Parametric analysis of Asymmetric Spur Gear Tooth

G. Mallesh[1*], Dr. V B Math[2], Venkatesh[3], Shankarmurthy H J [3], Shiva Prasad P[3], Aravinda K[3]

[1] Assistant Professor, Sri Jayachamarajendra College of Engineering, Mysore-570006, India
[2] Professor, Basaveshwar Engineering College, Bagalkot-587101, India
[3] Research Group, Sri Jayachamarajendra College of Engineering, Mysore-570006, India

G.Mallesh (Email: mallesh_gowder@yahoo.co.in)

Abstract

Gear is a machine element used to transmit motion and power between rotating shafts by means of progressive engagement of projections called teeth. Gears are classified according to the relative position of the axes of the shaft, type of gearing, peripheral velocity of the gears and position of teeth on gear surface. Presently gears are suffered by **backlash** the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles, **undercut** a condition in generated gear teeth when any part of the fillet curve lies inside of a line drawn tangent to the working profile at its lowest point and **interference** is an important aspect of kinematics of gearing. When the gear tooth tries to dig below the base circle of mating gear then the gear tooth action shall be non conjugate and violate the fundamental law of gearing this non conjugate action is called the interference .

These defects can be eliminated by increasing the pressure angle, by increasing the addendum of mating gear and another way of increasing the load capacity of transmissions is to modify the involute geometry. This has been a standard practice in sophisticated gear design for many years. The nomenclature describing these types of gear modifications can be quite confusing with reference to addendum modification or profile shift. An additional alteration that is very rarely used is to make the gears asymmetric with different pressure angles for each side of the tooth.

An asymmetric spur gear drive means that larger and smaller pressure angles are applied for the driving and coast sides. The two profiles of a gear tooth are functionally different for most gear drives. The workload on one side of profile is significantly higher than the other Gears.

The main objective of this paper is to generate asymmetric spur gear tooth geometry for different pressure angles on drive and coast side using computer programme. Developed programme is used to create a finite element model of gear tooth to study the effect of bending stress at the critical section for different pressure angles, different number of teeth and module. To study the effect of above asymmetric spur tooth parameters Finite Element Analysis software ANSYS was used.

Keywords: Module, Number of Teeth, Asymmetric Spur Gear Tooth, Finite Element Analysis

1 Introduction

With the advent of computer engineering analysis is getting more dependent on computer day by day. In a product design process involving engineering analysis, design alternative has been developed in the geometric modeling process. The analysis may take the form of stress strain calculations and dynamic simulation. The computations are complex and time consuming before advant of digital computer. Here the gear tooth is assumed to be isotropic and homogeneous and elastic material, interest is restricted to elastic materials in which deformation and stress regains to its original status with the removal of internal forces provided that external force do not cross certain limit.

But somehow these stress analysis problems are still suffering from a lot of short comings and thus are being constantly looked into by many researchers. One of the major issues is that reduction of bending stress at the critical section; this can be reduced by increasing the size of the gear tooth and number of teeth along with the following alterations.

- Increasing the pressure angle.
- Increasing the addendum of mating gear.
- Modify the involute geometry.

An additional alteration that is very rarely used is to make the gears asymmetric with different pressure angles for each side of the tooth.

In this paper we are generating asymmetric spur gear tooth geometry for different pressure angles on drive and coast side by varying the number of teeth and module using computer programme. Developed programme was used to create a finite element model of gear tooth, to study the effect of bending stress at the critical section and deflection at the centerline of a gear tooth at different locations.

2 Asymmetric spur gear teeth

The two profiles (sides) of a gear tooth are functionally different for many gears. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of the asymmetric tooth shape reflects this functional difference.

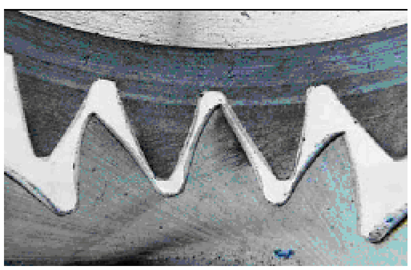


Fig. 1: Asymmetric spur gear.

