Performance Analysis of Reliability Filling on Quasi-Static Fading Channels

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***Abstract*— Cooperative communication techniques are network-based approaches to achieve spatial diversity in systems in which each node only has a single antenna. Many such techniques are based on relaying, which is effective in terms of error performance but requires a large information exchange among the cooperating nodes. Cooperative reception techniques that offer near-optimal performance with a smaller information exchange are an area of ongoing research. One promising approach is to investigate combining techniques that can be used as a model for designing efficient cooperative reception schemes. In this paper, we consider one such technique, called reliability filling, that combines only as much information as needed to meet some reliability threshold. We analyze the performance of this technique for several scenarios of interest. Analytical estimates of the overhead involved in reliability filling are also given. Analysis and simulation results show that reliability filling can offer performance close to maximal-ratio combining while combining fewer symbols.**

I. Introduction

The physical size of modern radios do not permit the use of multiple transmit antennas, and hence network-based alternatives are required to achieve spatial diversity. The idea of users cooperating to achieve spatial diversity has received a lot of attention from researchers in recent years [1], [2], [3], [4], [5], [6]. Diversity achieved when users in a network collaborate by sharing information to form a virtual antenna array has been termed *cooperative diversity*. Information-theoretic cooperation techniques based on the relay channel have been proposed and studied in [1], [2], [7], [8]. Some more-practical approaches for relaying with two cooperating nodes have also been proposed in [2], [3], [5]. However, these schemes do not easily scale to more than two cooperating nodes. In addition, several of the proposed techniques require correct reception at a relaying nodes.

One relaying technique that does not rely on correct decoding at a relay is the amplify-and-forward technique [2]. In practice many transmitters use a fixed digital modulation scheme, which means that the soft-decision for each received symbol would have to be quantized and transmitted as a sequence of bits. This results in a large overhead if each symbol must be quantized and transmitted by the relay. If the relay has a reliable communication channel to the intended receiver, then the amplify-and-forward scheme is basically a distributed approach to maximal-ratio combining (MRC).

In [4], a collaborative reception scheme is proposed that uses the reliabilities of the decisions at the output of a soft-input, soft-output decoder to reduce the amount of information that needs to be exchanged. In [9], several approaches are considered that can achieve good performance while reducing the amount of collaborative exchange by using reliability information generated by soft-output decoders. Although these techniques are found to be effect for AWGN channels, they are not as effective on quasi-static fading channels. Furthermore, it is not easy to analyze the performance of these schemes, and hence it is also difficult to design better approaches based on these previous schemes.

In order to overcome these probems with previous collaborative reception techniques, a new approach was considered in [6], [10]. In these papers, the design of collaborative reception techniques is simplified by decomposing the problem into two steps. In the first step, an alternative combining scheme to MRC is considered that can achieve performance close to MRC while combining far fewer symbols. In the second step [10], a practical collaborative reception scheme is developed based on the principles of the combining scheme selected in the first step.

An efficient combining technique that meets the requirement of the first step described above is the *reliability filling* technique proposed in [6], [10]. This technique combines a subset of all of the received symbols based on reliabilities generated by soft-output decoders. A single parameter, called the reliability threshold, can be used to trade-off block error rate and number of symbols combined. In [10], a practical collaborative reception scheme based on reliability filling is developed and shown to be effective for quasi-static fading channels.

In this paper, we analyze the performance of the reliability filling scheme both in terms of block error rate and number of symbols combined. We present bounds on the block error probability for reliability filling with two cooperating

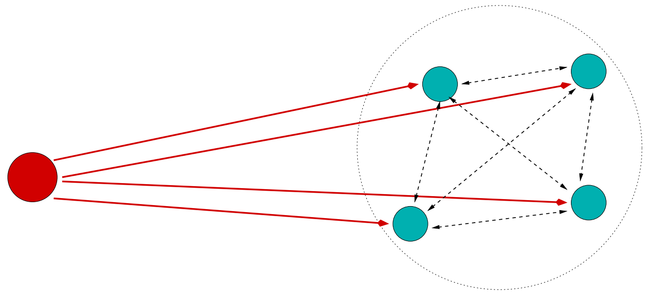

Fig. 1. System Topology for Reliability Filling

nodes and also apply this bound to determine the block error probability for a hybrid selection-combining and reliability-filling technique with more than two cooperating nodes. The analytical results on block error probability and expected number of symbols combined are compared with simulation results. We show how the analytical results may be applied to select the reliability threshold based on a target value for block error probability or average number of symbols to be combined.

This paper is organized as follows. In Section II, we introduce the concept of reliability filling. Section III contains the performance analysis. Numerical results and design criteria are given in Section IV. The paper is concluded in Section V.

