping well, L;
r/B_{ij}, dimensionless leakage parameter, equal to $r(K_j'/K_ib_ib_j')^{1/2}$;
s_D, dimensionless drawdown, equal to $4\pi T_is/Q_i$;
s_i, drawdown in ith aquifer, L;
s_j', drawdown in jth aquitard, L;
S_i, storage coefficient of ith aquifer, equal to $S_{s_i}b_i$;
S_{s_i}, specific storage of ith aquifer, L^{-1};
S_{s_j}', specific storage of jth aquitard, L^{-1};
t, pumping time, T;
t_{D_i}, dimensionless time for pumped ith aquifer, equal to $K_it/S_{s_i}r^2$;
t_{D_j}', dimensionless time for jth aquitard, equal to $K_j't/S_{s_j}'z^2$;
T_i, transmissibility of ith aquifer, equal to K_ib_i, L^2T^{-1};
z, vertical coordinate, L;
α_i, hydraulic diffusivity of ith aquifer, equal to K_i/S_{s_i}, L^2T^{-1};
α_j', hydraulic diffusivity of jth aquitard, equal to K_j'/S_{s_j}', L^2T^{-1};
β_{ij}, dimensionless leakage parameter, equal to $r/4b_i(K_j'S_{s_j}'/K_iS_{s_i})^{1/2}$;
γ_w, specific weight of water, $ML^{-2}T^{-2}$.

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