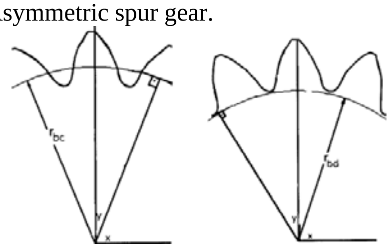


Fig. 2: Different tooth profiles on Coast and drive sides of asymmetric involute tooth.

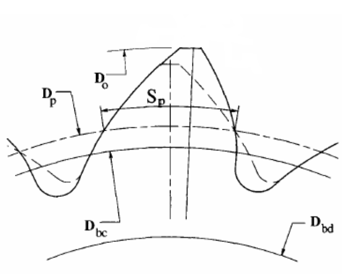


Fig. 3: Asymmetric spur gear with different base circles.

The design intent of asymmetric gear teeth is to improve the performance of the primary contacting profile. The opposite profile is typically unloaded or lightly loaded during relatively short work periods. The degree of asymmetry and drive profile selection for these gears depends on the application.

The difference between symmetric and asymmetric tooth is defined by two involutes of two different base circles D_{bd} and D_{bc}. The common base tooth thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter D_p that should be bigger than the largest base diameter.

Asymmetric gears simultaneously allow an increase in the transverse contact ratio and operating pressure angle beyond the conventional gear limits. Asymmetric gear profiles also make it possible to manage tooth stiffness and load sharing while keeping a desirable pressure angle and contact ratio on the drive profiles by changing the coast side profiles. This provides higher load capacity and lower noise and vibration levels compared with conventional symmetric gears.

3 Estimation of module and number of tooth to avoid interference and undercutting.

Module: It is defined as the ratio of the pitch diameter to the number of teeth of a gear. The module is one of the major and determining parameters of the gear it is required to estimate minimum value of module to transmit a power of 15 KW at rated speed of 1000rpm.

$$d_1 = 1.5d \quad (1)$$

$$d = 160\sqrt{\frac{P(KW)}{N(rpm)}} \quad (2)$$

$$M_{n(min)} = \frac{PCD\ of\ Pinion}{Minimum\ Number\ of\ teeth} \quad (3)$$

From the above data it is found that minimum module is 1.175 mm.

Number of teeth: There are two different types of gear tooth system

- Full depth system
- Stub tooth system

Full depth teeth system alleviate the interference and undercutting problems, and are also of broader and stronger root section as compared to the system having smaller pressure angles, where as stub tooth system is having lesser contact ratio compared to the full depth system hence there is a chance of slip, so full depth tooth system is preferred for further analysis.

For gears with standard tooth proportions, the minimum number of teeth which a pinion can have to mate with a rack without interference and undercutting can be calculated using equation.

$$z = \frac{2}{\sin^2\alpha} \quad (4)$$

For 20^0 pressure angle the minimum number of teeth on pinion is 17.09. Based on the above results following gear tooth parameters are listed in table:1 and the same is considered for the analysis and profile generation.

Table-1: Gear tooth parameters

Sl. No.	Description	Value		
1	Number of teeth on pinion	18	25	32
2	Module	2mm	4mm	6mm
3	Pressure angle, Coast side	20^0 fixed		
4	Pressure angle, Drive side	20^0-35^0 increment by 1^0		
5	Back up ratio	1.2		
6	Profile shift factor	0		

4 Equation used to generate spur gear tooth profile

Following equations are used to generate symmetric and asymmetric spur gear tooth profile on drive and coast side and to calculate RHPSTC, load angle, critical section thickness and distance from critical section to intersection of the tooth centerline and the line of action for load at HPSTC.