II. Reliability Filling

The system topology that we consider is shown in Figure 1. A distant transmitter broadcasts a packet to a cluster of receiving nodes. The message at the source is packetized and encoded with a code that permits soft-input, soft-output (SISO) decoding. The codeword is then broadcast to a cluster of receiving nodes over an imperfect channel. Typical scenarios could be military applications in which a battleship broadcasts a message to a platoon of soldiers on the mainland or commercial applications wherein a base station communicates with a cluster of mobile users. The distance to the transmitter and the power limitations of the receiving nodes do not permit the use of ARQ and traditional code combining [11] techniques. As an alternative, the nodes in the cluster can collaborate with each other to resolve any ambiguity about the transmitted message. We assume that communication for collaboration within the cluster is error-free owing to the proximity of the nodes. This assumption keeps our results general without being tied to a specific modulation and coding scheme that is employed in the cluster.

With error-free collaboration channels, the amplify-and-forward relaying strategy [2] is equivalent to MRC with combining at a central node. However, this requires that all but one of the cooperating nodes send copies of all of their received symbols, thereby resulting in a bandwidth-expensive collaboration procedure. We assume that the nodes are constrained to a fixed digital modulation scheme, which results in an even greater overhead, as each received symbol must be quantized and transmitted as a sequence of bits to the combining node. The information exchanged by the nodes will be referred to as the cooperation overhead. Though the combining in MRC is optimal in terms of error performance, it is inefficient in terms of the cooperation overhead.

In [6], [10], an idealized technique called reliability filling is proposed that achieves performance similar to MRC with a much lower overhead. Reliability filling relies on the use of error correction codes and SISO decoders to identify trellis sections that could potentially benefit from information from other nodes in the cluster. In order to perform reliability filling, each receiver uses a SISO maximum *a posteriori* (MAP) decoder. *A priori* probabilities and received channel values are the typical inputs to such decoders. At the output, the decoders produce *a posteriori* probabilities (APPs). If the decoders operate in the log domain (log-MAP decoders), the outputs consists of log-likelihood ratios (LLRs), $L(X_i|\mathbf{Y}=\mathbf{y})$,of the APPs and are referred to as soft information (outputs). In particular, we use convolutional codes for encoding and the Max-Log-MAP implementation of the BCJR [12] algorithm in the decoder. The LLR for information bit X_i is given by

$$L(X_i|\mathbf{Y}=\mathbf{y}) = \log \frac{\Pr(X_i=+1|\mathbf{Y}=\mathbf{y})}{\Pr(X_i=-1|\mathbf{Y}=\mathbf{y})}, \quad (1)$$

where $\mathbf{y}$ represents the vector of received symbols.

The magnitude of the soft output is called the reliability of the decision and is a measure of the correctness of the hard-decision. The higher the reliability of a decision, the more likely the decoder already has sufficient information about that particular section of the code trellis to decode that bit correctly. Thus the use of SISO decoding helps identify bits (trellis sections) about which reliable decisions can be made without the exchange of information. There are other trellis sections that are a little unreliable but that only need information from a few other nodes to make reliable decisions, and there are very unreliable trellis sections that need information from all other nodes. Reliability filling is a technique based on water-filling in the reliability domain that takes into account the above observations. Note that MRC combines the same amount of information for all trellis sections, regardless of the reliability of the original decisions.

In reliability filling the number of coded symbols combined per trellis section is reduced based on the reliabilities of the decoded bit decisions. Assume that the decoding process is controlled by genie that knows the reliabilities $|L(X_i|\mathbf{y_j})|$ ($\mathbf{y}_j$ is the received vector at node j) of the information bits at all the nodes in the cooperating cluster. For each trellis section, the genie chooses the nodes from which coded symbols should be combined based on the reliability information. So even though reliabilities of the information bits are used to select the nodes for combining, the coded symbols are the quantities being combined, as in MRC.

The combining procedure works as follows. Let

$$\mathbf{S}_i = \{S \subset \{1, 2, \ldots, M\} : \sum_{j\in S} |L(X_i|\mathbf{y_j})| \geq T\}, \quad (2)$$

where M is the total number of cooperating nodes. Thus, $\mathbf{S}_i$ is the set of all possible combinations of nodes in the cluster such that the sum of the reliabilities of bit i at those nodes exceeds a threshold T. Let $N_i = \min_{S' \in \mathbf{S}_i} |S'|$. Thus, N_i is the set of minimum number of nodes required such that the sum of the reliabilities of bit i at those nodes exceeds a threshold T. Then the set of nodes $\mathbf{C}_i$ for which information will be combined is given by

$$\mathbf{C}_i = \begin{cases} \underset{S \in \mathbf{S}_i : |S| = N_i}{\text{argmax}} \left\{ \sum_{j \in S} |L(X_i|\mathbf{y_j})| \right\} & \text{, if } \mathbf{S}_i \neq \emptyset \\ \{1, 2, \ldots, M\} & \text{, if } \mathbf{S}_i = \emptyset \end{cases} \tag{3}$$

Thus, when $\mathbf{S}_i = \emptyset$, coded symbols are combined from all nodes in the cluster. When $\mathbf{S}_i \neq \emptyset$, the set of nodes $\mathbf{S}_i$ is chosen to maximize the sum of the reliabilities for bit i subject to $|\mathbf{S}_i| = N_i$. Note that for different trellis sections, a different number of nodes will be involved in the combining process.