$$\bar{r}(\theta) = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} \quad (5)$$

$$X(\theta) = N\frac{M_n}{2}\left\{\sin\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right]\cos(\phi + \theta)\right\} \quad (6)$$

$$Y(\theta) = N\frac{M_n}{2}\left\{\cos\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right]\sin(\phi + \theta)\right\} \quad (7)$$

$$\theta_{min} \le \theta \le \theta_{max} \quad (8)$$

$$\theta_{min} = \frac{2}{N}[U + (V + X)\cot\phi] \quad (9)$$

$$\theta_{max} = \frac{1}{N\cos\phi} \times \sqrt{(2 + N + 2X)^2 - N(\cos\phi)^2} - \left(1 + \frac{2X}{n}\right)\tan\phi - \frac{\pi}{2N} \quad (10)$$

$$U = \frac{\pi}{4} + (\alpha - \gamma)\tan\phi + \frac{\gamma}{\cos\phi} \quad (11)$$

$$V = \gamma - \alpha \quad (12)$$

$$X(\theta) = M_n(P\cos\theta + Q\sin\theta) \quad (13)$$

$$Y(\theta) = M_n(-P\sin\theta + Q\cos\theta) \quad (14)$$

$$\theta_{min} \le \theta \le \theta_{max} \quad (15)$$

$$\theta_{min} = \frac{2}{N}[U + (V + X)\cot\phi] \quad (16)$$

$$\theta_{max} = \frac{2U}{N} \quad (17)$$

$$P = \frac{\gamma}{i-\left(u-\frac{n\theta}{2}\right)} \quad (18)$$

$$Q = \frac{2\gamma}{i} \times \left(\frac{v+x}{2u-n\theta}\right) + V + \frac{N}{2} + X \quad (19)$$

$$m_B = \frac{t_R}{h_t} \quad (20)$$

$$i = \frac{z_2}{z_1} \quad (21)$$

$$d_1 = M_n z_1 \; d_2 = M_n z_2 \quad (22)$$

$$d_{a1} = d_1 + 2M_n + 2x_1 M_n \quad (23)$$

$$d_{a2} = d_2 + 2M_n + 2x_2 M_n \quad (24)$$

$$r_{a1} = \frac{d_{a1}}{2} \quad (25)$$

$$r_{a2} = \frac{d_{a2}}{2} \quad (26)$$

$$d_{b1} = d_1\cos\emptyset_n \quad (27)$$

$$d_{b2} = d_2\cos\emptyset_n \quad (28)$$

$$r_{b1} = \frac{d_{b1}}{2} \quad (29)$$

$$r_{b2} = \frac{d_{b2}}{2} \quad (30)$$

$$C_e = \left(\frac{d_1 + d_2}{2}\right) + (x_1 + x_2)M_n \quad (31)$$

$$\alpha' = \cos^{-1}\left(\frac{(d_1 + d_2)\cos\alpha_n}{2C_e}\right) \quad (32)$$

$$r_{HPSTC} = \left(\left(\left((r_{a1} + r_{a2})\tan\alpha' - \sqrt{(r_{a2}^2 - r_{b2}^2)}\right) + \pi M_n\cos\alpha_n\right)^2 + r_{b1}^2\right)^{\frac{1}{2}} \quad (33)$$

$$\alpha_L = \theta - \gamma \quad (34)$$

$$\cos\theta = \frac{r_b}{r_{HPSTC}} \quad (35)$$

$$\tan\gamma = \frac{y}{x} \quad (36)$$

Using above equations and by suitable input gear data the values of RHPSTC, load angle, critical section thickness and distance from critical section to intersection of the tooth centerline and the line of action for load at HPSTC can be obtained as shown in fig.4.

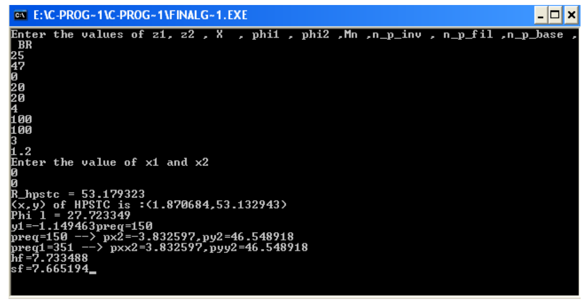


Fig. 4: Programme I/O data for gear tooth profile generation.

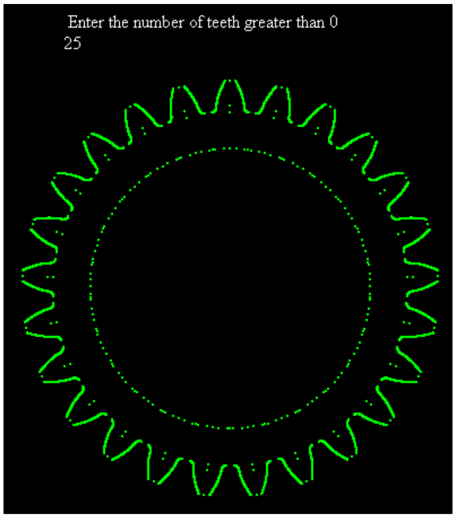


Fig. 5: Gear profile output from C-programme

5 Finite element analysis procedure

As a major part of present investigation a series of finite element analyses has been carried out for different sets of symmetric and asymmetric spur gears listed in table.1, subjected to a load at highest point of single tooth of contact (HPSTC) as shown in fig.6.

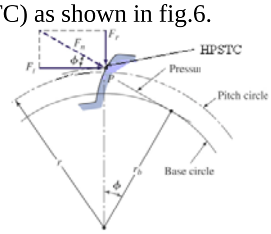


Fig. 6: Different types of forces acting at HPSTC

Gears are used to transmit a power of 15KW at 1000 rpm. Key points for involute spur gears were generated using "C "programme and same can be used for generating model for ANSYS as shown in fig.7.

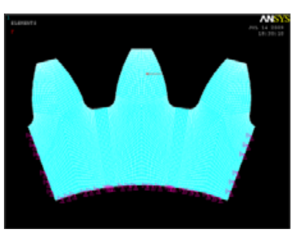


Fig. 7: Gear tooth system considered for finite element analysis with proper boundary conditions

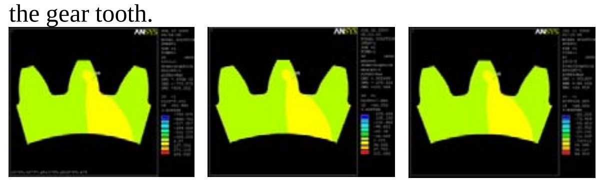
A finite element problem is treated as plane stress with thickness problem and a plane 182, 8-noded quadrilateral element are used to discritized the gear tooth domain.

The first investigation involved a two–dimensional plane stress analysis for 4 mm module and 20^0 pressure angle on both sides of the gears with 25 teeth and zero profile shifts. The gear tooth is considered to be a cantilever and it is constrained at the rim, an element supports the two degree of freedom and all the degrees of freedom are fixed. The gear tooth is loaded at HPSTC. The above meshed model, which is subjected to the boundary conditions and loading were statically analyzed and software performs the mathematical calculations and results are obtained in the post processing stage.

Similar analyses were carried out for different number of teeth and module. In the post- processor stage we accepts the results and generates the contour plots for bending stress at the critical section, and displacements at the tooth centerline at different location i.e. gear tooth root, tip and at the pitch point.

6 Results and discussions

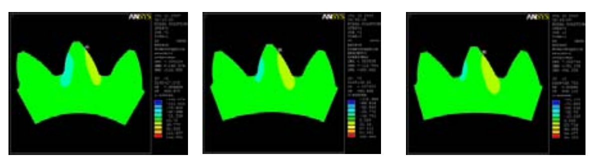
The results mainly consist of bending stress at critical section and displacements at the centre line of the gear tooth.



2mm 4 mm 6 mm

Fig. 8: Bending stress distribution in symmetric spur gear tooth.

It was found that the bending stress at the critical section reduces drastically with increases in the module this is the fact that as the module increases the gear tooth becomes bulkier, stress pattern is same for all profiles with different modules for same pressure angle and load.



18 teeth 25 teeth 32teeth

Fig. 9: Bending stress distribution in Asymmetric spur gear tooth.

As the number of tooth, module and pressure angle on the drive side increases the magnitude of the load, and its position shifts towards the gear tooth tip. Even the load and moment increases the bending stress reduces.