Thus, for bits (trellis sections) with low reliabilities, information from more nodes are combined so that the sum of the reliabilities of the bits combined is greater than the threshold. For bits with high reliabilities, information from only a few nodes is combined. For example, if before combining, the maximum reliability for a bit across all the nodes is greater than the threshold, the information for that trellis section at the node that achieved the maximum reliability is used without any information from other nodes. Thus, reliability filling combines fewer coded symbols per trellis section than MRC. It has been shown in [6], [10] that reliability filling achieves full diversity and *almost* all the coding gain of MRC while combining 45% fewer symbols than MRC. Since reliability filling relies on a genie, it cannot be implemented practically. However, it demonstrates that it is not necessary to combine information from all nodes in order to achieve good performance in bandwidth-limited scenarios. It shows that when every node has some information about decoding at other nodes, the overhead can be reduced significantly. It also proves that bit reliabilities convey useful information, and the principle of combining symbols based on reliability values can be used to design practical cooperation schemes with low overhead. A practical, iterative scheme based on the principles of reliability filling is proposed in [10]. The multiple iterations of the practical scheme makes it harder to analyze. The analysis and design criteria presented in Sections III and IV can be considered to be the first step towards the design and analysis of schemes like those presented in [10].

III. PERFORMANCE ANALYSIS

A review of standard results on error bounds for convolutional codes and a mathematical characterization of reliabilities at the output of the decoder is first presented. These results will then be used to derive bounds on the error probability of reliability filling.

A. Block error rate of convolutional codes over quasi-static fading channels

The performance of convolutionally encoded systems is usually analyzed by first calculating the pairwise error probability (PEP). The PEP is defined as the probability of choosing a codeword $\hat{\mathbf{x}}$ when codeword $\mathbf{x}$ was transmitted. For a linear binary code with antipodal modulation and coherent detection, the conditional PEP under maximum likelihood (ML) decoding can be expressed as

$$P(d|\boldsymbol{\alpha}) = P(\mathbf{x} \rightarrow \hat{\mathbf{x}}|\boldsymbol{\alpha}) = Q\left(\sqrt{\frac{2E_s}{N_0}\sum_{n\in\eta}\alpha_n^2}\right), \tag{4}$$

where d is the Hamming distance between $\mathbf{x}$ and $\hat{\mathbf{x}}$, $\frac{E_s}{N_0}$ represents the symbol energy-to-noise ratio, $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2 \ldots, \alpha_n\}$ is the set of fading coefficients, $\eta = \{n : x_n \neq \hat{x}_n\}$ (Note that $|\eta| = d$), and $Q(\cdot)$ represents the Gaussian Q-function. A particularly tight bound on the block error rate of linear codes over quasi-static fading channels is [13],

$$P_{block} \leq 1 - \int_{\boldsymbol{\alpha}} \left[1 - \min\left(1, \sum_{d=d_{min}}^{d_{max}} a(d) P(d|\boldsymbol{\alpha})\right)\right]^B f(\boldsymbol{\alpha}) d\boldsymbol{\alpha}, \tag{5}$$

where B is the block-size, $a(d)$ represents the multiplicity of error events with Hamming weight d, and $f(\boldsymbol{\alpha})$ represents the joint probability density function (PDF) of the fading coefficients. In a quasi-static or block-fading channel, the fading amplitude is constant over all symbols in a block and independent between blocks. The PDF characterizing a Rayleigh fading channel with unit energy is given by

$$f(\alpha) = 2\alpha e^{-\alpha^2} u(\alpha), \tag{6}$$

where $u(\alpha)$ is the unit step function. The joint PDF $f(\boldsymbol{\alpha})$ can then be obtained depending on the scenario under consideration.