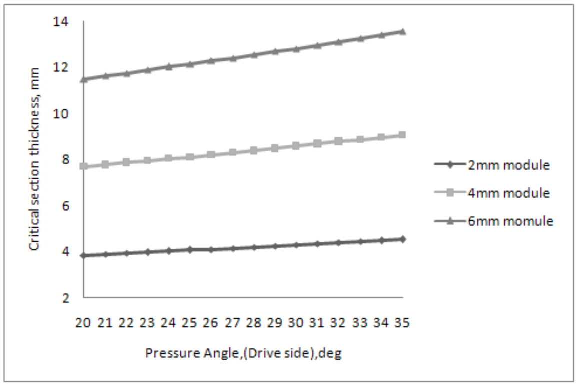


Fig. 10: Tooth thickness at the critical section for different modules.

From the above figure it is found that, thickness of the critical section increases slightly with increase in pressure angle, and directly depends on the module. It is the fact that as module increases the gear becomes bigger and having more resistance to the load.

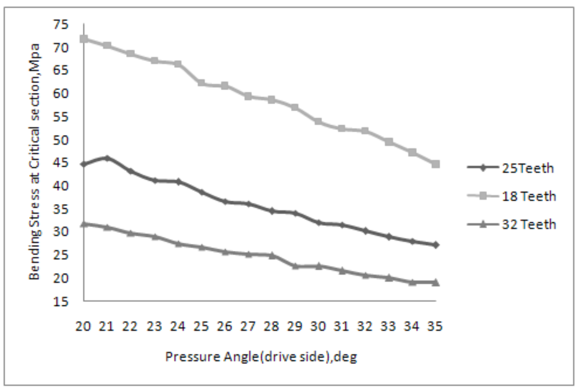


Fig. 11: Bending stress at the critical section for different number of teeth.

Critical section is one of the most important gear tooth parameter. Bending stress were reduced with increase in the pressure angle, by increasing the number of tooth the critical section shifts from the centre of the gear, which influence to reduce deflection of the gear tooth.

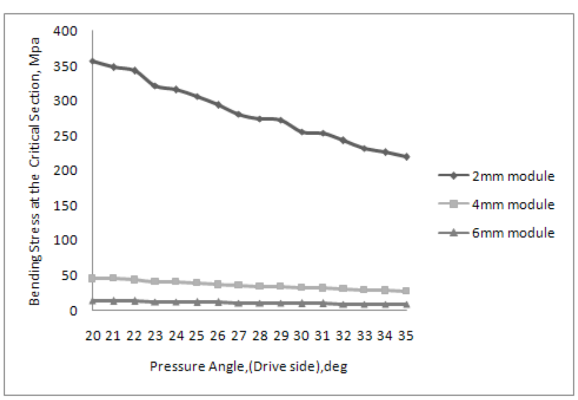


Fig. 12: Bending stress at the critical section for different modules.

Figure shows the variation of bending stress at the critical section for different modules, it was found that as the pressure angle on the drive side increases bending stress decreases.

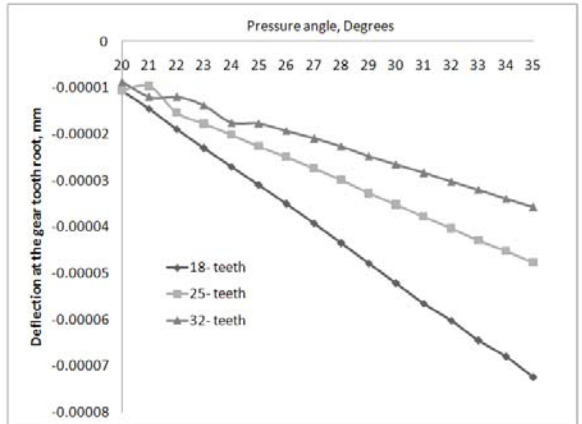


Fig. 12: Displacement at the gear tooth root for different number of teeth.

The fact that as the module increase, pitch diameter of the gear tooth increases causing the tooth become bulkier and stronger, from Fig.10 tooth thickness at the gear tooth root increases with increase in pressure angle on drive side due to the combined effect of module, number of teeth and pressure angle, hence deflection at the gear tooth root will decreases.

7 Conclusions

As the number of teeth and module increases the bending stresses decreases, while the other parameters are unchanged. This is due to the fact that with increase of module, the pitch diameters of the gear tooth increases causing the tooth to become bulkier and stronger. Again with the increase of module, the fillet radius of the tooth increases which would cause less impact in the root region (critical section) of the gear tooth. The increase of module means the increase of the tooth width from top to bottom, as a result the stress is observed to be less in the wider tooth for the same loading.

The bending stresses decreases with increases in the number of teeth and pressure angle on drive side, with the application of the same load for all gear teeth and one with more teeth, i.e. the bigger one will be stressed lesser. It is observed that as the pressure angle on the drive side increases bending stress decreases.

Acknowledgment

We thank the management, Principals and Heads of the

Department of Mechanical Engineering, Sri Jayachamarajendra College of Engineering, Mysore and Basaveshwar Engineering College, Bagalkot, for providing us an opportunity and encouraging to present this research paper.

References

[1] S. Vijayarangan And N. Ganesan , " A Static Analysis of Metal Matrix Composite Spur Gear by Three-Dimensional Finite Element Method", *Computers & Structure* Vol. 46, No. 6, PP,1021-1027, 1993.

[2] S. Vijayarangan And N. Ganesan ," Static Stress Analysis of a Composite Bevel Gear using a Three-Dimensional Finite Element Method" ,*Computer & Structure* Vol. No. 6. PP. 771-783. 1994.

[3] Ming-Haung Tsai, Ying-Chien Tsai ,"A Method for Calculating Static Transmission Errors of Plastic Spur Gears *Using* FEM Evaluation" , *Finite Elements in Analysis And Design* Vol. 27 (1997) 345-357.

[4] Toshiki HIROGAKI1, Eiichi AOYAMA1, Tsutao KATAYAMA1 Kazuya SUGIMURA2, Yoshinori YAGURA3"Development of Design System for FRP Gears for Power Transmissions",

[5] S.M. Mahbub Murshed, M. Arifeen Wahed M. A. Salam Akanda , "Parametric Solution Of Spur Gear Tooth Under Tip Loading",*MED , Bangladesh university and technology.*

[6] R. Muthukumar And M. R. Raghavan , "Estimation of Gear Tooth Deflection by the Finite Element Method*",Mech. Mach. Theory* Vol. 22, No. 2, PP. 171-181, 1987.

[7] Celik, M, "Comparison of Three Teeth and Whole Body Models in Spur Gear Analysis", *Mechanism and Machine Theory* Vol. 34, Issue: 8, November, 1999. PP. 1227-1235.

[8] Chung-Biau Tsay, Wen-Yao Liu, Yi-Cheng Chen, "Spur Gear Generation by Shaper Cutters", *Journal of Materials Processing Technology.* 104 (2000) 271-279.

[9] Chien-Hsing Li_, Hong-Shun Chiou_, Chinghua Hung, Yun-Yuan Chang, Cheng-Chung Yen , "Integration Of finite Element Analysis And Optimum Design On Gear Systems" ,*Finite Elements In Analysis And Design* 38 (2002) 179-192.

[10] Akanda, M. A. Salam Ahmed, S. Reaz; Uddin, M. Wahhaj , "Stress Analysis of Gear Teeth using Displacement Potential Function and Finite Differences", *International Journal for Numerical Methods in Engineering* , Vol. 53, Issue: 7, 10 March 2002. PP. 1629 – 1640.

[11] Brauer, Jesper ,"A General Finite Element Model of Involute Gears", *Finite Elements in Analysis and Design* Vol. 40, Issue: 13-14, August, 2004. PP. 1857-1872.

[12] Yang, Shyue-Cheng, "Mathematical Model of a Helical Gear With Asymmetric Involute Teeth and its Analysis", *The International Journal of Advanced Manufacturing Technology* Vol. 26, Issue: 5, September 2005. PP. 448 – 456.

[13] Senthilvelan, S.; Gnanamoorthy, R.,"Effect of Gear Tooth Fillet Radius on the Performance of Injection Molded Nylon 6/6 Gears", *Materials and Design* Vol. 27, Issue: 8, 2006. PP. 632-639.

[14] Jian D. Wang Ian M. Howard, "Error Analysis on Finite Element Modeling of Involute Spur Gears", *Transactions of the ASME* Vol. 128, January 2006.