B. Characterizing reliabilities at the output of a Max-Log-MAP decoder

Reliability filling selects coded symbols for combining based on the reliabilities of the decoded bits. Thus, the analysis of reliability filling requires a mathematical characterization of reliability that enables computation of various probabilities involving bit reliabilities. Soft-information was first characterized mathematically in [14]. Analytically tractable expressions for the cumulative density function (CDF) and PDF for bit reliabilities at the output of a Max-Log-MAP decoder are given in [15]. The reliability, Λ, is modeled as the absolute value of a Gaussian random variable with variance equal to twice the mean, $(\Lambda \sim \mathcal{N}(\mu, 2\mu))$ in [15]. The CDF and PDF can then be obtained as,

$$F_\Lambda(\lambda) = \left\{Q\left(\frac{\mu - \lambda}{\sqrt{2\mu}}\right) - Q\left(\frac{\mu + \lambda}{\sqrt{2\mu}}\right)\right\} u(\alpha), \tag{7}$$

and

$$f_\Lambda(\lambda) = \frac{u(\alpha)}{2\sqrt{\pi\mu}}\left\{e^{-\frac{(\mu-\lambda)^2}{4\mu}} + e^{-\frac{(\mu+\lambda)^2}{4\mu}}\right\}. \tag{8}$$

Note that μ in equations (7) and (8) represents the mean of the soft information. An expression to compute the mean of the reliabilities for transmission over an additive white Gaussian noise (AWGN) channel was also given in [15] as,

$$\mu(\sigma^2) = \int_0^\infty \prod_{d=d_{min}}^{d_{max}} \left\{ Q\left(\frac{2\sigma^2\lambda - 4d}{\sqrt{16d\sigma^2}} \right) \right\}^{a(d)} d\lambda, \quad (9)$$

where σ^2 denotes the noise variance and $a(d)$ represents the condensed event multiplicity (cf. Table 1 in[15]). Starting from the definition of reliability in [15], it is straight-forward to show that the mean of the reliabilities conditioned on the fading coefficients of a quasi-static fading channel is given by

$$\mu = E[\Lambda|\alpha] = \mu(\sigma^2/\alpha^2). \quad (10)$$

C. Reliability filling with two cooperating nodes

If there are only two cooperating nodes (node 1 and node 2), the genie controlling the reliability filling process can select coded symbols for combining from either node 1 or node 2 or from both nodes depending on the reliability of the bit decisions. The genie combines coded symbols for trellis section i from both nodes when the reliability of bit i at both nodes is less than the reliability filling threshold T, i.e., $\max(\Lambda_{i,1}, \Lambda_{i,2}) < T$. If the reliability of bit i at node 1 is greater than both T and the reliability of bit i at node 2, then the genie picks coded symbols corresponding to trellis section i from only node 1. That is, if $\Lambda_{i,1} \geq \max(\Lambda_{i,2}, T)$, the genie picks coded symbols for bit i from node 1 only. Similarly, if $\Lambda_{i,2} \geq \max(\Lambda_{i,1}, T)$, the genie picks coded symbols from node 2 only. Note that $\Lambda_{i,j} \sim \mathcal{N}(\mu(\sigma^2/\alpha_j^2), 2\mu(\sigma^2/\alpha_j^2))$ is the reliability of bit i at node j, where $j \in \{1, 2\}$ and α_j represents the fading coefficient at node j. Since we are considering a block-fading channel wherein all the bits in a block experience the same fading amplitude, the fading can be characterized by a scalar α_j at each node. For simplicity of exposition, we will henceforth use μ_j to represent $\mu(\sigma^2/\alpha_j^2)$. Thus, given the fading coefficients at the two nodes, α_1 and α_2, the value of the signal-to-noise ratio (SNR) for bit i after combining can be expressed by the random variable $\Phi_i E_s/N_0$ where,

$$\Phi_i = \begin{cases} \alpha_1^2, & \Lambda_{i,1} \geq \max(\Lambda_{i,2}, T) \\ \alpha_2^2, & \Lambda_{i,2} \geq \max(\Lambda_{i,1}, T) \\ (\alpha_1^2 + \alpha_2^2), & T > \max(\Lambda_{i,1}, \Lambda_{i,2}). \end{cases} \quad (11)$$

The conditional PEP in (4) can then be obtained as

$$P(d|\boldsymbol{\alpha}) = \sum_{\gamma} Q\left(\sqrt{\frac{2E_s}{N_0}\gamma} \right) \mathrm{P}_\Gamma(\gamma), \quad (12)$$

where $\gamma = \sum_{n\in\eta} \Phi_n$ and $\mathrm{P}_\Gamma(\gamma)$ denotes the PDF of γ. Since Φ is a discrete random variable, we can use the multinomial probability law to express the conditional PEP in (12) as

$$P(d|\boldsymbol{\alpha}) \approx \sum_{\substack{a,b \\ a,b\geq 0 \\ a+b\leq d}} Q\left(\sqrt{\frac{2E_s}{N_0}(a\alpha_1^2 + b\alpha_2^2 + c(\alpha_1^2+\alpha_2^2))} \right) \times \frac{d!}{a!b!c!} P^a(\Phi = \alpha_1^2) P^b(\Phi = \alpha_2^2) P^c(\Phi = \alpha_1^2 + \alpha_2^2), \quad (13)$$

where $c = d - a - b$. Note that the approximation in (13) comes from the assumption that the Φ_i's (and hence reliability

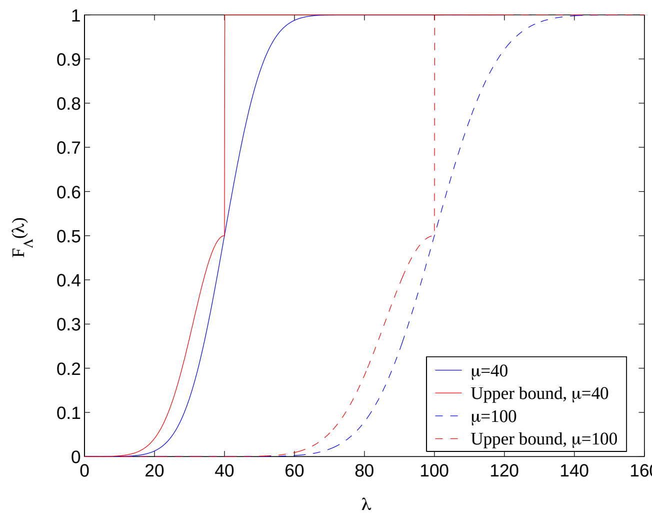

Fig. 2. Upper bound on $F_\Lambda(\lambda)$

of different bits) are conditionally independent given α_1 and α_2. The computation of the PEP in (13) requires knowledge of the probability mass function (PMF) of Φ. The PMF of Φ can be computed as follows. Consider the probability of combining received symbols from both nodes,

$$\begin{aligned} \text{Prob}\{\Phi_i = \alpha_1^2 + \alpha_2^2\} &= \text{Prob}\{T > \max(\Lambda_{i,1}, \Lambda_{i,2})\} \\ &= \text{Prob}\{\Lambda_{i,1} < T\} \cdot \text{Prob}\{\Lambda_{i,2} < T\} \\ &= F_{\Lambda_1}(T) \cdot F_{\Lambda_2}(T). \end{aligned} \quad (14)$$

Consider the probability of the genie choosing the coded symbols of node 1 only,

$$\begin{aligned} \text{Prob}\{\Phi_i = \alpha_1^2\} &= \text{Prob}\{\Lambda_{i,1} \geq \max(\Lambda_{i,2}, T)\} \\ &= \text{Prob}\{\Lambda_{i,1} \geq \Lambda_{i,2}\} \cdot \text{Prob}\{\Lambda_{i,1} \geq T\} \\ &= \int_T^\infty \int_0^{\lambda 1} f_{\Lambda_1,\Lambda_2}(\lambda_1, \lambda_2) d\lambda_1 d\lambda 2 \\ &= \int_T^\infty f_{\Lambda_1}(\lambda_1) \int_0^{\lambda 1} f_{\Lambda_2}(\lambda_2) d\lambda_2 d\lambda_1 \\ &= \int_T^\infty f_{\Lambda_1}(\lambda_1) F_{\Lambda_2}(\lambda_1) d\lambda_1. \end{aligned} \quad (15)$$

The expressions for $f_{\Lambda_1}(\lambda_1)$ and $F_{\Lambda_2}(\lambda_1)$ makes the integral in (15) hard to evaluate. We obtain an upper bound on $\text{Prob}\{\Phi_i = \alpha_1^2\}$ by using an upper bound for $F_{\Lambda_2}(\lambda)$. We upper bound $F_{\Lambda_2}(\lambda)$ by 1 for $\lambda > \mu_2$. For $\lambda \leq \mu_2$, we use the improved Chernoff bound for $Q()$, yielding

$$F_{\Lambda_2}(\lambda) \leq Q\left(\frac{\mu_2 - \lambda}{\sqrt{2\mu_2}}\right) \leq \frac{1}{2} e^{-\frac{(\mu_2-\lambda)^2}{4\mu_2}}, \quad \lambda \leq \mu 2. \quad (16)$$

The upper bound on $F_{\Lambda_2}(\lambda)$ is shown in Fig. 2 for two different values of μ_2. Using (16) in (15), an upper bound on $\text{Prob}\{\Phi_i = \alpha_1^2\}$ can be obtained as follows:

$$\text{Case 1}: \; T > \mu_2$$
$$\text{Prob}\{\Phi_i = \alpha_1^2\} \leq \int_T^\infty f_{\Lambda 1}(\lambda) d\lambda = 1 - F_{\Lambda_1}(T). \quad (17)$$

Case 2 : $T \leq \mu_2$

$$\begin{aligned}\text{Prob}\{\Phi_i = \alpha_1^2\} &\leq \int_T^{\mu_2} f_{\Lambda_1}(\lambda)\frac{1}{2}e^{-\frac{(\mu_2-\lambda)^2}{4\mu_2}}d\lambda \\ &\quad + \int_{\mu_2}^{\infty} f_{\Lambda_1}(\lambda)d\lambda \\ &= \frac{1}{4\sqrt{\pi\mu_1}}\Bigg[\int_T^{\mu_2} e^{-\frac{(\mu_1-\lambda)^2}{4\mu_1}}e^{-\frac{(\mu_2-\lambda)^2}{4\mu_2}}d\lambda \\ &\quad + \int_T^{\mu_2} e^{-\frac{(\mu_1+\lambda)^2}{4\mu_1}}e^{-\frac{(\mu_2-\lambda)^2}{4\mu_2}}d\lambda\Bigg] + 1 - F_{\Lambda_1}(\mu_2) \\ &= \frac{ke^{-\frac{\mu_1+\mu_2}{2}}}{2\sqrt{2\mu_1}}\left[e^{\frac{k^2}{2}}\left\{Q\left(\frac{T}{k}-k\right)-Q\left(\frac{\mu_2}{k}-k\right)\right\}\right. \\ &\quad \left. + Q\left(\frac{T}{k}\right) - Q\left(\frac{\mu_2}{k}\right)\right] + 1 - F_{\Lambda_1}(\mu_2). \end{aligned} \tag{18}$$

where $k = \sqrt{\frac{2\mu_1\mu_2}{\mu_1+\mu_2}}$. Similarly a bound on $\text{Prob}\{\Phi_i = \alpha_2^2\}$ can be obtained.

An upper bound on the block error rate can then be obtained using (14), (17) and (18) in (13) and (5). We assume that the two nodes experience independent fading and hence the joint density function in (5) can be expressed as $f(\alpha_1, \alpha_2) = f(\alpha_1) \cdot f(\alpha_2)$, where $f(\cdot)$ denotes the density function of a quasi-static Rayleigh fading channel.

D. Hybrid Selection Combining and reliability filling

An extension of our analysis to $M > 2$ cooperating nodes, though straight forward, becomes computationally prohibitive. However, we can extend the previous analysis to a system of practical interest with more than two cooperating nodes. In systems with many cooperating nodes, it may be most efficient to apply some combination of selection diversity along with combining to constrain the amount of information that must be combined. We consider a hybrid *selection combining* and reliability filling scheme that works as follows. If there are more than two cooperating nodes, the genie controlling the combining process chooses two nodes (selection) with the highest signal-to-noise ratios (SNRs). Then reliability filling is performed using the information at the two selected nodes.This is an instance of a *generalized selection combining* (GSC) [16] scheme wherein combining is performed according to the rules of reliability filling with two of the available M nodes. In keeping with the notation on GSC [16], we denote this scheme as SC/RF(T)-2/M, where RF represents reliability filling and T is the associated reliability threshold. Because reliability filling is performed with only two nodes, all the equations derived earlier can be used with no modifications. The only change would be in the region of integration in (5) and in the density function of the fading coefficients $f(\boldsymbol{\alpha})$.

For a block fading channel, the node with the highest SNR is the node with the highest fading amplitude. Let $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \ldots, \alpha_M\}$ represent the set of independent and identically (i.i.d) distributed Rayleigh fading amplitudes at the M cooperating nodes, and let $\alpha_{1:M} \geq \alpha_{2:M} \geq \ldots \alpha_{M:M} \geq 0$ be the order statistics obtained by arranging the elements of $\boldsymbol{\alpha}$ in decreasing order. Since the fading amplitudes ($\boldsymbol{\alpha}$) are i.i.d, the joint PDF $f_{\alpha_{1:M},\alpha_{2:M},\ldots\alpha_{L:M}}(\alpha_{1:M}, \alpha_{2:M}, \ldots \alpha_{L:M})$ of the $L \leq M$ highest fading amplitudes is given by [16, p. 381, Eq.(9.311)],

$$f_{\alpha_{1:M},\alpha_{2:M},\ldots\alpha_{L:M}}(\alpha_{1:M}, \alpha_{2:M}, \ldots \alpha_{L:M}) = L!\binom{M}{L}[F(\alpha_{L:M})]^{M-L}\prod_{i=1}^{L} f(\alpha_{i:M}), \tag{19}$$

$$\alpha_{1:M} \geq \alpha_{2:M} \geq \ldots \alpha_{L:M}, \tag{20}$$

where $F(\cdot)$ in (19) denotes the CDF of the Rayleigh random variable and is given by $F(\alpha) = \int_0^{\infty} f(\alpha)d\alpha = 1 - e^{-\alpha^2}$. Using (19) in (5) and using $L = 2$, we can bound the block error rate for SC/RF(T)-2/M as

$$\begin{aligned}P_{block} \leq 1 - &\int_0^{\infty}\int_0^{\alpha_1}\left[1 - \min\left(1, \sum_{d=d_{min}}^{d_{max}} a(d)P(d|\alpha_1, \alpha_2)\right)\right]^B \\ &\times M(M-1)[F(\alpha_2)]^{M-2}f(\alpha_1)f(\alpha_2)d\alpha_2 d\alpha_1,\end{aligned} \tag{21}$$

where the region of integration comes from (20) and $P(d|\alpha_1, \alpha_2)$ is given by (13).

E. Overhead of reliability-filling

We now derive expressions for the overhead of reliability filling. Recall that the genie combines symbols from both the nodes only if the reliability for the corresponding bit is less than the threshold (T) at both the nodes. Assuming independent fading at the receivers the probability that a bit has a reliability less than T at both nodes is given in (14) as

$$p \triangleq \text{Prob}(\{\Lambda_{i,1} < T\} \cap \{\Lambda_{i,2} < T\}) = F_{\Lambda_1}(T)F_{\Lambda_2}(T) \tag{22}$$

Thus, the average number of bits for which the genie combines information from both nodes is given by Np, where N is the blocksize. For each information bit $\frac{1}{R}$ coded symbols from each node are combined, where R is the code-rate. Assuming q bits are required to quantize each coded symbol, the overhead due to the genie combining symbols from both nodes is given by

$$\Theta_{\text{both nodes}}|\alpha = \frac{2Npq}{R}. \tag{23}$$

The factor of two arises in the above equation since information is combined from two nodes. The conditioning on α arises because F_{Λ_i} is a conditional distribution and hence p is a conditional probability. Note that we refer to the number of bits combined per transmitted block as the cooperation overhead.

Similarly the average number of bit for which the genie selects coded symbols from only one node is given by $N(1-p)$. The overhead for such bits is

$$\Theta_{\text{one node}}|\alpha = \frac{N(1-p)q}{R}. \tag{24}$$

Thus, the total overhead conditioned on the fading coefficient is given by

$$\Theta|\alpha = \Theta_{\text{both nodes}}|\alpha + \Theta_{\text{one node}}|\alpha = \frac{N(1+p)q}{R}. \tag{25}$$

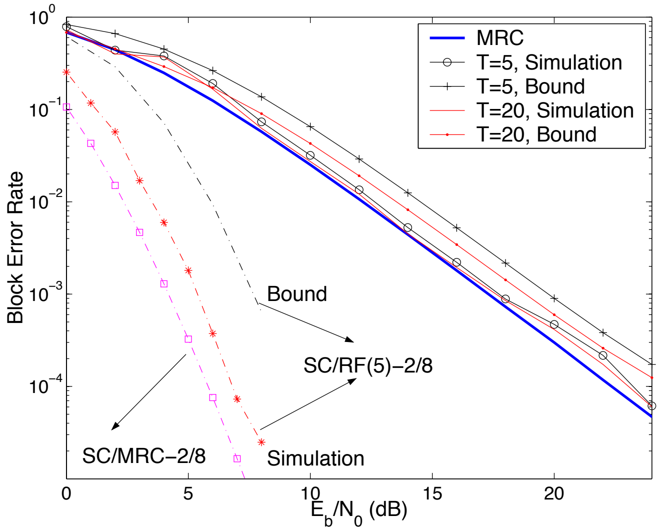

Fig. 3. Simulated block error rates and corresponding analytical bounds. Results are shown for reliability filling with two thresholds, 5 and 20. Results are also presented for the hybrid selection combining and reliability filling scheme, SC/RF(5)-$2/8$.

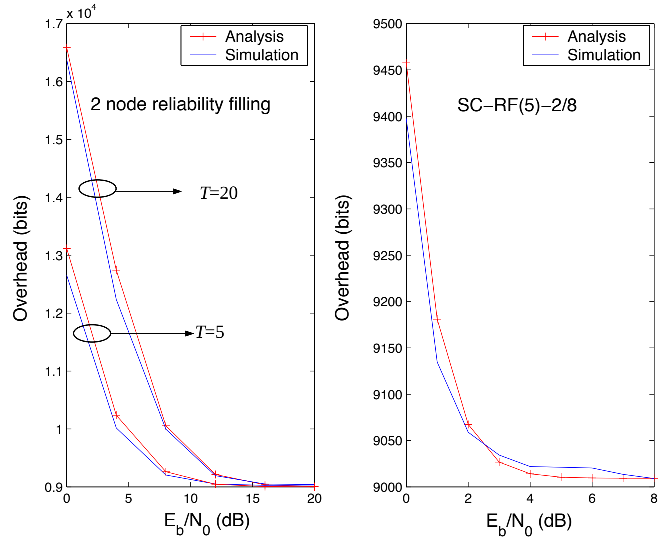

Fig. 4. Simulated overhead for reliability and corresponding analytical results. Results are shown for reliability filling with two thresholds, 5 and 20, in the figure on the left. Results are also presented for the hybrid selection combining and reliability filling scheme, SC/RF(5)-$2/8$ in the figure on the right.

The net overhead of reliability filling can then be obtained by integrating (25) over the density of the fading coefficients, $f(\alpha_1, \alpha_2)$.

IV. Numerical Results

For all the results shown in this paper, a rate $R = \frac{1}{2}$, constraint-length 3 convolutional code with generator polynomials $1 + D^2$ and $1 + D + D^2$ ($(5, 7)$ in octal) is used at the distant transmitter to encode the message sequence. The information at the transmitter is segmented into $N = 900$ bit fragments before encoding it with the channel code. The summation in (5) is performed with $d_{min} = 5$ and $d_{max} = 15$. The performance of reliability filling is compared with the performance of MRC. MRC is the best the nodes can do in terms of diversity, but it is bandwidth expensive. The block error rate of reliability filling with thresholds of $T = 20$ and $T = 5$ is shown in Figure 3. The performance of the two schemes are very close to that of MRC. Both thresholds produce block error rates parallel to MRC and hence achieve full diversity. Observe that there is only a small loss in coding gain when reducing the threshold, T, from 20 to 5. Simulation results also show similar behavior. A lower threshold leads to a smaller overhead because coded symbols from fewer nodes can make the sum of the bit reliabilities at those nodes exceed the threshold, T. Since fewer symbols are combined per trellis section, there is a loss in coding gain as can been seen from both the simulation results and the analytical bounds. Analytical bounds and simulation results for the hybrid selection combining and reliability filling scheme, SC/RF(5)-$2/8$, is also shown in Figure 3. Observe that the effect of the upper bound on the CDF is more pronounced in the hybrid scheme when compared to the original reliability filling scheme.

The overhead of the different reliability filling schemes are shown in Figure 4. For these results, it is assumed that $q = 5$ bits [17], [18] is enough to accurately represent the received coded symbols at each node. The analytical results for the overhead of reliability filling with two nodes are by integrating (25) over the density function $f(\alpha_1, \alpha_2) = f(\alpha_1) \cdot f(\alpha_2)$, where $f(\cdot)$ is given in (6). Analytical results are also presented for the hybrid selection combining and reliability filling scheme, SC/RF(5)-$2/8$ in the figure on the right. This result was obtained by integrating (25) over the density function given in (19). Note that the overhead of MRC can be obtained using $p = 1$ in (24). For the given parameters, the overhead for MRC is obtained as 1.8×10^4 bits. Thus, it is seen that the overhead of all the reliability filling schemes shown in this section is less than that of MRC.

The block error rate achieved by reliability filling is shown in Table I for various thresholds. The overhead required to achieve the target block error rate is also shown as a percentage with respect to the overhead of MRC. A threshold of ∞ forces the genie to combine information from all nodes for every bit and thus, it represents the performance of MRC. Thus, it is seen that performance of reliability filling approaches the performance of MRC as the threshold increases. However, the overhead also increases with an increase in the threshold. Appropriate thresholds for reliability filling can be chosen depending on the overhead a collaborative system can tolerate.

V. Conclusions

Reliability filling was introduced as a model for designing cooperative protocols for use in scenarios that are limited in bandwidth. An analysis of the performance of reliability filling for transmission over a block-fading channel is presented.

TABLE I

BLOCK ERROR RATE AND OVERHEAD OF RELIABILITY FILLING AT $E_b/N_0 = 5$ DB

Threshold (T)	Percentage overhead relative to MRC	Block error rate
5	54.78%	0.318
10	58.37%	0.302
20	65.83%	0.213
∞ (MRC)	100%	0.176

Bounds on the block-error rate of reliability filling with two cooperating nodes are given. Bounds are also presented for the block error rate of a hybrid selection-combining and reliability filling technique. We also present analytical estimates of the overhead involved in reliability filling. The proximity between the simulation and analytical results, and their similar responses to change in system parameters (namely, T) make the bounds a computationally more attractive tool to study and understand reliability filling. The analytical results show that a high value of the threshold leads to better performance at the cost of additional overhead. It is shown that close-to-optimal performance can be achieved with only a fraction of the overhead. The analysis validates the fact that the bit reliabilities can be exploited in the design of low-overhead cooperation protocols, and that all the symbols in a packet from all cooperating nodes need not be combined in order to produce the best results. Thus, practical schemes can be designed based on the principles of reliability filling that can achieve good performance in bandwidth-constrained applications.

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